Beamspace Approach for Detection of the Number of Coherent Sources

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***Abstract*—A beamspace approach is presented to detect the multiplicity of impinging sources on the sensor array for the direction of arrival estimation problem. The method applies to the coherent signals and can also be used to detect the presence of interfering multipath signal. In many applications, the signal to noise ratio is not sufficient to resolve multiple sources with close spatial frequencies. The suggested method is to detect the presence of an interfering secondary source which can not be possibly resolved.**

***Index Terms*—Beamspace, Direction of Arrival, Low Altitude Radar Tracking, Low Angle Radar Tracking, Detection of the number of sources.**

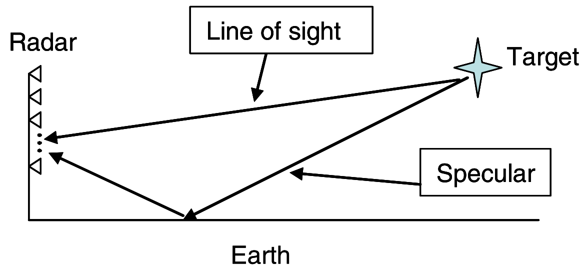


Fig. 1. Geometry for the angle of arrival estimation for low altitude targets

I. Introduction

Direction of arrival estimation in the presence of multipath is a challenging radar signal processing problem. Figure 1 shows a typical geometry for which the problem arises. Here a radar system is to detect and estimate the elevation angle of the target. We assume that target echo arrives at the radar site with a sufficient signal-to-noise ratio (SNR) so that detection can be reliably established. In spite of large SNR, the direction of arrival estimate may not be accurate, i.e. severely biased, due to the interference by the multipath signal. The problem is especially critical for low flying targets at large distances.

Following the seminal work of Barton in 1974, [1]; various methods to combat the effect of multipath interference have been proposed. Readers can examine [2] and [3] for a summary of the recent works on this problem. It should be noted that for low altitude targets at large distances, the multipath echo is not separable in time or in space, that is both the main and specular component impinge on the array lined in the elevation direction at the same time and have an angular separation that is in the order of a fraction of beamwidth, [3]. For the illustrated flat earth geometry, the angle difference between the main and specular path is approximately $2h_t/R$, where h_t and R are the altitude and the range of the target respectively. An additional difficulty is the coherency of the line-of-sight and the multipath components.

In this paper our goal is not to present an estimation technique, but to describe a method for the detection of a second (possibly the multipath) component. The goal is to issue a warning to the user when the angle of arrival estimates are produced under an interfering multipath signal. The presented method is closely connected with the angle of arrival estimation method proposed by Zoltowski et al. in [3]. In [3], a beamspace approach is presented to estimate the frequency of complex exponentials whose separation is smaller than the Rayleigh resolution. The performance of this method degrades when the angle of arrivals are too close. Our goal is to detect such instances in which the angle of arrival estimates are not accurate, i.e. heavily biased.

More specifically, Zoltowski *et al.* have shown that the accuracy of the frequency estimation through beamspace maximum likelihood (ML) method depends on the angular separation of signals and SNR. We believe that the detection of number of sources is important especially when the sources can not be resolved. In target tracking applications, there may be instances in which the received snapshot is free of multipath interference and a proper use of such instances can improve the tracking performance of the system.

In the literature, several methods on the number of signals detection have been proposed. Some methods are based on the information theoretical principles such as the minimum description length, Akaike's information criterion etc. and others rely on parametric models. A review of these approaches can be found in [4]. It should be noted that the application of these techniques becomes non-trivial when the input is a single snapshot vector possibly arising from coherent sources.

In this paper, we present a simple method for the detection of the number of sources specific for the direction of arrival problem. The presented method can be easily coupled with the beamspace ML estimation approach of Zoltowski *et al.*

II. Proposed Method

Without any loss of generality, we assume the following signal model:

$$r[n] = \alpha_1 s_{\theta_1}[n] + \alpha_2 s_{\theta_2}[n] + w[n], \quad n = \{0, \ldots, N-1\}$$

Here $r[n]$ shows the elements of the snapshot vector, $w[n]$ is white Gaussian noise with variance σ_w^2 and $\{\theta_1, \theta_2\}$ are the parameters to be estimated. The parameters α_1 and α_2 are complex scalars representing the signal amplitude. We assume that the complex signal amplitudes are non-random and possibly coherent, that is the phase difference between α_1 and α_2 is constant for all snapshots.

In the following we consider a model with two signals, one of which is due to line-of-sight path and the other due to the specular path. However, the method remains applicable even if there are more than two signals. The signal $s_{\theta_k}[n]$ is expressed as follows:

$$s_{\theta_k}[n] = \exp(j\theta_k n), \quad k = \{1, 2\} \tag{1}$$

The signal model corresponds to a system with uniform linear array in which $\theta_k = \frac{2\pi}{\lambda} d \sin(\phi_k)$ where ϕ_k is the angle of arrival measured from the array boresight, λ is the wavelength and d is the inter-element spacing of the array. It should be noted that the model is also valid for the Doppler frequency estimation or in general to the parameter estimation of uniformly sampled complex exponential signals.

Manifold Alignment: The presented method depends on a simple idea which we call the manifold alignment. The central idea is based on the fact that both the real and imaginary parts of $s_{\theta_k}[n]$ contain the same information about the signals which can be used for the detection of the number of sources.

The $N \times 1$ column vector $\mathbf{s}_\theta$, with the entries $s_\theta[n]$ for $n = \{0, \ldots, N-1\}$, is the array manifold vector. We say that a manifold is aligned, if there is a unitary transformation $\mathbf{U}$ for which every entry of $\mathbf{U}\mathbf{s}_\theta$ has the same phase angle (which may depend on the parameter to be estimated).

It should be noted that the matrix $\mathbf{U}$ is unitary and therefore invertible. Hence the alignment by the application of $\mathbf{U}$ does not impose an additional difficulty to the estimator. Furthermore, the unitarity of $\mathbf{U}$ implies that the white noise corrupting the snapshot vector remains white after its application:

$$\widehat{\mathbf{r}} = \mathbf{U}\mathbf{r} = \widehat{\alpha}_1 \widehat{\mathbf{s}}_{\theta_1} + \widehat{\alpha}_2 \widehat{\mathbf{s}}_{\theta_2} + \widehat{\mathbf{w}} \tag{2}$$

Here the vectors with hat show the resultant vector after the application of $\mathbf{U}$. Without any loss of generality, we assume that $\widehat{\mathbf{s}}_{\theta_1}$ and $\widehat{\mathbf{s}}_{\theta_2}$ are real valued vectors (aligned manifold); if the manifold vectors are not real, their common phase angle can be absorbed into α_1 and α_2 resulting in $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$.

For the sake of clarity, the proposed method is explained for the noiseless case first and then the effect of noise is examined. With the assumption of noise-free $\widehat{\mathbf{r}}$, we note that if there is only one signal contributing to $\widehat{\mathbf{r}}$ (say $\alpha_2 = 0$, i.e. no multipath), then the real and imaginary parts of the snapshot vector $\widehat{\mathbf{r}}$ are parallel, after the alignment procedure:

$$\text{Noise Free, Single Signal:} \tag{3}$$
$$\begin{aligned} \text{Re}\{\widehat{\mathbf{r}}\} &= \text{Re}\{\widehat{\alpha}_1\}\widehat{\mathbf{s}}_{\theta_1} \\ \text{Im}\{\widehat{\mathbf{r}}\} &= \text{Im}\{\widehat{\alpha}_1\}\widehat{\mathbf{s}}_{\theta_1} \end{aligned}$$

If there are two signal components, the same equation is:

$$\text{Noise Free, Two Signals:} \tag{4}$$
$$\begin{aligned} \text{Re}\{\widehat{\mathbf{r}}\} &= \text{Re}\{\widehat{\alpha}_1\}\widehat{\mathbf{s}}_{\theta_1} + \text{Re}\{\widehat{\alpha}_2\}\widehat{\mathbf{s}}_{\theta_2} \\ \text{Im}\{\widehat{\mathbf{r}}\} &= \text{Im}\{\widehat{\alpha}_1\}\widehat{\mathbf{s}}_{\theta_1} + \text{Im}\{\widehat{\alpha}_2\}\widehat{\mathbf{s}}_{\theta_2} \end{aligned}$$

It can be noted that unless $\alpha_1 = \pm\alpha_2$, the real and imaginary parts of the received signal after alignment are not parallel in the two signal case. Our goal is to examine the angle between $\text{Re}\{\widehat{\mathbf{r}}\}$ and $\text{Im}\{\widehat{\mathbf{r}}\}$ for the detection of the number of sources.

For the noisy case, we introduce a threshold for the angle between the real and imaginary parts of the snapshot vector. A two-signal decision is made whenever the angle of the snapshot exceeds the threshold. It should be noted that this procedure has no assumptions on the coherence, separation of signals, i.e. $\theta_2 - \theta_1$ and it is valid for all manifolds that can be aligned.

For the selection of the threshold level, we define the event of two-signal declaration given that there is a single signal as false multiple signal decision and denote its probability as P_{FMSD}. The threshold is adjusted to meet the desired P_{FMSD}.

In the subsequent parts of the paper, we present the probability density function (pdf) for the calculation of the threshold. Unfortunately, an analytical expression for the pdf is not available to us. Instead, we present an approximation to the density function which is accurate at high SNR conditions. We believe that the approximation is sufficiently accurate for radar signal processing applications.

Manifold Alignment for Complex Exponential Signals: The manifold alignment is the critical step for the proposed method. For uniformly sampled complex exponential signals, the manifold alignment is accomplished by the discrete Fourier transformation (DFT) followed by a diagonal matrix multiplication.

The DFT of $s_\theta[n]$ given in (1) can be written as follows:

$$\begin{aligned} \text{DFT}\{s_\theta[n]\}(k) &= \left.\sum_{n=0}^{N-1} e^{j(\theta - \frac{2\pi k}{N})n}\right|_{\theta = 2\pi k_\theta / N} \\ &= \frac{1 - e^{j2\pi(k_\theta - k)}}{1 - e^{j\frac{2\pi}{N}(k_\theta - k)}} \\ &= e^{j\pi(k_\theta - k)(1 - 1/N)} \frac{\sin(\pi(k_\theta - k))}{\sin(\frac{\pi(k_\theta - k)}{N})} \end{aligned} \tag{5}$$

Here we introduce the variable $k_\theta = N\theta/2\pi$ to represent the frequency of $s_\theta[n]$ in the units of DFT bins. For example, k_θ of 3.25 refers to a signal located at 0.25 DFT bin offset from the 3rd DFT bin. The frequency in radians per sample is $\theta = 3.25 \times 2\pi/N$ where N is length of the snapshot vector. When the k'th DFT bin output, i.e. $\text{DFT}\{s_\theta[n]\}(k)$, is multiplied with $\exp(-j\pi k/N)$, the alignment is completed:

$$\text{DFT}\{s_\theta[n]\}(k)e^{-j\pi k/N} = (-1)^k \underbrace{e^{j\pi k_\theta \frac{N-1}{N}}}_{c} \frac{\sin(\pi(k_\theta - k))}{\sin(\frac{\pi(k_\theta - k)}{N})}$$

In the equation above, the only complex valued quantity is the constant c. It should be noted that the constant c does not depend on the bin index k. Hence it appears as a global multiplier for all entries of the aligned manifold vector.

We summarize the alignment procedure: The snapshot vector $\mathbf{r}$ of dimensions $N \times 1$ is transformed to the DFT domain. Then the DFT output is multiplied by a diagonal matrix whose diagonal entries are $\exp(-j\pi k/N)$ for $k = \{0, \ldots, N-1\}$. The cascade of DFT matrix and the diagonal matrix constitutes the unitary matrix $\mathbf{U}$ required for the manifold alignment.

Estimation in Beamspace using Three Beams: Following the methodology of Zoltowski *et al.* [3], we use only 3 beams to estimate the number and the parameters of sources. The method also applies to higher dimensional beamspace problems; but the derivation of the probability density function required for the threshold calculation becomes even more involved. Figure 2 shows the motivation of this approach. Here the magnitude of the discrete time Fourier transform (DTFT) output (continuous curve) and DFT samples are illustrated.

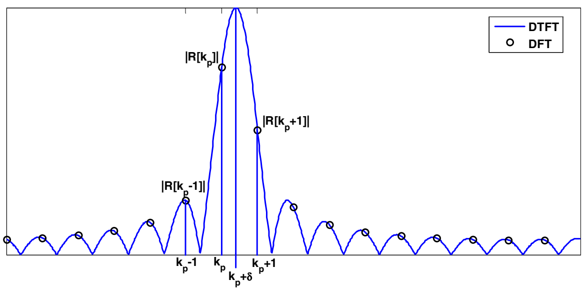


Fig. 2. Processing with three beams

In Figure 2, the signal has the frequency of $k_\theta = k_p + \delta$ where k_p is the integer closest to k_θ and δ is its fractional part, $|\delta| \leq 1/2$.

As can be seen from Figure 2, the DFT output is concentrated around a few bins surrounding the true frequency of the signal. A similar concentration of energy is always expected when the operational SNR of the system is sufficiently high. It should be noted that for radar signal processing problems, the parameter estimation stage is preceded with the detection stage. Therefore, high SNR conditions, i.e. the output SNR $>$ 20 dB, is a typical mode of operation for direction of arrival estimation in radar applications.

Zoltowski *et al.* makes use of only three beams surrounding the peak in DFT spectrum to estimate the complex exponential parameters. The usage of three beams, instead of N beams, can be interpreted as the restriction of the parameter estimation problem to a 3 dimensional subspace. It can be shown that there is little loss in estimation accuracy if it is possible to find a subspace in which most of the signal energy resides. Furthermore, the subspace operation can enable efficient methods for the computation of the ML estimates, as described in [3]. In this paper, we continue to use three DFT bins around the true frequency and denote the indices of these bins with $\{k_{p-1}, k_p, k_{p+1}\}$, as shown in Figure 2.

It is important to estimate the fractional part of k_θ accurately in the presence of multipath, that is when the frequency separation between two signals is less than 1 DFT bin. In the literature, this problem is known as the resolution problem, [5], [6]. It is known that to accurately resolve both signals within the same beam, the SNR should be sufficiently high. As expected, the required SNR for resolution increases as the frequency separation decreases.

In this paper, our goal is to construct a mechanism that can be used when the SNR is not sufficient to resolve both complex exponentials. The mechanism is to detect the presence of multiple complex exponentials and issue a warning stating that, unless the operational SNR is exceptionally high, the estimation results are not reliable.

Threshold Calculation: The proposed method is based on thresholding the angle between the real and imaginary parts of the aligned snapshot vector. For the single signal case, the angle is expected to be small; while for the two signal case, it is expected to be large. Here we suggest to set a threshold for the angle such that the threshold is exceeded with the probability of P_{FMSD} when there is a single signal.

The operation with three beams reduces the snapshot vector to a 3 dimensional vector. Here we denote this vector by $\mathbf{v}$ and its real and imaginary parts are represented by $\mathbf{v_r}$ and $\mathbf{v_i}$ respectively. Under single signal hypothesis (H_1), we have:

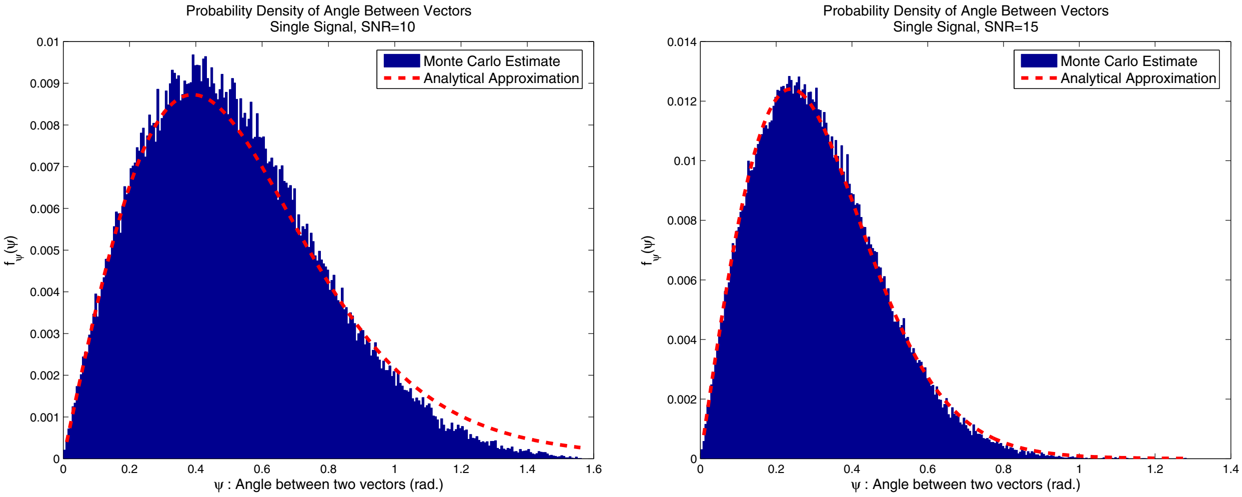
$$\begin{aligned} \mathbf{v_r} &= \text{Re}\{\mathbf{v}\} = \mathbf{s}\cos(\angle\widehat{\alpha}_1) + \mathbf{n_r} \\ \mathbf{v_i} &= \text{Im}\{\mathbf{v}\} = \mathbf{s}\sin(\angle\widehat{\alpha}_1) + \mathbf{n_i} \end{aligned} \quad (6)$$

In (6), $\mathbf{s}$ is a real valued vector (or equivalently a vector with aligned manifold) which corresponds to $|\widehat{\alpha}|\widehat{\mathbf{s}}_{\theta_1}$. The cosine and sine terms appearing in (6) are due to real and imaginary part calculation in (3). Due to unitary transformation, the elements of the noise vectors are Gaussian distributed with zero mean, σ_n^2 variance and independent and identically distributed.

For threshold selection, the probability density function (pdf) of the angle between $\mathbf{v_r}$ and $\mathbf{v_i}$ should be calculated. An analytical expression for this density is not known to the authors of this paper. The most related work in this line of research is the two dimensional version of this problem presented by Pawula *et al.* [7]. This work is highly original and based on a non-standard approach only valid for the two dimensional problem. As noted in a more recent paper by Pawula, there is almost no significant contribution in this line of research since his original work in 1982, [8].

In this paper, we present a density function for a related problem and then adapt the original problem to the related problem whose density is available. The adaptation is valid for the high SNR conditions.

The related problem is the calculation of the angle between a fixed and known vector and its noisy version. In other words, the pdf of the angle between a known vector $\mathbf{s}$ with dimensions 3×1 and its noisy version $\mathbf{s_1} = \mathbf{s}+\mathbf{n_1}$, where the noise is white and Gaussian distributed, is of interest. The pdf for the angle



(a) SNR = 10 dB

(b) SNR = 15 dB

Fig. 3. Accuracy of analytical approximation for SNR = {10, 15} dB

can be derived by expressing the joint distribution of $\mathbf{s_1}$ in the spherical coordinates and then integrating out all variables except the azimuth variable, shown with ψ in this paper. Once this is done, the density function shown in (7) (on the top of the following page) is retrieved. In (7), ρ denotes the SNR which is defined as $\rho = ||\mathbf{v}||^2/\sigma_n^2$.

To adapt the original problem to the related problem, the cosine and sine multipliers appearing in (6) should be eliminated. The cosine and sine terms scale the signal component of the snapshot vector. Hence the SNR values for the real and imaginary part vector are different from each other. To equate the SNR values of the real and imaginary parts, a unit norm scalar is introduced:

$$\beta = \exp\left(j\frac{1}{2}\arctan\left(\frac{||\mathbf{v_r}||^2 - ||\mathbf{v_i}||^2}{2\mathbf{v_r^T}\mathbf{v_i}}\right)\right) \tag{8}$$

It can be shown that β is the unique unit magnitude scalar which minimizes $||\text{Re}\{\beta\mathbf{v}\} - \text{Im}\{\beta\mathbf{v}\}||^2$. It can be noted that the variable β acts an artificial phase multiplier on the snapshot vector which is used to "rotate" the snapshot vector $\mathbf{v} = \mathbf{v_r} + j\mathbf{v_i}$, so that its real and imaginary parts are aligned to the "center" of the first octant.

Once the snapshot vector is multiplied by β, we get

$$\begin{aligned} \mathbf{q_r} &= \text{Re}\{\beta\mathbf{v}\} \\ \mathbf{q_i} &= \text{Im}\{\beta\mathbf{v}\} \end{aligned} \tag{9}$$

Here $\mathbf{v}$ is a 3×1 vector whose elements are the snapshot vector $\widehat{\mathbf{r}}$ given in (2) with the indices $\{k_{p-1}, k_p, k_{p+1}\}$.

Under the high SNR assumption, q_r and q_i vectors can be considered as the additive white Gaussian noise corrupted versions of a vector and the pdf for the angle between q_r and q_i can be written as $f_\psi(\psi; \rho/2)$ where ρ is the operational SNR.

Decision Statistic: The declaration of single (H_1) or multiple sources (H_2) is based on the following test:

$$\arccos\left(\frac{|\mathbf{q_r^T}\mathbf{q_i}|}{||\mathbf{q_r}||||\mathbf{q_i}||}\right) \underset{\text{H}_1}{\overset{\text{H}_2}{\gtrless}} \lambda \tag{10}$$

where $\mathbf{q_r}$ and $\mathbf{q_i}$ are as defined in (9) and λ is the threshold value to meet a desired false multiple target decision probability which is calculated through

$$\int_\lambda^\pi f_\psi(\psi; \frac{\rho}{2})d\psi = P_{\text{FMSD}}. \tag{11}$$

Table I summarizes the proposed algorithm.

TABLE I
ALGORITHM STEPS

1.	Transform the snapshot vector to the beamspace and align the manifold via (2).
2.	Select three DFT bins around the peak value in the spectrum. Call 3×1 vector composed of the selected entries as $\mathbf{v}$.
3.	Calculate β from (8) using the real and imaginary parts of $\mathbf{v}$.
4.	Calculate $\mathbf{q_r}$ and $\mathbf{q_i}$ vector via (9).
5.	Calculate the threshold, λ, using (11). In (11), ρ is the operational SNR, which is the average signal energy per sensor over the sensor noise variance σ_n^2.
6.	Calculate the decision statistics from (10). Declare a multiple-source decision if the threshold is exceeded, otherwise declare a single-source decision.

III. Numerical Results

We first present a set of numerical results to examine the validity of high SNR assumption for the calculation of the threshold and then examine the performance of the proposed algorithm.

The Validity of High SNR Approximation: Figure 3 compares the suggested high SNR approximation for the density with the true density calculated through Monte Carlo simulations. These figures show the distribution of the angle between $\mathbf{q_r}$ and $\mathbf{q_i}$, that is the decision statistics calculated in (10). The SNR values are 10 and 15 dB. It can be noted that the approximation does not follow the pdf in the tails at the SNR = 10 dB, while the results for 15 dB are in very good agreement.

Since the detection threshold in many radar applications is higher than 15 dB, the suggested approximation can be reliably

$$f_{\psi}(\psi;\rho) = \sqrt{\frac{\rho}{8\pi}}\exp\left(-\frac{\rho}{2}\sin(2\psi)\right) + \exp\left(-\frac{\rho}{2}\sin^2(\psi)\right)\operatorname{erfc}\left(-\sqrt{\frac{\rho}{2}}\cos(\psi)\right) \times \left(\frac{\rho}{8}\left[\sin(\psi)+\sin(3\psi)\right] + \frac{1}{2}\sin(\psi)\right), \qquad 0 \le \psi \le \pi \tag{7}$$

utilized. It should be noted that the approximation significantly improves as SNR increases.

Detection of the Number of Targets: We examine the scenario where two equal power sources are located within the Rayleigh resolution limit. For the illustration simplicity, we use the units of DFT bins to represent the signal frequencies. For example, a signal located at 3.2 DFT bin has the frequency of $2\pi/N \times 3.2$ radians per sample. (Here N is the number of observations or sensors for the directional of arrival problem.) The Rayleigh resolution with N observations is equivalent to 1 DFT bin with this convention.

In this experiment, we consider two complex exponentials separated by 0.3 DFT bins. The signals have the frequencies of 2.9 and 3.2 DFT bins. Under sufficiently high SNR conditions, it is expected to have large signal values for the DFT bins of $\{2, 3, 4\}$. The beamspace operation reduces N element snapshot vector to the 3-element vector $\mathbf{v}$ which is composed of 2nd, 3rd and 4th DFT output bins, as explained in Table I. The estimation goal is to determine the fractional parts of the frequencies, i.e. -0.1 and 0.2, by processing $\mathbf{v}$ vector.

For the estimation of frequencies, we use the maximum likelihood (ML) estimation approach in the beamspace domain. It has been shown in [3] that ML estimates of the frequencies can be generated through solving a quadratic relation involving the entries of 3×1 $\mathbf{v}$ vector. Here we use this method for the estimation of the frequencies. Due to space limitations we can not present the details on the estimation procedure. Readers are invited to examine [3] for further details.

It should be noted that the resolution of close sources is only possible when the SNR is above a certain level. In the first experiment, we set the SNR level to 25 dB. The Monte Carlo runs are generated by superposing two complex exponential vectors of length 16 having the mentioned frequencies and having complex amplitudes with random phases and constant magnitude of $\sqrt{\text{SNR}}$. The snapshot vector is formed by adding zero mean, unit variance Gaussian vector to the superposition of source vectors.

Figure 4a shows the distribution of the frequency estimates under these conditions. The true values for the fractional parts are indicated on the x-axis of the same Figure. From this figure, it can be observed that SNR of 25 dB is not sufficient to resolve both signals. Furthermore, the estimates are concentrated around $\widehat{\delta} = 0.05$ (which is the midpoint of two source frequencies) indicating that the estimates are heavily biased. The proposed method is to detect the existence of interfering multipath signal and issue a warning on the estimator bias.

Figure 4b shows the distribution of the angle between real and imaginary parts of the resultant vector after following the steps given in Table I. The red dashed curve is the analytical approximation of the probability density function under the hypothesis of single source. Previously it has been shown that the analytical approximation closely follows the true distribution for SNRs greater than 15 dB. In order not to complicate the figure, we present the analytical curve instead of the Monte Carlo results. On the same figure, the threshold location for $P_{\text{FMSD}} = 0.01$ is also indicated. The threshold guarantees that only 1 percent of the single signal arrivals are erroneously declared as multiple sources.

The blue histogram in Figure 4b shows the Monte Carlo estimate of the angle between real and imaginary parts when there are two signals. From this figure, it can be noted that the threshold set for $P_{\text{FMSD}} = 0.01$ is exceeded for the majority of the trials when two signals are present. The probability of two signal declarations is 0.67 for this experiment.

Figure 5 shows the same experiment for SNR = 35 dB. It can be seen from Figure 5a that both signals are successfully resolved at this SNR level. Figure 5b shows that the threshold level is much lower than the level for SNR = 25 dB. Furthermore the angle between the vectors for the two signal case is spread up to $\pi/2$ radians. The probability of declaring multiple targets becomes 0.89 for the SNR of 35 dB.

Figure 6 shows the probability of declaring multiple sources in the presence of two sources for different SNR and frequency separations at the $P_{\text{FMSD}} = 0.01$. The required SNR to resolve and detect the interfering targets decreases when interfering target moves away from each other.

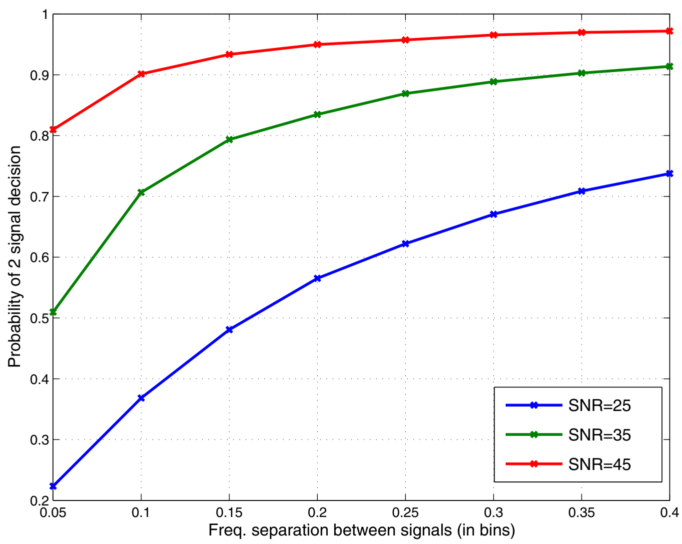
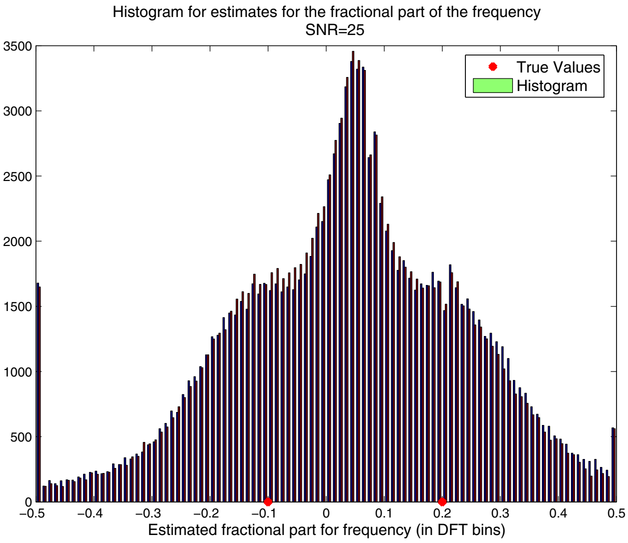
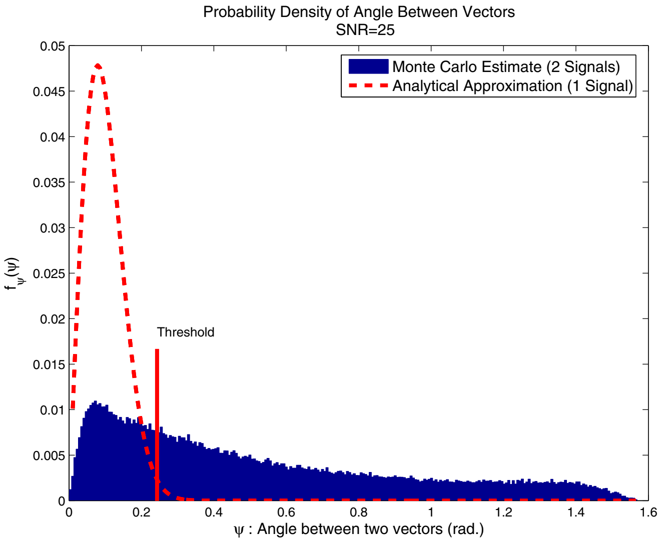


Fig. 6. Probability of Two Signal Decision at $P_{\text{FMSD}} = 0.01$

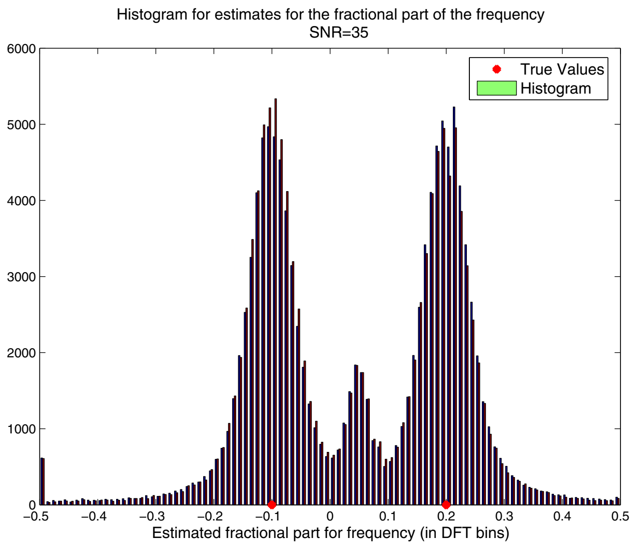


(a) The distribution of frequency estimates

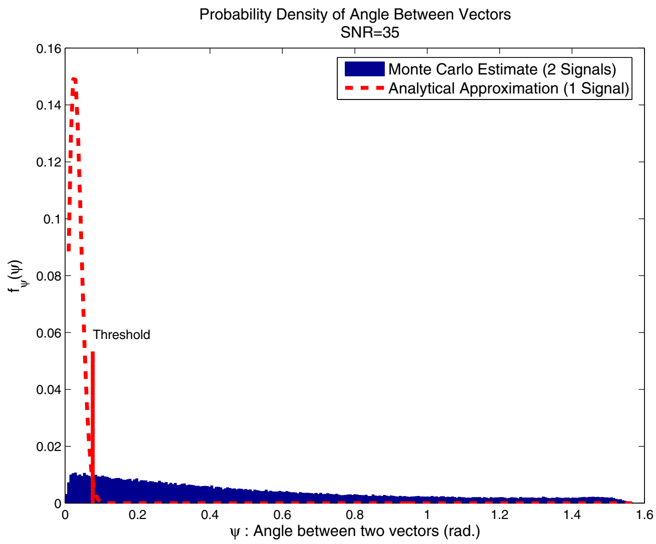


(b) Histograms for one and two signal hypotheses

Fig. 4. Frequency Estimates and Histograms for SNR = 25 dB



(a) The distribution of frequency estimates



(b) Histograms for one and two signal hypotheses

Fig. 5. Frequency Estimates and Histograms for SNR = 35 dB

IV. Conclusion

We present a method for detecting the presence of a second signal which possibly introduces a bias in the direction of arrival estimates unless the operational SNR is exceptionally high. The method is based on the manifold alignment procedure introduced in this paper. The present work can be coupled with the beamspace ML estimation techniques introduced by Zoltowski *et al.* [3] and can be incorporated into the existing direction finding systems with little effort.

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