Optimization of Feedback in a MISO Downlink with Energy Harvesting Users

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Abstract—We study the optimization of the number of bits allocated by energy harvesting users for sending feedback to a common multiple-antenna access point (AP). The nodes need to distribute their feedback transmissions judiciously across time (and channel states) in order to maximize certain throughput goals. While the MISO channel capacity from the AP to a user is a strictly increasing function of the number of feedback bits sent by the user to the AP for providing channel state information, the energy consumption for sending this feedback is (assumed to be) directly proportional to the number of feedback bits. Considering long term throughput, the nodes need to adapt the number of bits of feedback to their energy harvesting profiles.

I. INTRODUCTION

For large scale distributed device networks such as sensor networks and M2M networks, energy harvesting (EH) is a very promising technology, especially when the networked devices use low transmit power and send at very low data rates. Consider, for example, the following scenario: in an M2M network, nodes need to be kept updated with information of other nodes in a central AP continuously. Control signals require a significant amount of data transmission in the downlink. As the AP is connected to the power grid, it has no major energy constraint. However the sensor nodes, designed for energy-neutral operation (spending about as much energy as they harvest from the environment) have to schedule their operations based on the state of their batteries.

One of the main reasons why Energy Harvesting introduces a challenge for communication networks is that operations at different networking layers, i.e. coding, power control, scheduling, etc. need to be adapted to the rate of energy harvesting, which may be sporadic and hard to predict [1] [2] [3]. In particular, in a multiple antenna communication link where channel state information is crucial for approaching channel capacity, the quantized link state information sent by users to a base station may have to be adapted to the energy budget of the users. Note that in the Multi Input Single Output (MISO) channel (with \( m \) antennas at the transmitter and 1 at the receiver), with full channel state information at transmitter (CSIT), maximum transmission rate can be achieved by beamforming along the \( m \times 1 \) channel gain vector \( h \) (whose magnitude is \( ||h|| \)) and the ergodic capacity of the channel for a given transmit power \( P \) is [4]

\[
C_{CSIT}(P) = E_h \left[ \log \left( 1 + P ||h||^2 \right) \right] \tag{1}
\]

When the transmitter has no CSIT, and only knowledge of channel statistics, the optimum transmission policy uses the same power at each transmitting antenna. The ergodic channel capacity for this case is as below.

\[
C_{NO-CSIT}(P) = E_h \left[ \log \left( 1 + \frac{P}{m} ||h||^2 \right) \right] \tag{2}
\]

an unlimited number of bits, the problem of optimal quantization of channel state vector for feedback has been the topic of many papers [5] [6][7].

When the transmitter has partial CSIT (gained through feedback through \( b \) bits generated by random vector quantization), and the number of transmitting antennas is large, an approximation\(^1\) of the ergodic rate is given by [8].

\[
R_{FB}(P) = E_h \left[ \log \left( 1 + P ||h||^2 \left( 1 - 2 \left( \frac{b}{mb+1} \right) \right) \right) \right] \tag{3}
\]

According to the above formula by sending more bits of feedback the channel throughput will be improved. However, in [9], the authors show that feedback transmission needs to be designed judiciously since it reduces the allocated transmission time in each frame and also it consumes energy at user. [10] is the closest work to ours, the authors consider a point to point MISO channel with an energy harvesting receiver, who needs to send feedback about its channel state to the multiple antenna transmitter. They formulate the problem of maximizing throughput achieved in a finite amount of time under energy (harvesting) constraints at the receiver side. They show that the optimization problem is concave and devise an algorithm to reach the optimal throughput.

In this work we consider the optimization of the amount of feedback in a multiuser MISO system with en-

\(^1\)Numerical results show that the approximation is tight if \( b \) and/or \( m \) be large enough.
ergy harvesting users to maximize the expected throughput of the system. Hereafter, our optimization on the number of bits allocated for feedback, \( b \), will be based on the approximated rate given in eq. 3. The aim of this optimization is optimal usage of energy harvesting users with low energy harvesting rate such as piezoelectric, electromagnetic or indoor solar cell etc[11].

Here is an outline of the rest of this paper: In section II the problem statement is given. Next, in section III, the case of fixed number of bits in feedback is studied, with the goal of optimizing the times at which a node sends feedback to the access point to achieve maximum expected throughput. In section III an algorithm for doing this is proposed and its optimality is shown. Later, in section IV, the case of variable size feedback is studied. A policy for reaching maximum expected throughput is derived, and the performance of the proposed policy is evaluated. In section V we conclude the paper and highlight some directions for further work.

II. SYSTEM MODEL

We consider a multiuser MISO system that consists of an access point with \( m \) antennas and \( M \) single-antenna users. In each time frame, a portion of time is allocated to each user to send its quantized channel state information to the AP. Users decide whether or not to send feedback in their allocated time with respect to their energy level and channel gain. The remaining portion of the frame is allocated to one of the users for receiving data from the AP. We assume a simple rate maximizing time division strategy where the AP selects a user with the largest channel gain magnitude, among those that sent channel state feedback (see Fig. 1).

The users rely on energy harvested from the environment (Fig. 2), whereas the AP is assumed to be connected to a reliable power source supplying average power \( P \). We assume the channels’ gain take values from a discrete set of \( N \) possible states, IID in each timeslot. The objective of the users is to optimize their own long term throughput. As noted in Remark 1, if nodes act to attain this objective, they also maximize the expected long term throughput of the whole system.

Remark 1: As energy arrivals and channel states of nodes are independently and identically distributed, if nodes apply a policy that maximizes their expected throughput, they maximize the throughput of the system.

\begin{align*}
\text{Proof follows from the linearity of expectation.}
\end{align*}

III. CASE 1: FIXED NUMBER OF BITS OF FEEDBACK

We begin by studying the case that feedbacks contains a fixed number of bits (\( b \) bits). That is, the user has two choices: whether to quantize its current channel state to one of \( 2^b \) levels or decide not to send feedback in the current time frame. In this part in a finite time horizon we address an offline scheduling. We assume all future energy arrivals and future channel conditions are known in advance at starting time by each node. We propose a policy to maximize the expected throughput of nodes. cf. Remark 1, maximizing the expected throughput of nodes separately leads to maximizing the expected throughput of the system. Some symbols which are used in the following expression for the expected throughput of a node, are defined in below:

\( \omega_j \in 0,1 \) indicates whether a user sends feedback in time frame \( j \) or not.

\( R_{S_i} \) is the reward a node will get if it be selected by the access point when channel is in state \( S_i \) (refers to the channel capacity when the channel state is \( S_i \))

\( P_{C}(S_i) \) is the probability that a channel in state \( S_i \) be chosen by the access point.

\( E_j \) is the amount of harvested energy by a node at the beginning frame \( j \).

\( E_b \) is the amount of energy a node consumes for sending one bit of feedback.

The expected throughput of a node over a problem horizon of \( T \) frames is:

\begin{align*}
E[R_{tot}] &= \max_{\omega_i \in 0,1} \sum_{i=1}^{T} \omega_i R_{S_i} P_{C}(S_i) \\
\text{s. t.} & \\
\sum_{j=1}^{i} \omega_j &\leq \sum_{j=1}^{i} E_j \quad i = 1, 2, ..., T
\end{align*}

Definition: Candidate vector of a node contains channel capacity of the node for timeslots it sends feedback.
(i.e. \( i \) s.t. \( \omega_i = 1 \)). It is sorted by decreasing order of channel capacities.

**Definition**: For \( A, B \in \mathbb{R}^n \), \( A \geq B \) if \( A(i) \geq B(i) \) for \( i = 1, \ldots, n \)

**Remark 2**: If candidate vector of a policy is greater than candidate vector of other policies, the policy achieves maximum expected throughput.

**Proof**: Assume candidate vector \( A \) is greater than candidate vector \( B \). As \( A(i) \geq B(i) \) so clearly \( P_C(A(i)) \geq P_C(B(i)) \), and \( \sum_{i=1}^{n} A(i)P_C(A(i)) \geq \sum_{i=1}^{n} B(i)P_C(B(i)) \), which means that the expected throughput of candidate vector \( A \) is greater than candidate vector \( B \).

Let \( S_t \) defines the channel state of a node at timeslot \( t \). One energy packet is defined as amount of energy which is sufficient for transmission of one feedback packet, and assume that the studied node will harvest \( E_{\text{tot}} \) energy packet until the end of the whole transmissions duration. Hence a node should send feedback in \( E_{\text{tot}} \) of the frames, which motivates the following algorithm for the solution of the offline problem.

### A. The Offline Solution

This is an offline algorithm which assumes that energy arrivals and channel states are known ahead of time. For a given node, first the node chooses the \( E_{\text{tot}} \) best frames in terms of channel state, and sets them as the candidate instants for sending feedback. Next, it checks the first time that energy causality is disturbed and name it \( T_1 \) (we name these instants as decision epochs). Assume that until time \( T_1 \) the node will harvest \( E_1 \) energy packets, but the algorithm suggests it to consume \( E'_1 \) energy packets which is greater than \( E_1 \). The node should choose instants the channel is in one of its \( E_1 \) best instants in the interval of starting time to \( T_1 \). The node repeats this procedure for time after \( T_1 \). It can consume \( E_{\text{tot}} - E_1 \) energy packets for its future feedback transmission. Therefore it selects the instants the channel is in one of its \( E_{\text{tot}} - E_1 \) best instants for time after \( T_1 \) and chooses them as candidate instants. Again it sees the first time after \( T_1 \) that energy causality is disturbed (name it \( T_2 \)), assume that the node in the interval of \( T_1 \) to \( T_2 \) harvest \( E_2 \) energy packets but the algorithm suggests to consumes \( E'_2 \) packets of energy which is greater than \( E_2 \). Again the node chooses the times the channel is in one of its \( E_2 \) best instants in the interval of \( T_2 \) to \( T_2 \). Now the node makes decision on its feedback scheduling for time after \( T_2 \) and repeat the same procedure.

In the rest, it is shown that the candidate vector hosen by the proposed policy dominate the candidate vectors of other policies. Therefore according to Remarks 1 and 2, the policy maximizes throughput.

### B. Proof of correctness of the algorithm

Let \( A \) be a candidate vector achieving the highest expected throughput and \( B \) be the candidate vector determined by the proposed algorithm. Let timeslot \( x \) be the first time which candidate vector \( A \) decides to send feedback but candidate vector \( B \) does not. Let assume \( T_y \) be the closest future decision time to timeslot \( x \). The proposed algorithm transmits \( \sum_{j=1}^{y} E_j \) bits of feedback up to time \( T_y \). Hence, there should be at least one timeslot before \( T_y \), where \( B \) decides to send but \( A \) does not (name it timeslot \( y \)). But the proposed algorithm chooses timeslots with the highest channel capacity so all the chosen timeslots must observe higher channel capacity than timeslot \( x \). This means that a timeslot which has a higher channel capacity than channel capacity in time slot \( x \) exists, and is not selected in \( A \). Therefore \( A \) could be improved by incorporating this time slot. This improvement continues until \( A \) choses time slots with the same capacity values as \( B \).

### C. Simulation results

In this part the performance of the proposed algorithm is compared with a greedy algorithm and the case the users have access to an unlimited amount of energy (Fig. 3). In the greedy algorithm, users use up all of the harvested energy in the next time slot after harvesting. The channels gain is modeled by rayleigh fading.

In the simulation it is assumed the system contains 5 users and the access point will choose one of them for transmission. The results is gathered for over \( 10^6 \) timeslots for each case and energy harvesting distribution is modeled by an exponential distribution with different mean values.
IV. VARIABLE FEEDBACK LENGTH

In this part of the study we consider a more general case where number of feedback bits is variable. So users will decide on time and number of feedback bits. For simplicity we assume the system contains only two users and one of the users will be selected by the access point in each timeslot. The results can be generalized to more users as well.

Let us assume users are initialized with finite amounts of energies, and they do not harvest during the problem horizon. The amount of initial energy of nodes are equal and this is known to them. Also it is assumed the number of timeslots is much greater than the number of possible channel states. We assume channel state processes are ergodic and the problem horizon is sufficiently long such that the frequency of occurrence of each channel state for any user converges to the probability of occurrence of this state, within the problem horizon. Expected reward of each node is:

\[
\max_{b_i} E(R_{tot}) = \max_{b_i} \sum_{i=1}^{N} R(h_i, b_i) P_C(R(h_i, b_i)) P(h_i)
\]

subject to:

\[
TE_b \sum_{i=1}^{N} b_i P(h_i) = B \times E_b
\]

Where \(R(h_i, b_i)\) is the approximation of transmission rate of channel when the channel gain is \(h_i\) and user sends \(b_i\) bits of feedback.

\(N\) is the number of possible magnitude of \(h\).

\(T\) is the number of timeslots.

\(B\) refers to number of bits a user can send according to its initial energy.

\(P_C(R(h_i, b_i))\) is the probability that user with transmission rate \(R(h_i, b_i)\) be chosen by the access point.

\(P(h_i)\) is the probability the channel state be in state \(h_i\).

\(E_b\) is the amount of energy for transmitting 1 bit.

Remark 3: In the optimal policy for maximizing the expected throughput of a user to achieve maximum expected throughput, \(R(h_i, b_i)\) is an increasing function with respect to \(||h_i||\).

The proof is in sec VI-A

According to remark 3 \(R(h_i, b_i)\) is an increasing function with respect to \(||h||\). Therefore \(P(R(h_i, b_i) > R(h_j, b_j)) = P(||h_i|| > ||h_j||)\). Therefore the expected throughput can be written as below:

\[
\max_{b_i} E(R_{tot}) = \max_{b_i} \sum_{i=1}^{N} R(h_i, b_i) F(||h_i||) P(h_i)
\]

subject to:

\[
TE_b \sum_{i=1}^{N} b_i P(h_i) = B \times E_b
\]

Where \(F(||h_i||) = P(||h|| < ||h_i||) + \frac{1}{2}P(||h|| = ||h_i||)\). The derived statement is concave function of \(b_i\) and constraint is linear. Therefore, the optimal bit allocation can be found through straightforward algebra.

A. Simulation results

In this part the performance with variable sized feedback is evaluated.

In Fig. 4 the bit allocation of feedbacks for average energy consumption 1, 7, and 15 packets of energy in each timeslot is plotted. There are two users and \(||h|| \in \{1, 2, ..., 10\}\) and the channel absolute gain has an exponential distribution with mean of 3 (values greater than 10 are mapped to 10). AP has 10 transmitting antennas.

Next, the performance of fixed feedback size and variable feedback size are compared. The expected throughput of each user in the system with 2, 3, and 4 users for two cases of fixed feedback size and variable feedback size are plotted in Fig. 5. AP has 10 transmitting antennas and \(SNR = 0dB\). The simulation was performed over \(10^7\) timeslots and the absolute channel gain is assumed to be the truncated version of an exponential with mean 3, where values greater than 10 are mapped to 10. Performance is studied with respect to average energy consumption. It appears that when the available energy for feedback is low, allowing variable size feedback fares significantly better than fixing the number bits of feedback.

V. CONCLUSIONS

We considered a system with a common multi-antenna access point (or center) and energy harvesting users. With a throughput objective, we study the optimization of the number of bits allocated by energy harvesting users for sending feedback to the common transmitter. The nodes need to distribute their feedback transmissions judiciously across time. In the first part
of this study it is assumed the feedback’s number of bits is fixed and in the second part we consider the case which the users decide on the number of bits to allocate for feedback, as well. Among other things, it is observed that when the available energy for feedback is limited, allowing variable sized feedback (as opposed to a fixed number of feedback bits) improves performance significantly.

VI. APPENDIX

A. Proof of Remark 3

Let us assume that by policy $A$ users achieve the maximum expected throughput. Assume channel states are sorted by their channel capacity with regard to bits number allocated for them according policy $A$ like below: $C(h_1, b_1) < C(h_2, b_2) < \ldots < C(h_i, b_i) < C(h_j, b_j) < \ldots$. Assume that channel state $i$ is the first channel state which has a larger absolute value channel gain but has less channel capacity than its next state in the sorted list. $C(h_i, b_i) < C(h_j, b_j)$. $\|h_i\| > \|h_j\|$. We define $C(h_i, b'_i)$ and $C(h_j, b'_j)$ in a way that $C(h_i, b'_i) = C(h_j, b'_j)$ and $C(h_i, b'_i) = C(h_i, b_i)$. Here we want to show that with bit allocation $b'_i$ and $b'_j$ for states $h_i$ and $h_j$ respectively, the expected throughput remain same but $b'_i + b'_j < b_i + b_j$. So by using those free bits the expected throughput will be increased and this is in contrast with our first assumption.

\[
C(h_i, b'_i) = C(h_j, b'_j)
\]

\[
\Rightarrow \log_2 \left(1 + P \||h_i||^2 \left(1 - \frac{b'_i}{2^{\|h_i\|^2}}\right)\right)
\]

\[
= \log_2 \left(1 + P \||h_j||^2 \left(1 - \frac{b'_j}{2^{\|h_j\|^2}}\right)\right)
\]

\[
\Rightarrow \|h_i\|^2 \left(1 - \frac{b'_i}{2^{\|h_i\|^2}}\right) = \|h_j\|^2 \left(1 - \frac{b'_j}{2^{\|h_j\|^2}}\right)
\]

\[
\Rightarrow 1 - \frac{b'_i}{2^{\|h_i\|^2}} = \frac{\|h_j\|^2 \left(1 - \frac{b'_j}{2^{\|h_j\|^2}}\right)}{\|h_i\|^2 \left(1 - \frac{b'_i}{2^{\|h_i\|^2}}\right)}
\]

Similarly

\[
1 - \frac{b'_j}{2^{\|h_j\|^2}} = \frac{\|h_i\|^2 \left(1 - \frac{b'_i}{2^{\|h_i\|^2}}\right)}{\|h_j\|^2 \left(1 - \frac{b'_j}{2^{\|h_j\|^2}}\right)}
\]

$b_i < b'_i, b'_j < b_j$ Let’s define $f(x) = 1 - 2^{\frac{-x}{2^{\|h_i\|^2}}}$. $f(x)$ is a concave function. In below equations $L = \frac{\|h_i\|^2}{\|h_j\|^2}$. So

\[
f(b'_i) = \frac{1}{L} f(b_i)
\]

\[
f(b'_j) = Lf(h_i)
\]

$f(x)$ is a monotonically increasing concave function so $b'_j - b_i < b_j - b'_i$ which means $b'_i + b'_j < b_i + b_j$.

If $P(h_{i_1}) > P(h_{i_2})$, in first we separate channel state $h_i$ into two groups $h_{i_1}$ and $h_{i_2}$ where $h_{i_1} = h_{i_2} = h_i$ and $P(h_{i_1}) = P(h_{i_2})$ and $P(h_{i_1}) + P(h_{i_2}) = P(h_i)$.

ACKNOWLEDGEMENT

This research was supported by The Scientific and Technological Research Council of Turkey, via grant 112E175 which provides the local funding for the Chist-EERA E-CROPS project.

REFERENCES


