Optimal Offline Packet Scheduling on an Energy Harvesting Broadcast Link

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Abstract—We consider the minimization of packet transmission duration on an energy harvesting broadcast channel (BC). Energy and data arrivals are assumed to occur at arbitrary but known instants. An achievable rate region with structural properties satisfied by the two-user AWGN BC capacity region is assumed. Structural properties of power and rate allocation in an optimal policy are established, as well as the uniqueness of the optimal policy under the condition that all the data of the “weaker” user are available at the beginning. An iterative algorithm, DuOpt, based on block coordinate descent that achieves the same structural properties as the optimal is described.

I. INTRODUCTION

The basic offline problem of energy-efficient packet transmission scheduling [1], [2], [3], [4] is to assign code rates (consequently transmission durations) to a set of packets whose arrival times are known beforehand, so that they are all transmitted within a given time window with minimum total energy. The solution needs to strike a tradeoff between energy and delay based on the observation that energy per bit with many ideal and suboptimal coding schemes is convex and monotone decreasing with rate. Recently, the problem has been reformulated with a model where energy gets “harvested” or replenished at certain known instants [5]. The formulation has been extended to an AWGN BC in [6], [7], considering a static pool of data to be sent at the beginning of the schedule.

In this paper the problem in [6] and [7] is reformulated relaxing the assumption that data is ready at the beginning of the schedule. In particular, this paper considers a broadcast problem where, given an average transmit power constraint, rates are picked from an achievable rate region which obeys certain structural properties satisfied by the AWGN BC. The sender (transmitter) gets replenished with arbitrary amounts of energy as well as information bits destined to each user at arbitrary points in time. The choices of power level and the rates to individual receivers across time is called a schedule.

An optimal scheduling policy is defined to be one that transmits all the bits that have arrived within a certain time window, in the minimum possible amount of time $T^{opt}$. The policy is allowed to use as many energy harvests as it needs, provided no energy is used before it is harvested.

The main contribution of this paper is to establish structural properties of the optimal schedule, and the uniqueness of the optimal policy under certain conditions. It is shown (Section II) that in an optimal policy, transmit power used is constant within each epoch, and may only rise from one epoch to the next, so that once it starts, the transmitter never lowers its power until it finally goes silent. On the other hand, the transmitter should increase its power only under certain conditions. These conditions, along with other structural properties of power and rate are established in Section III. Next, the uniqueness of the optimal policy is established under the condition that all of the weaker user’s bits are available at the beginning. Finally, an iterative algorithm (that we refer to as DuOpt) based on the nonlinear block descent method.
which returns a feasible schedule carrying the same structural properties that the optimal is shown to have, is described. We start by giving the problem statement in the next section.

II. SYSTEM MODEL

Consider a broadcast channel with one transmitter and two receivers. Arbitrary amounts of energy, \( \{E_i < \infty, i = 1, 2, \ldots \} \), as well as data for either user \( \{B_{i1}^{(1)}, B_{i2}^{(2)} < \infty, i = 1, 2, \ldots \} \) become available to the sender at arbitrary times \( t_i \).

Throughout the paper, \( E(t) \) denotes the total energy that has been harvested in \([0, t]\) (regardless of how much of it has been used.) Similarly, \( B_1(t) \) and \( B_2(t) \) denote the total numbers of bits destined to the first and second user, respectively, available at the sender by time \( t \). The interval between any two arrival events (regardless of energy or data) will be called an inter-arrival epoch. The length of the \( i \)th epoch is \( \xi_i = t_{i+1} - t_i \).

In this offline problem, all the future arrival times and amounts of energy and bits are known by the sender at \( t = 0 \). It is also assumed that harvested energy and data are available for use instantaneously as they arrive, and code rate and transmission power decisions can be changed instantaneously. However, codeword blocklengths will be chosen such that each codeword is sent completely within a single epoch (note that starting and ending times of epochs are known ahead of time), so that no arrival event occurs during a codeword. Consequently, the power and rate pair decision will be fixed throughout each codeword. We are interested in minimizing the total transmission time for packets arriving by a certain time \( W < \infty \), so W.L.O.G., set \( B_i(t) = B_i(W) \) for \( t \geq W \), \( i = 1, 2 \). A schedule, which is a sequence of power and rate allocations, is feasible if it sends \( B_1(W) < \infty \) bits to the \( 1 \)st user and \( B_2(W) < \infty \) to the \( 2 \)nd user (with a certain level of reliability\(^1\)), without violating causality (at any time, using available energy and data by that time). We are interested in finding a schedule with the smallest completion time, \( T^{\text{opt}} \).

The structure of the achievable rate region will be based on the two-user AWGN BC. The capacity region of a two-user discrete time AWGN BC with average power constraint \( P \), noise variance \( \sigma^2 \), where the \( 1 \)st user’s channel gain \( (s_1 > 0) \) is larger than the \( 2 \)nd user’s \( (s_2 > 0) \), consists of rate pairs \((r_1, r_2)\) satisfying:

\[
    r_1 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\alpha s_1 P}{\sigma^2} \right), \quad r_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \alpha) s_2 P}{\alpha s_2 P + \sigma^2} \right) \tag{1}
\]

where \( \alpha \), \( (0 \leq \alpha \leq 1) \), denotes the fraction of \( P \) used for the \( 1 \)st user. Since \( s_1 > s_2 \), the \( 1 \)st user will be referred as the “stronger user”, and the \( 2 \)nd as the “weaker user”. From (1) each user’s rate can be expressed as a function of the other’s and \( P \), as \( r_1 = h_1(P, r_2) \), \( r_2 = h_2(P, r_1) \). The rate functions \( h_1 \) and \( h_2 \) defined on \( \mathbb{R}^+ \times \mathbb{R}^+ \) will be assumed to satisfy the following properties [6]:

1) Nonnegativity: \( h_1(P, r) \geq 0, h_2(P, r) \geq 0 \).

2) Monotonicity: \( h_1(P, r) \), \( h_2(P, r) \) are both monotone decreasing in \( r \), and monotone increasing in \( P \).

3) Concavity: \( h_1(P, r) \), \( h_2(P, r) \) are concave in \( P \) and \( r \):

\[
\frac{\partial^2 h_i(P, r)}{\partial P^2} \leq 0, \quad \frac{\partial^2 h_i(P, r)}{\partial r^2} \leq 0, \quad \text{for } i = 1, 2.
\]

4) \( \frac{\partial h_1(P, r)}{\partial P} \) \( \frac{\partial h_1(P, r)}{\partial r} \) \( \leq 0 \).

5) \( \frac{\partial^2 h_2(P, r)}{\partial P^2} = 0, \quad \frac{\partial^2 h_2(P, r)}{\partial r^2} = 0 \).

The results in the rest of the paper will hold for any coding scheme obeying (1)-(5).

It is straightforward to show that one can restrict attention to feasible schedules that do not change their power and rate allocations within epochs.

Lemma 1: In an optimal schedule, the power and rate pair remain constant within all epochs, except for the epoch during which the schedule ends.

Proof. During an epoch, there are no energy or data arrivals and the claim is identical with the one stated and proved

\(^1\)The achievable rate regions will be implicitly assumed to correspond to a certain constant tolerable error probability respecting which it is possible to transmit a finite number of bits with a finite amount of energy per bit.
Problem 1: Transmission Time Minimization of Data Arriving at Arbitrary Points on an Energy Harvesting BC:

Minimize: \( T = T\{\{(P_i, r_{i1})\}_{1 \leq i \leq k^{up}}\} \)

subject to: \( P_i > 0, 0 \leq r_{i1} \leq h_1(P_i, 0), r_{2i} = h_2(P_i, r_{i1}) \)

\[ \sum_{i=1}^{k} P_i \xi_i \leq E(t_k) \]

\[ \sum_{i=1}^{k^*} P_i \xi_i + P_i(k^*+1)(T - \sum_{i=1}^{k^*} \xi_i) \leq E(T) \quad (2) \]

\[ \sum_{i=1}^{k} r_{1i} \xi_i \leq B_1(t_k), \sum_{i=1}^{k} r_{2i} \xi_i \leq B_2(t_k) \quad (3) \]

\[ k = 1, 2, ..., k^* = \max\{i : \sum_{j=1}^{i} \xi_j \leq T\} \]

\[ \sum_{i=1}^{k^*} r_{1i} \xi_i + r_{1(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_1(T) \]

\[ \sum_{i=1}^{k^*} r_{2i} \xi_i + r_{2(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_2(T) \quad (4) \]

In problem 1 we have stated the problem in terms of power and rate allocations to epochs, more precisely, an assignment of power and user 1’s rate to each epoch (user 2’s rate is thus determined). It has been assumed that there is some \( k^{up} < \infty \) such that there is at least one feasible schedule that ends within the first \( k^{up} \) epochs. In other words, \( k^{up} \) is an upper bound for epochs to be considered. In problem statement, \( k^* \) denotes the last epoch of an optimal schedule, where \( k^* \leq k^{up} \). We will refer to (2) and (3) as energy and data causality constraints, respectively, as these ensure no energy or bit is transmitted before becoming available. The feasibility constraint (4) ensures all the data destined to each user is transmitted.

III. STRUCTURE OF AN OPTIMAL POLICY

Lemma 1 recorded that in an optimal schedule power can only change upon a data arrival or energy harvest. The next result states that when power changes, it can only increase. The key to the proof is that due to convexity, more “bits per joule” can be sent by maintaining a constant power level. An even distribution of power is possible unless this is prevented by causality constraints [8]. On the other hand, transferring energy or bits to the right, which corresponds to postponing energy or bits for future use never violates causality, so power never decreases in time.

Lemma 2: (For the proof see [9]) Consider an optimal schedule that ends during epoch \( k^* \). Power is non-decreasing with epoch number \( i \), i.e, \( P_i \leq P(i+1) \) for \( i = 1, 2, \ldots k^* - 1 \).

Lemma 3: (For the proof see [9]) In an optimal policy, power rises at \( t_i \) (start of epoch \( i \)) only if at least one of the conditions below holds:

a) Energy constraint is active at point \( t_i \).

b) Bit constraints for both users are active at point \( t_i \).

c) 2\textsuperscript{nd} user’s bit constraint is active and data arrival to 2\textsuperscript{nd} user occurs at time \( t_i \).

The next set of results illustrate the structure of rate allocation in conjunction with the power allocation in an optimal policy.

Corollary 1: (For the proof see [9]) In an optimal policy,

1) If power increases upon a data arrival for the second user, data to be sent to the 2\textsuperscript{nd} user have been finished by this event.

2) If power rises upon a data arrival for the 1\textsuperscript{st} user, all available bits have been sent by this event.

3) If power increases upon an energy harvest, all energy available at the beginning of the former constant power band has been consumed by this energy harvest.

A. Uniqueness of the Optimum Schedule

In the following Lemma we note that an optimal schedule uses all energy harvested by the time the schedule ends completely.
Lemma 4: (For the proof see [9]) The energy consumed by an optimal schedule that ends at $T_{opt}$ is equal to $E(T_{opt})$.

Next, we show the uniqueness of the optimal schedule in case the weaker user’s bits are all ready at the beginning.

**Theorem 1:** If $B_2(W) = B_2(0)$, there is a unique optimum schedule, i.e., a unique power-rate allocation achieving $T_{opt}$.

**Proof.** Suppose that there are two distinct optimal schedules, $S^A$ and $S^B$. Consider the first epoch, $s$, on which these schedules differ. Suppose the respective power allocation vectors, $P^A$ and $P^B$, differ, such that $P^A_i = P^B_i$, $\forall i \in \{1, 2, ..., s-1\}$, and $P^A_s < P^B_s$. Both schedules end at $T_{opt}$ by definition.

By Lemma 4, they use nonzero power in exactly the same sequence of epochs, $1, 2, ..., k^*$. If $P^A_i = P^B_i$ for all $i > s$, then $\sum_{i=s}^{k^*} P^A_i \xi_i < \sum_{i=s}^{k^*} P^B_i \xi_i$ which contradicts Lemma 4.

From Lemma 2, power is nondecreasing; therefore, if power cannot stay constant till the end of transmission, it can only increase, i.e., $P^A_s < P^A_{s+1}$, $u \in \{s, s+1, ..., k^*-1\}$. As there are no data arrivals for the $2^{nd}$ user, the increase in is either due to energy constraint being met or due to all the bits available by $t_u$ having been transmitted (cf. conditions (a) or (c) in Lemma 3.) As $\sum_{i=s}^{u} P^A_i < \sum_{i=s}^{u} P^B_i$, there is still feasible energy for $S^A$ at $t_u$, therefore, $S^A$ should have transmitted all the bits arrived until $t_u$. Then, $S^A$ has transmitted at least the same number of bits to both users while consuming less energy than $S^B$ between $t_0$ and $t_u$, which contradicts with the optimality of $S^B$. Therefore if there are two distinct optimal schedules, $S^A$ and $S^B$, their power allocation vectors cannot be different, i.e., $P^A = P^B$.

Now, consider two rate pair vectors, $R^A$ and $R^B$, where $(r^A_{1i}, r^A_{2i}) = (r^B_{1i}, r^B_{2i}), \forall i \in \{1, 2, ..., s-1\}$ and $(r^A_{1s} < r^B_{1s})$. If the rate of the stronger user, $r^A_{1j}$ in $S^A$ stays constant after epoch $s$, then $S^A$ would transmit fewer bits to the stronger user than $S^B$ does, which contradicts its feasibility, i.e., $r^A_{1(j+1)} = r^A_{1j} < r^B_{1j} \forall j \in \{s, s+1, ..., k-1\}$. On the other hand, if $r^A_{1j}$ increases at some point after epoch $s$, i.e., $r^A_{1u} < r^A_{1(u+1)}$, for some $u \in \{s, s+1, ..., k-1\}$, then either all the bits of stronger user have been transmitted or $r^A_{2u} = 0$ and all the harvested energy has been consumed by $t_u$, otherwise we could find a better schedule by bringing the stronger user’s rates closer to each other by Lemma 6 of [6]. However, $r^A_{2u} = h_2(P_u, r^A_{1u}) > h_2(P_u, r^B_{1u}) \geq 0$ and $B_1(t_u)^A = \sum_{i=1}^{s} u_i r^A_{1i} \xi_i < \sum_{i=1}^{s} u_i r^B_{1i} \xi_i = B_1(t_u)^B$, which contradicts the optimality of schedule $A$. Finally, $\{r^A_{1i}\}$ cannot decrease as this would also contradict optimality (see Part 1 of Theorem 2 of [6], rate of the stronger cannot be non-decreasing as there is no bit constraint on the weaker user.) Hence there cannot be two optimal schedules with different rate pair vectors. As both the power allocation vector and the rate pair vector of an optimal schedule are unique, the optimal schedule is unique.

**IV. THE DuOPT ALGORITHM**

Problem 1 is a nonlinear optimization problem that is not necessarily convex, but has a global minimum, $T_{opt}$. A suitably chosen block coordinate descent algorithm (a.k.a. Gauss-Siedel method) can converge in the solution of this problem provided that the algorithm satisfies certain conditions provided by Zangwill’s theorem [10] for global convergence. We propose an iterative block coordinate descent algorithm, DuOpt, to solve Problem 1. DuOpt starts with an initial feasible schedule of $k^{up}$ epochs and transmission completion time of $T^{up}$, then updates rates and powers of two consecutive epoch pairs; i.e., epochs $(1, 2), (2, 3), ... (k-1, k)$. After $k-1$ sequential optimizations, DuOpt strictly improves the initial schedule unless the initial schedule is optimal. Thereafter, DuOpt restarts from the first epoch pair and does another iteration. It stops after $N$ iterations such that $N = \min\{n : T^n = T^{n-1}, i = 1, ..., k^{n}, j = 1, 2\}$, where $T^n$ is the transmission completion time and $k^n \leq k^{up}$ is the number
of epochs used at the end of $n^{th}$ iteration.

The local optimizations on epoch pairs have the objective of minimizing the transmission time of bits currently transmitted in these epochs, using the same energy, by reallocating power and rate between the two epochs. This optimization respects the constraints, such that when minimizing completion time would result in a gap due to a bit constraint, it is not done (rather, the minimum time that does not cause a gap is selected, with the minimum energy to achieve that.) Consequently, if all the feasible bits transmitted during an optimization on $i^{th}$ epoch pair, $i < k^{n} - 1$, and Flag $< i$ then Flag is set to $i$ and in the next iteration the local optimizations will perform energy minimization up to epoch pair $i + 1$. The pseudocode in [9] outlines DuOpt.

**Theorem 2:** (For the proof see [9]) When DuOpt stops,

1) Rate pairs remain constant within epochs.

2) Powers assigned to epochs are monotonically non-decreasing, i.e., $P_1 \leq P_2 \leq \ldots \leq P_n$.

3) Power can only rise if at least one of the conditions stated in Lemma 3 holds.

V. NUMERICAL EXAMPLE

Consider a two-user AWGN BC with 1Khz bandwidth and $N_0 = 10^{-12}$ Watts/Hz. Path loss factors on the links of stronger and weaker user are assumed to be 70dB and 75dB, respectively. Amounts and instants of energy harvests and bit arrivals are depicted in Fig. 1. Under these circumstances DuOpt algorithm is run and final schedule is calculated as drawn in Fig. 1

VI. CONCLUSION

Structural properties about the optimal offline broadcast packet scheduling policy for an energy harvesting transmitter have been established. The uniqueness of the optimal policy under the condition that the weaker user’s bits are available at the beginning has been shown. An iterative algorithm, DuOpt, that returns a feasible schedule which possesses the same structural policies that the optimal was shown to have has been described. Proving the uniqueness of the optimal policy in general, as well as the optimality of the schedule returned by DuOpt are two goals of ongoing work.

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