Q1. Consider the system given by

\[
\dot{x} = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,
\]

\[y = [1 \ 2]x\]

a. Check if the system is controllable or not?

b. Can you find a state feedback law \(u = r - Kx\) where \(K = [k_1, k_2]\) to place the closed loop poles at \(s = -4\) and \(s = -4\)?

c. Can you find a state feedback law \(u = r - Kx\) where \(K = [k_1, k_2]\) to place the closed loop poles at \(s = -4\) and \(s = -6\)?

Q2. Consider the system given by

\[
\dot{x} = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,
\]

\[y = [0 \ 1 \ 0]x\]

a. Find the transfer function of the system.

b. Check if the system is completely observable or not.

c. Design an observer for the system by placing the observer error poles at \(s = -1, -1 \pm j\).

Q3. Consider the system given by

\[
\dot{x} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,
\]

\[y = [0 \ -20 \ -40]x\]

a. Check if the system is completely controllable or not.

b. Check if the system is completely observable or not.

c. Remembering that state feedback does not change zeros of the open loop system, determine a feedback law \(u = r - Kx\) so that the resulting system is reduced to a second order system with a settling time of \(t_s = 1\) second (2%) and damped natural frequency of \(\omega_d = 2\) rad/sec.

Hint: You might consider canceling one of the zeros of the system to reduce the order to 2.

d. Check if the closed loop system obtained in part-c is observable or not.