chapter

14

Frequency Response

Dost thou love Life? Then do not squander Time; for that is the stuff Life is made.

—Benjamin Franklin

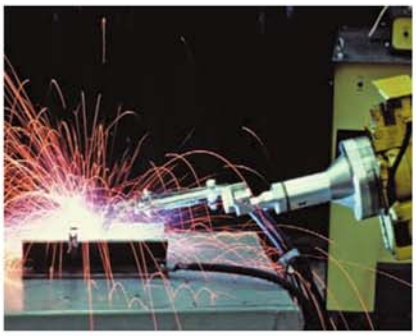
Enhancing Your Career

Career in Control Systems

Control systems are another area of electrical engineering where circuit analysis is used. A control system is designed to regulate the behavior of one or more variables in some desired manner. Control systems play major roles in our everyday life. Household appliances such as heating and air-conditioning systems, switch-controlled thermostats, washers and dryers, cruise controllers in automobiles, elevators, traffic lights, manufacturing plants, navigation systems—all utilize control systems. In the aerospace field, precision guidance of space probes, the wide range of operational modes of the space shuttle, and the ability to maneuver space vehicles remotely from earth all require knowledge of control systems. In the manufacturing sector, repetitive production line operations are increasingly performed by robots, which are programmable control systems designed to operate for many hours without fatigue.

Control engineering integrates circuit theory and communication theory. It is not limited to any specific engineering discipline but may involve environmental, chemical, aeronautical, mechanical, civil, and electrical engineering. For example, a typical task for a control system engineer might be to design a speed regulator for a disk drive head.

A thorough understanding of control systems techniques is essential to the electrical engineer and is of great value for designing control systems to perform the desired task.



A welding robot. © Vol. 1 PhotoDisc/ Getty Images

14.1 Introduction

In our sinusoidal circuit analysis, we have learned how to find voltages and currents in a circuit with a constant frequency source. If we let the amplitude of the sinusoidal source remain constant and vary the frequency, we obtain the circuit's *frequency response*. The frequency response may be regarded as a complete description of the sinusoidal steady-state behavior of a circuit as a function of frequency.

> The **frequency response** of a circuit is the variation in its behavior with change in signal frequency.

The frequency response of a circuit may also be considered as the variation of the gain and phase with frequency.

The sinusoidal steady-state frequency responses of circuits are of significance in many applications, especially in communications and control systems. A specific application is in electric filters that block out or eliminate signals with unwanted frequencies and pass signals of the desired frequencies. Filters are used in radio, TV, and telephone systems to separate one broadcast frequency from another.

We begin this chapter by considering the frequency response of simple circuits using their transfer functions. We then consider Bode plots, which are the industry-standard way of presenting frequency response. We also consider series and parallel resonant circuits and encounter important concepts such as resonance, quality factor, cutoff frequency, and bandwidth. We discuss different kinds of filters and network scaling. In the last section, we consider one practical application of resonant circuits and two applications of filters.

14.2 Transfer Function

The transfer function $\mathbf{H}(\omega)$ (also called the *network function*) is a useful analytical tool for finding the frequency response of a circuit. In fact, the frequency response of a circuit is the plot of the circuit's transfer function $\mathbf{H}(\omega)$ versus ω, with ω varying from $\omega = 0$ to $\omega = \infty$.

A transfer function is the frequency-dependent ratio of a forced function to a forcing function (or of an output to an input). The idea of a transfer function was implicit when we used the concepts of impedance and admittance to relate voltage and current. In general, a linear network can be represented by the block diagram shown in Fig. 14.1.

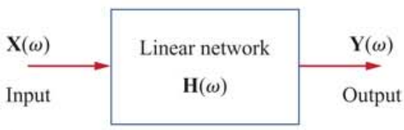


Figure 14.1
A block diagram representation of a linear network.

> The **transfer function** $\mathbf{H}(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\omega)$ (an element voltage or current) to a phasor input $\mathbf{X}(\omega)$ (source voltage or current).

In this context, $\mathbf{X}(\omega)$ and $\mathbf{Y}(\omega)$ denote the input and output phasors of a network; they should not be confused with the same symbolism used for reactance and admittance. The multiple usage of symbols is conventionally permissible due to lack of enough letters in the English language to express all circuit variables distinctly.

Thus,

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)} \tag{14.1}$$

assuming zero initial conditions. Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions:

Some authors use $\mathbf{H}(j\omega)$ for transfer instead of $\mathbf{H}(\omega)$, since ω and j are an inseparable pair.

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} \tag{14.2a}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} \tag{14.2b}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)} \tag{14.2c}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)} \tag{14.2d}$$

where subscripts i and o denote input and output values. Being a complex quantity, $\mathbf{H}(\omega)$ has a magnitude $H(\omega)$ and a phase ϕ; that is, $\mathbf{H}(\omega) = H(\omega)\underline{/\phi}$.

To obtain the transfer function using Eq. (14.2), we first obtain the frequency-domain equivalent of the circuit by replacing resistors, inductors, and capacitors with their impedances R, $j\omega L$, and $1/j\omega C$. We then use any circuit technique(s) to obtain the appropriate quantity in Eq. (14.2). We can obtain the frequency response of the circuit by plotting the magnitude and phase of the transfer function as the frequency varies. A computer is a real time-saver for plotting the transfer function.

The transfer function $\mathbf{H}(\omega)$ can be expressed in terms of its numerator polynomial $\mathbf{N}(\omega)$ and denominator polynomial $\mathbf{D}(\omega)$ as

$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)}} \tag{14.3}$$

where $\mathbf{N}(\omega)$ and $\mathbf{D}(\omega)$ are not necessarily the same expressions for the input and output functions, respectively. The representation of $\mathbf{H}(\omega)$ in Eq. (14.3) assumes that common numerator and denominator factors in $\mathbf{H}(\omega)$ have canceled, reducing the ratio to lowest terms. The roots of $\mathbf{N}(\omega) = 0$ are called the *zeros* of $\mathbf{H}(\omega)$ and are usually represented as $j\omega = z_1, z_2, \ldots$. Similarly, the roots of $\mathbf{D}(\omega) = 0$ are the *poles* of $\mathbf{H}(\omega)$ and are represented as $j\omega = p_1, p_2, \ldots$.

A **zero**, as a *root* of the numerator polynomial, is a value that results in a zero value of the function. A **pole**, as a *root* of the denominator polynomial, is a value for which the function is infinite.

A zero may also be regarded as the value of $s = j\omega$ that makes $\mathbf{H}(s)$ zero, and a pole as the value of $s = j\omega$ that makes $\mathbf{H}(s)$ infinite.

To avoid complex algebra, it is expedient to replace $j\omega$ temporarily with s when working with $\mathbf{H}(\omega)$ and replace s with $j\omega$ at the end.

Example 14.1

For the RC circuit in Fig. 14.2(a), obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and its frequency response. Let $v_s = V_m \cos \omega t$.

Solution:

The frequency-domain equivalent of the circuit is in Fig. 14.2(b). By voltage division, the transfer function is given by

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

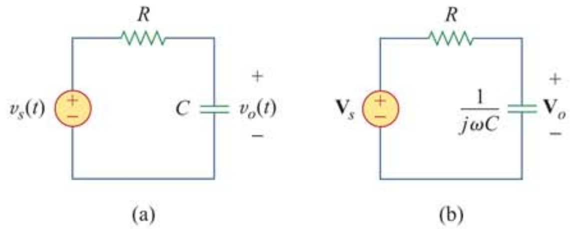


Figure 14.2
For Example 14.1: (a) time-domain *RC* circuit, (b) frequency-domain *RC* circuit.

Comparing this with Eq. (9.18e), we obtain the magnitude and phase of $\mathbf{H}(\omega)$ as

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0}$$

where $\omega_0 = 1/RC$. To plot H and ϕ for $0 < \omega < \infty$, we obtain their values at some critical points and then sketch.

At $\omega = 0$, $H = 1$ and $\phi = 0$. At $\omega = \infty$, $H = 0$ and $\phi = -90°$. Also, at $\omega = \omega_0$, $H = 1/\sqrt{2}$ and $\phi = -45°$. With these and a few more points as shown in Table 14.1, we find that the frequency response is as shown in Fig. 14.3. Additional features of the frequency response in Fig. 14.3 will be explained in Section 14.6.1 on lowpass filters.

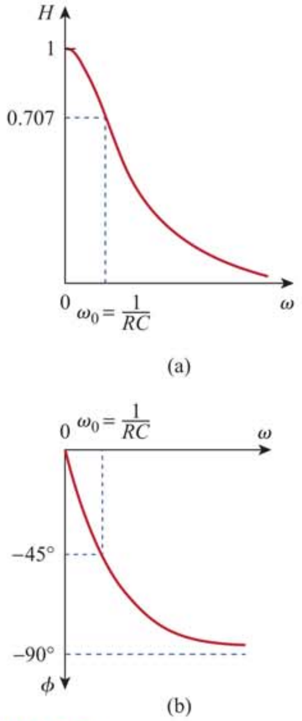


Figure 14.3
Frequency response of the *RC* circuit: (a) amplitude response, (b) phase response.

TABLE 14.1
For Example 14.1.

ω/ω_0	H	ϕ	ω/ω_0	H	ϕ
0	1	0	10	0.1	−84°
1	0.71	−45°	20	0.05	−87°
2	0.45	−63°	100	0.01	−89°
3	0.32	−72°	∞	0	−90°

Practice Problem 14.1

Obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the *RL* circuit in Fig. 14.4, assuming $v_s = V_m \cos\omega t$. Sketch its frequency response.

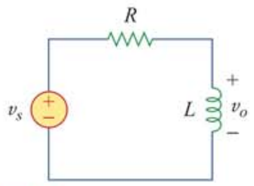


Figure 14.4
RL circuit for Practice Prob. 14.1.

Answer: $j\omega L/(R + j\omega L)$; see Fig. 14.5 for the response.

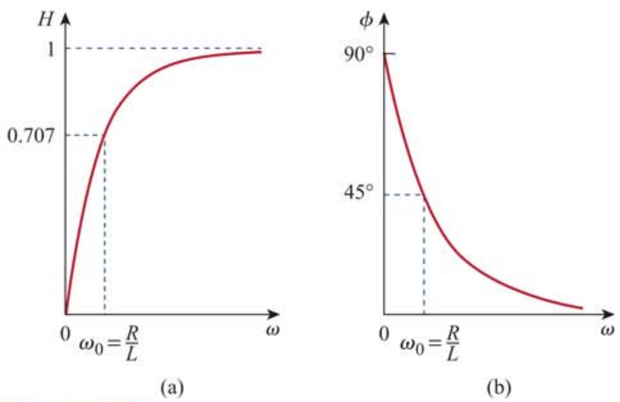


Figure 14.5
Frequency response of the *RL* circuit in Fig. 14.4.

Example 14.2

For the circuit in Fig. 14.6, calculate the gain $\mathbf{I}_o(\omega)/\mathbf{I}_i(\omega)$ and its poles and zeros.

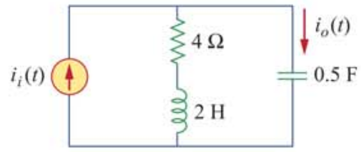


Figure 14.6
For Example 14.2.

Solution:
By current division,

$$\mathbf{I}_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + 1/j0.5\omega}\mathbf{I}_i(\omega)$$

or

$$\frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s + 2)}{s^2 + 2s + 1}, \qquad s = j\omega$$

The zeros are at

$$s(s + 2) = 0 \qquad \Rightarrow \qquad z_1 = 0, z_2 = -2$$

The poles are at

$$s^2 + 2s + 1 = (s + 1)^2 = 0$$

Thus, there is a repeated pole (or double pole) at $p = -1$.

Practice Problem 14.2

Find the transfer function $\mathbf{V}_o(\omega)/\mathbf{I}_i(\omega)$ for the circuit in Fig. 14.7. Obtain its zeros and poles.

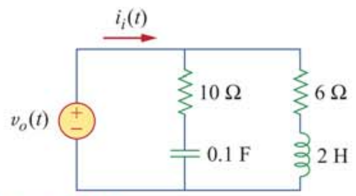


Figure 14.7
For Practice Prob. 14.2.

Answer: $\dfrac{10(s + 1)(s + 3)}{s^2 + 8s + 5}$, $s = j\omega$; zeros: $-1, -3$; poles: -0.683, -7.317.

14.3 †The Decibel Scale

It is not always easy to get a quick plot of the magnitude and phase of the transfer function as we did above. A more systematic way of obtaining the frequency response is to use Bode plots. Before we begin to construct Bode plots, we should take care of two important issues: the use of logarithms and decibels in expressing gain.

Since Bode plots are based on logarithms, it is important that we keep the following properties of logarithms in mind:

1. $\log P_1P_2 = \log P_1 + \log P_2$
2. $\log P_1/P_2 = \log P_1 - \log P_2$
3. $\log P^n = n \log P$
4. $\log 1 = 0$

In communications systems, gain is measured in *bels*. Historically, the bel is used to measure the ratio of two levels of power or power gain G; that is,

Historical note: The *bel* is named after Alexander Graham Bell, the inventor of the telephone.

$$G = \text{Number of bels} = \log_{10}\frac{P_2}{P_1} \tag{14.4}$$

Historical

Alexander Graham Bell (1847–1922) inventor of the telephone, was a Scottish-American scientist.

Bell was born in Edinburgh, Scotland, a son of Alexander Melville Bell, a well-known speech teacher. Alexander the younger also became a speech teacher after graduating from the University of Edinburgh and the University of London. In 1866 he became interested in transmitting speech electrically. After his older brother died of tuberculosis, his father decided to move to Canada. Alexander was asked to come to Boston to work at the School for the Deaf. There he met Thomas A. Watson, who became his assistant in his electromagnetic transmitter experiment. On March 10, 1876, Alexander sent the famous first telephone message: "Watson, come here I want you." The bel, the logarithmic unit introduced in Chapter 14, is named in his honor.

The *decibel* (dB) provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

$$G_{\mathrm{dB}} = 10 \log_{10} \frac{P_2}{P_1} \tag{14.5}$$

When $P_1 = P_2$, there is no change in power and the gain is 0 dB. If $P_2 = 2P_1$, the gain is

$$G_{\mathrm{dB}} = 10 \log_{10} 2 \simeq 3 \text{ dB} \tag{14.6}$$

and when $P_2 = 0.5P_1$, the gain is

$$G_{\mathrm{dB}} = 10 \log_{10} 0.5 \simeq -3 \text{ dB} \tag{14.7}$$

Equations (14.6) and (14.7) show another reason why logarithms are greatly used: The logarithm of the reciprocal of a quantity is simply negative the logarithm of that quantity.

Alternatively, the gain G can be expressed in terms of voltage or current ratio. To do so, consider the network shown in Fig. 14.8. If P_1 is the input power, P_2 is the output (load) power, R_1 is the input resistance, and R_2 is the load resistance, then $P_1 = 0.5V_1^2/R_1$ and $P_2 = 0.5V_2^2/R_2$, and Eq. (14.5) becomes

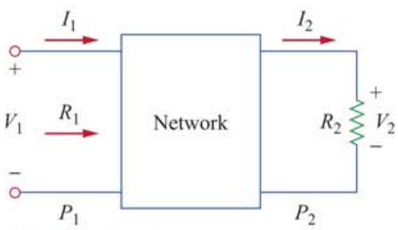


Figure 14.8
Voltage-current relationships for a four-terminal network.

$$\begin{aligned} G_{\mathrm{dB}} &= 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_2}{V_1^2/R_1} \\ &= 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2 + 10 \log_{10} \frac{R_1}{R_2} \end{aligned} \tag{14.8}$$

$$G_{\mathrm{dB}} = 20 \log_{10} \frac{V_2}{V_1} - 10 \log_{10} \frac{R_2}{R_1} \tag{14.9}$$

For the case when $R_2 = R_1$, a condition that is often assumed when comparing voltage levels, Eq. (14.9) becomes

$$G_{\mathrm{dB}} = 20 \log_{10} \frac{V_2}{V_1} \tag{14.10}$$

Instead, if $P_1 = I_1^2 R_1$ and $P_2 = I_2^2 R_2$, for $R_1 = R_2$, we obtain

$$G_{dB} = 20 \log_{10} \frac{I_2}{I_1} \quad \textbf{(14.11)}$$

Three things are important to note from Eqs. (14.5), (14.10), and (14.11):

1. That 10 $\log_{10}$ is used for power, while 20 $\log_{10}$ is used for voltage or current, because of the square relationship between them ($P = V^2/R = I^2 R$).
2. That the dB value is a logarithmic measurement of the *ratio* of one variable to another *of the same type*. Therefore, it applies in expressing the transfer function H in Eqs. (14.2a) and (14.2b), which are dimensionless quantities, but not in expressing H in Eqs. (14.2c) and (14.2d).
3. It is important to note that we only use voltage and current magnitudes in Eqs. (14.10) and (14.11). Negative signs and angles will be handled independently as we will see in Section 14.4.

With this in mind, we now apply the concepts of logarithms and decibels to construct Bode plots.

14.4 Bode Plots

Obtaining the frequency response from the transfer function as we did in Section 14.2 is an uphill task. The frequency range required in frequency response is often so wide that it is inconvenient to use a linear scale for the frequency axis. Also, there is a more systematic way of locating the important features of the magnitude and phase plots of the transfer function. For these reasons, it has become standard practice to plot the transfer function on a pair of semilogarithmic plots: the magnitude in decibels is plotted against the logarithm of the frequency; on a separate plot, the phase in degrees is plotted against the logarithm of the frequency. Such semilogarithmic plots of the transfer function—known as *Bode plots*—have become the industry standard.

Historical note: Named after Hendrik W. Bode (1905–1982), an engineer with the Bell Telephone Laboratories, for his pioneering work in the 1930s and 1940s.

> **Bode plots** are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

Bode plots contain the same information as the nonlogarithmic plots discussed in the previous section, but they are much easier to construct, as we shall see shortly.

The transfer function can be written as

$$\mathbf{H} = H\underline{/\phi} = He^{j\phi} \quad \textbf{(14.12)}$$

Taking the natural logarithm of both sides,

$$\ln \mathbf{H} = \ln H + \ln e^{j\phi} = \ln H + j\phi \quad \textbf{(14.13)}$$

Thus, the real part of ln **H** is a function of the magnitude while the imaginary part is the phase. In a Bode magnitude plot, the gain

$$\boxed{H_{dB} = 20 \log_{10} H} \quad \textbf{(14.14)}$$

TABLE 14.2

Specific gain and their decibel values.*

Magnitude H	$20 \log_{10} H$ (dB)
0.001	−60
0.01	−40
0.1	−20
0.5	−6
$1/\sqrt{2}$	−3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

* Some of these values are approximate.

The origin is where $\omega = 1$ or $\log \omega = 0$ and the gain is zero.

A decade is an interval between two frequencies with a ratio of 10; e.g., between ω_0 and $10\omega_0$, or between 10 and 100 Hz. Thus, 20 dB/decade means that the magnitude changes 20 dB whenever the frequency changes tenfold or one decade.

is plotted in decibels (dB) versus frequency. Table 14.2 provides a few values of H with the corresponding values in decibels. In a Bode phase plot, ϕ is plotted in degrees versus frequency. Both magnitude and phase plots are made on semilog graph paper.

A transfer function in the form of Eq. (14.3) may be written in terms of factors that have real and imaginary parts. One such representation might be

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots} \quad (14.15)$$

which is obtained by dividing out the poles and zeros in $\mathbf{H}(\omega)$. The representation of $\mathbf{H}(\omega)$ as in Eq. (14.15) is called the *standard form*. $\mathbf{H}(\omega)$ may include up to seven types of different factors that can appear in various combinations in a transfer function. These are:

1. A gain K
2. A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin
3. A simple pole $1/(1 + j\omega/p_1)$ or zero $(1 + j\omega/z_1)$
4. A quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ or zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

In constructing a Bode plot, we plot each factor separately and then add them graphically. The factors can be considered one at a time and then combined additively because of the logarithms involved. It is this mathematical convenience of the logarithm that makes Bode plots a powerful engineering tool.

We will now make straight-line plots of the factors listed above. We shall find that these straight-line plots known as Bode plots approximate the actual plots to a reasonable degree of accuracy.

Constant term: For the gain K, the magnitude is $20 \log_{10} K$ and the phase is 0°; both are constant with frequency. Thus, the magnitude and phase plots of the gain are shown in Fig. 14.9. If K is negative, the magnitude remains $20 \log_{10} |K|$ but the phase is ±180°.

Pole/zero at the origin: For the zero $(j\omega)$ at the origin, the magnitude is $20 \log_{10} \omega$ and the phase is 90°. These are plotted in Fig. 14.10, where we notice that the slope of the magnitude plot is 20 dB/decade, while the phase is constant with frequency.

The Bode plots for the pole $(j\omega)^{-1}$ are similar except that the slope of the magnitude plot is −20 dB/decade while the phase is −90°. In general, for $(j\omega)^N$, where N is an integer, the magnitude plot will have a slope of $20N$ dB/decade, while the phase is $90N$ degrees.

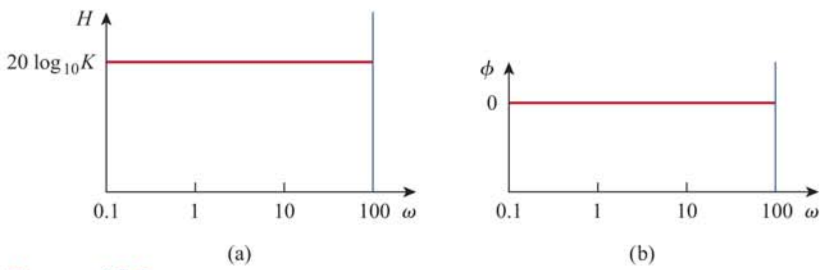


Figure 14.9
Bode plots for gain K: (a) magnitude plot, (b) phase plot.

Simple pole/zero: For the simple zero $(1 + j\omega/z_1)$, the magnitude is $20 \log_{10} |1 + j\omega/z_1|$ and the phase is $\tan^{-1} \omega/z_1$. We notice that

$$H_{\text{dB}} = 20 \log_{10} \left| 1 + \frac{j\omega}{z_1} \right| \quad \Rightarrow \quad 20 \log_{10} 1 = 0 \quad \text{as} \quad \omega \to 0 \tag{14.16}$$

$$H_{\text{dB}} = 20 \log_{10} \left| 1 + \frac{j\omega}{z_1} \right| \quad \Rightarrow \quad 20 \log_{10} \frac{\omega}{z_1} \quad \text{as} \quad \omega \to \infty \tag{14.17}$$

showing that we can approximate the magnitude as zero (a straight line with zero slope) for small values of ω and by a straight line with slope 20 dB/decade for large values of ω. The frequency $\omega = z_1$ where the two asymptotic lines meet is called the *corner frequency* or *break frequency*. Thus the approximate magnitude plot is shown in Fig. 14.11(a), where the actual plot is also shown. Notice that the approximate plot is close to the actual plot except at the break frequency, where $\omega = z_1$ and the deviation is $20 \log_{10} |(1 + j1)| = 20 \log_{10} \sqrt{2} \simeq 3$ dB.

The phase $\tan^{-1}(\omega/z_1)$ can be expressed as

$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \to \infty \end{cases} \tag{14.18}$$

As a straight-line approximation, we let $\phi \simeq 0$ for $\omega \le z_1/10$, $\phi \simeq 45^\circ$ for $\omega = z_1$, and $\phi \simeq 90^\circ$ for $\omega \ge 10z_1$. As shown in Fig. 14.11(b) along with the actual plot, the straight-line plot has a slope of 45° per decade.

The Bode plots for the pole $1/(1 + j\omega/p_1)$ are similar to those in Fig. 14.11 except that the corner frequency is at $\omega = p_1$, the magnitude has a slope of -20 dB/decade, and the phase has a slope of -45° per decade.

Quadratic pole/zero: The magnitude of the quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ is $-20 \log_{10} |1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2|$ and the phase is $-\tan^{-1}(2\zeta_2\omega/\omega_n)/(1 - \omega^2/\omega_n^2)$. But

$$H_{\text{dB}} = -20 \log_{10} \left| 1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2 \right| \quad \Rightarrow \quad 0 \quad \text{as} \quad \omega \to 0 \tag{14.19}$$

The special case of dc ($\omega = 0$) does not appear on Bode plots because log $0 = -\infty$, implying that zero frequency is infinitely far to the left of the origin of Bode plots.

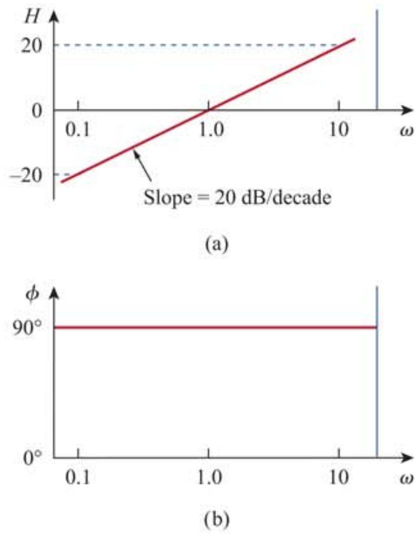


Figure 14.10
Bode plot for a zero $(j\omega)$ at the origin: (a) magnitude plot, (b) phase plot.

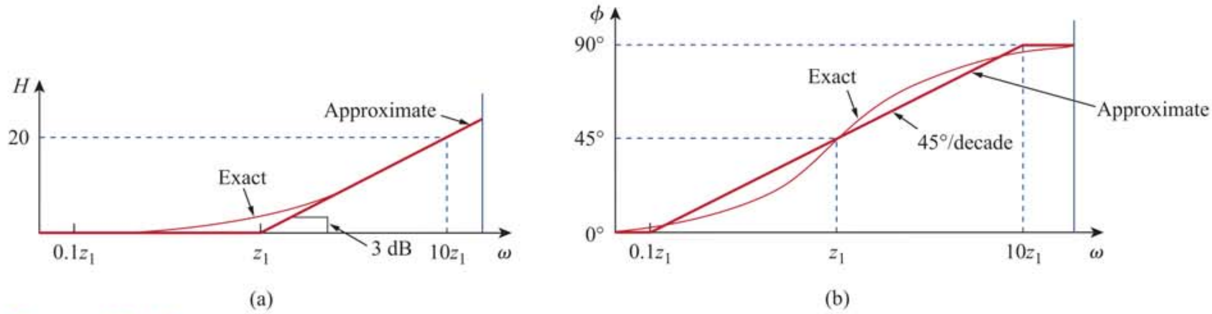


Figure 14.11
Bode plots of zero $(1 + j\omega/z_1)$: (a) magnitude plot, (b) phase plot.

and

$$H_{dB} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \quad\Rightarrow\quad -40\log_{10}\frac{\omega}{\omega_n} \quad \text{as} \quad \omega \to \infty \tag{14.20}$$

Thus, the amplitude plot consists of two straight asymptotic lines: one with zero slope for $\omega < \omega_n$ and the other with slope -40 dB/decade for $\omega > \omega_n$, with ω_n as the corner frequency. Figure 14.12(a) shows the approximate and actual amplitude plots. Note that the actual plot depends on the damping factor ζ_2 as well as the corner frequency ω_n. The significant peaking in the neighborhood of the corner frequency should be added to the straight-line approximation if a high level of accuracy is desired. However, we will use the straight-line approximation for the sake of simplicity.

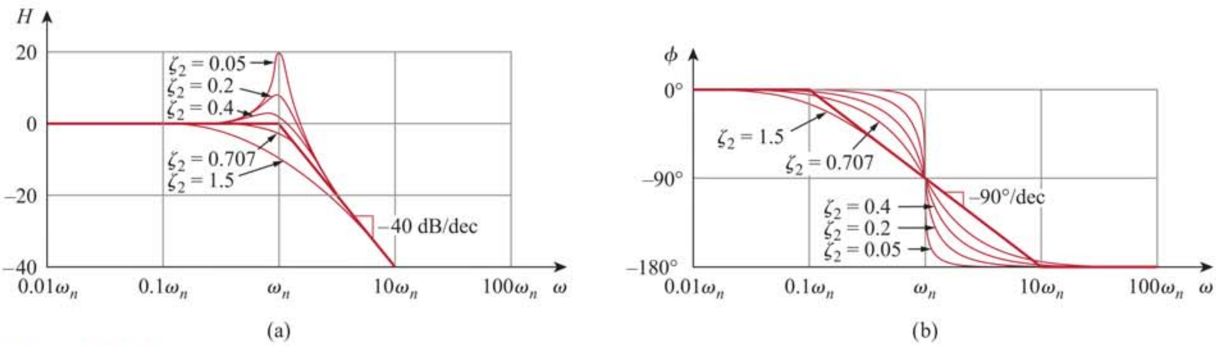


Figure 14.12
Bode plots of quadratic pole $[1 + j2\zeta\omega/\omega_n - \omega^2/\omega_n^2]^{-1}$: (a) magnitude plot, (b) phase plot.

There is another procedure for obtaining Bode plots that is faster and perhaps more efficient than the one we have just discussed. It consists in realizing that zeros cause an increase in slope, while poles cause a decrease. By starting with the low-frequency asymptote of the Bode plot, moving along the frequency axis, and increasing or decreasing the slope at each corner frequency, one can sketch the Bode plot immediately from the transfer function without the effort of making individual plots and adding them. This procedure can be used once you become proficient in the one discussed here.

Digital computers have rendered the procedure discussed here almost obsolete. Several software packages such as *PSpice*, *MATLAB*, Mathcad, and Micro-Cap can be used to generate frequency response plots. We will discuss *PSpice* later in the chapter.

The phase can be expressed as

$$\phi = -\tan^{-1}\frac{2\zeta_2\omega/\omega_n}{1 - \omega^2/\omega_n^2} = \begin{cases} 0, & \omega = 0 \\ -90°, & \omega = \omega_n \\ -180°, & \omega \to \infty \end{cases} \tag{14.21}$$

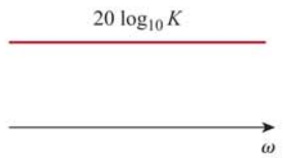
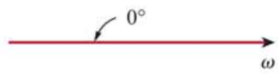
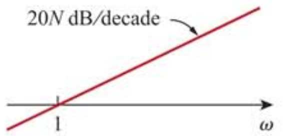
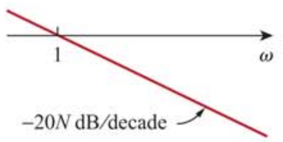
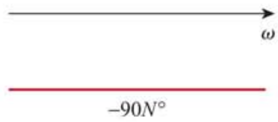
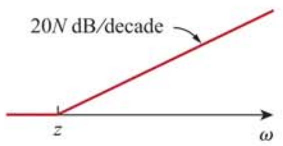
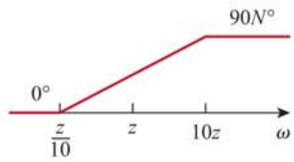
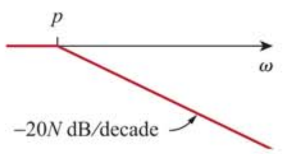
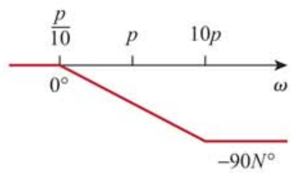
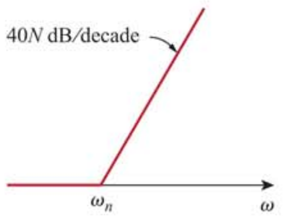
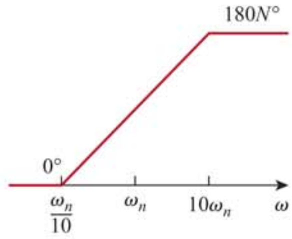
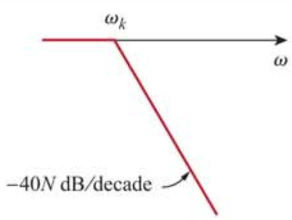
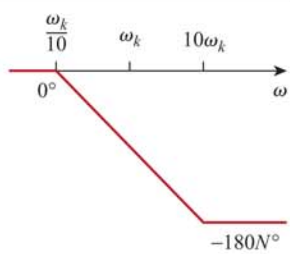
The phase plot is a straight line with a slope of $-90°$ per decade starting at $\omega_n/10$ and ending at $10\omega_n$, as shown in Fig. 14.12(b). We see again that the difference between the actual plot and the straight-line plot is due to the damping factor. Notice that the straight-line approximations for both magnitude and phase plots for the quadratic pole are the same as those for a double pole, i.e. $(1 + j\omega/\omega_n)^{-2}$. We should expect this because the double pole $(1 + j\omega/\omega_n)^{-2}$ equals the quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ when $\zeta_2 = 1$. Thus, the quadratic pole can be treated as a double pole as far as straight-line approximation is concerned.

For the quadratic zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$, the plots in Fig. 14.12 are inverted because the magnitude plot has a slope of 40 dB/decade while the phase plot has a slope of 90° per decade.

Table 14.3 presents a summary of Bode plots for the seven factors. Of course, not every transfer function has all seven factors. To sketch the Bode plots for a function $\mathbf{H}(\omega)$ in the form of Eq. (14.15), for example, we first record the corner frequencies on the semilog graph paper, sketch the factors one at a time as discussed above, and then combine

TABLE 14.3

Summary of Bode straight-line magnitude and phase plots.

Factor	Magnitude	Phase
K		
$(j\omega)^N$		
$\dfrac{1}{(j\omega)^N}$		
$\left(1+\dfrac{j\omega}{z}\right)^N$		
$\dfrac{1}{(1+j\omega/p)^N}$		
$\left[1+\dfrac{2j\omega\zeta}{\omega_n}+\left(\dfrac{j\omega}{\omega_n}\right)^2\right]^N$		
$\dfrac{1}{[1+2j\omega\zeta/\omega_k+(j\omega/\omega_k)^2]^N}$		

additively the graphs of the factors. The combined graph is often drawn from left to right, changing slopes appropriately each time a corner frequency is encountered. The following examples illustrate this procedure.

Example 14.3

Construct the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)}$$

Solution:

We first put $\mathbf{H}(\omega)$ in the standard form by dividing out the poles and zeros. Thus,

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{10j\omega}{(1 + j\omega/2)(1 + j\omega/10)} \\ &= \frac{10|j\omega|}{|1 + j\omega/2||1 + j\omega/10|} \underline{/90^\circ - \tan^{-1}\omega/2 - \tan^{-1}\omega/10}\end{aligned}$$

Hence, the magnitude and phase are

$$\begin{aligned}H_{\text{dB}} &= 20\log_{10}10 + 20\log_{10}|j\omega| - 20\log_{10}\left|1 + \frac{j\omega}{2}\right| \\ &\quad - 20\log_{10}\left|1 + \frac{j\omega}{10}\right| \\ \phi &= 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}\end{aligned}$$

We notice that there are two corner frequencies at $\omega = 2, 10$. For both the magnitude and phase plots, we sketch each term as shown by the dotted lines in Fig. 14.13. We add them up graphically to obtain the overall plots shown by the solid curves.

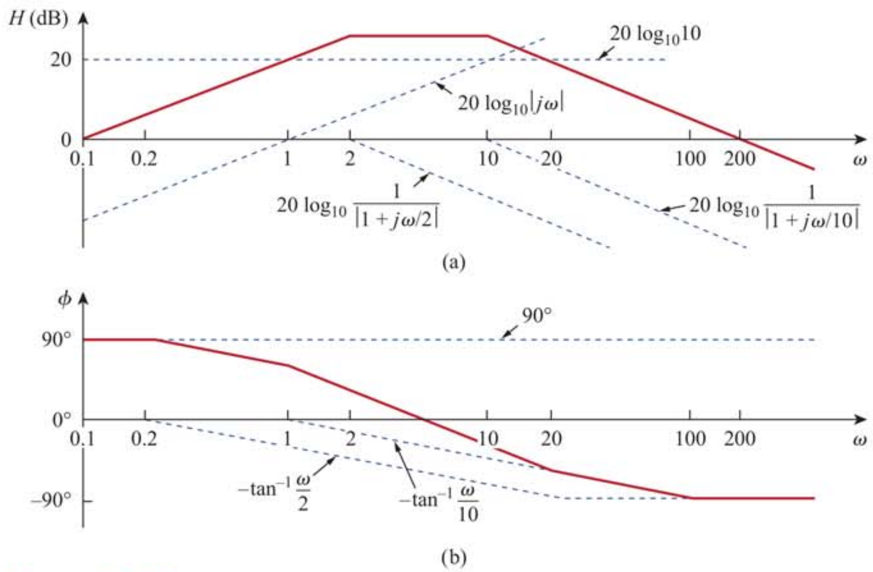


Figure 14.13
For Example 14.3: (a) magnitude plot, (b) phase plot.

Practice Problem 14.3

Draw the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

Answer: See Fig. 14.14.

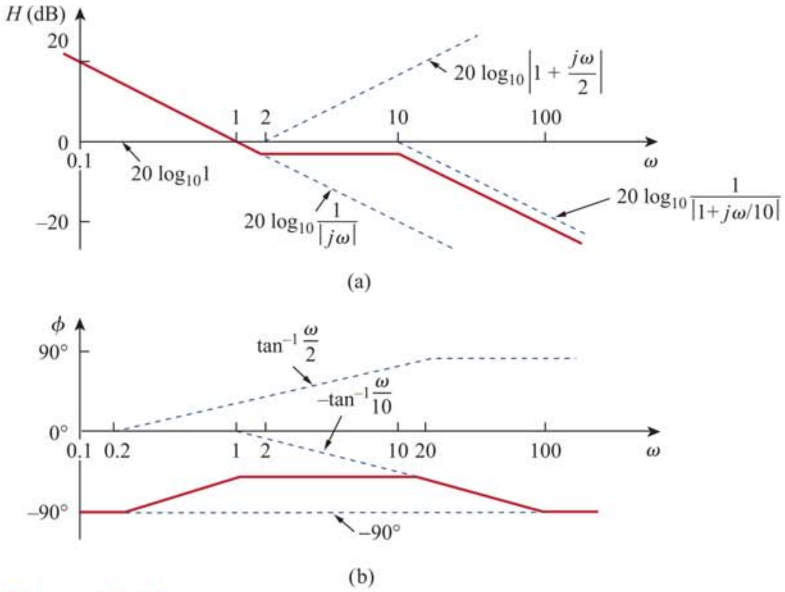


Figure 14.14
For Practice Prob. 14.3: (a) magnitude plot, (b) phase plot.

Example 14.4

Obtain the Bode plots for

$$\mathbf{H}(\omega) = \frac{j\omega + 10}{j\omega(j\omega + 5)^2}$$

Solution:
Putting $\mathbf{H}(\omega)$ in the standard form, we get

$$\mathbf{H}(\omega) = \frac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$$

From this, we obtain the magnitude and phase as

$$H_{\text{dB}} = 20\log_{10}0.4 + 20\log_{10}\left|1 + \frac{j\omega}{10}\right| - 20\log_{10}|j\omega| - 40\log_{10}\left|1 + \frac{j\omega}{5}\right|$$

$$\phi = 0° + \tan^{-1}\frac{\omega}{10} - 90° - 2\tan^{-1}\frac{\omega}{5}$$

There are two corner frequencies at $\omega = 5, 10$ rad/s. For the pole with corner frequency at $\omega = 5$, the slope of the magnitude plot is -40 dB/decade and that of the phase plot is $-90°$ per decade due to the power of 2. The magnitude and the phase plots for the individual terms (in dotted lines) and the entire $\mathbf{H}(j\omega)$ (in solid lines) are in Fig. 14.15.

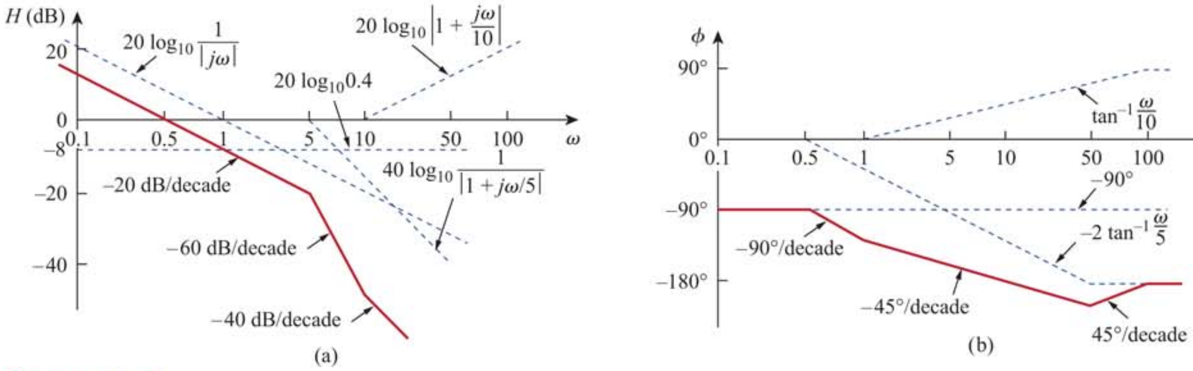


Figure 14.15
Bode plots for Example 14.4: (a) magnitude plot, (b) phase plot.

Practice Problem 14.4

Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{50j\omega}{(j\omega + 4)(j\omega + 10)^2}$$

Answer: See Fig. 14.16.

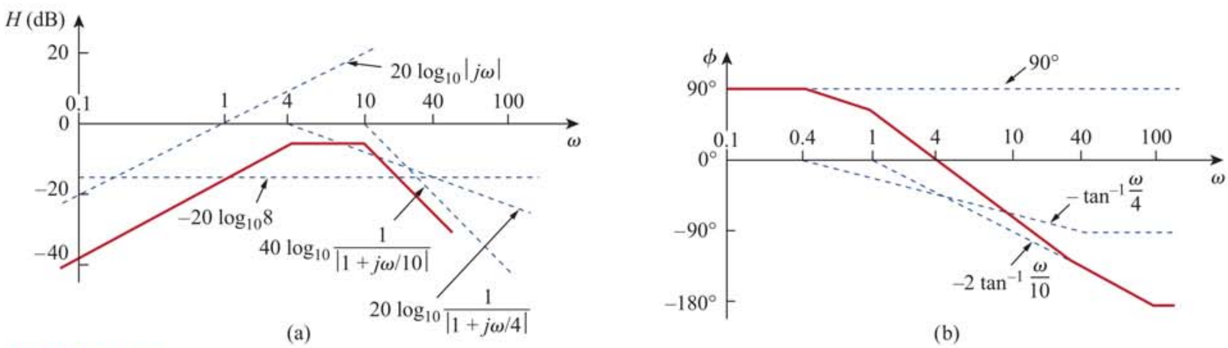


Figure 14.16
For Practice Prob. 14.4: (a) magnitude plot, (b) phase plot.

Example 14.5

Draw the Bode plots for

$$\mathbf{H}(s) = \frac{s + 1}{s^2 + 12s + 100}$$

Solution:

1. **Define.** The problem is clearly stated and we follow the technique outlined in the chapter.
2. **Present.** We are to develop the approximate bode plot for the given function, $\mathbf{H}(s)$.
3. **Alternative.** The two most effective choices would be the approximation technique outlined in the chapter, which we will

use here, and *MATLAB*, which can actually give us the exact Bode plots.

4. **Attempt.** We express $\mathbf{H}(s)$ as

$$\mathbf{H}(\omega) = \frac{1/100(1 + j\omega)}{1 + j\omega 1.2/10 + (j\omega/10)^2}$$

For the quadratic pole, $\omega_n = 10$ rad/s, which serves as the corner frequency. The magnitude and phase are

$$H_{\text{dB}} = -20 \log_{10} 100 + 20 \log_{10} |1 + j\omega| - 20 \log_{10} \left| 1 + \frac{j\omega 1.2}{10} - \frac{\omega^2}{100} \right|$$

$$\phi = 0° + \tan^{-1} \omega - \tan^{-1} \left[\frac{\omega 1.2/10}{1 - \omega^2/100} \right]$$

Figure 14.17 shows the Bode plots. Notice that the quadratic pole is treated as a repeated pole at ω_k, that is, $(1 + j\omega/\omega_k)^2$, which is an approximation.

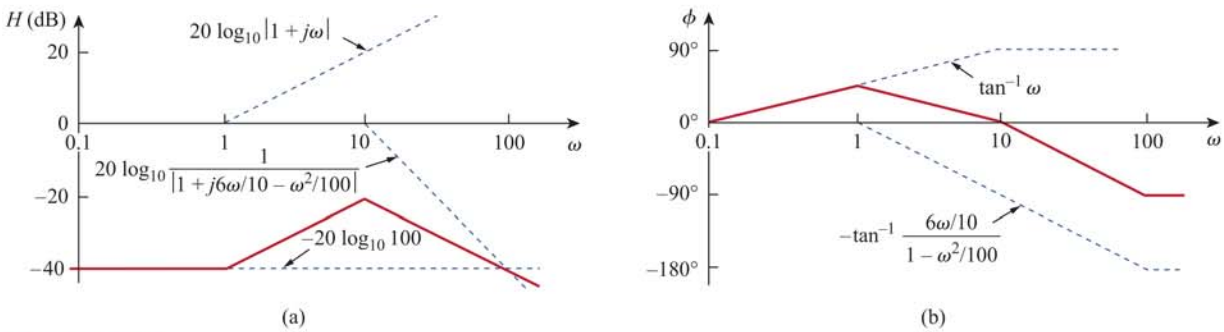


Figure 14.17
Bode plots for Example 14.5: (a) magnitude plot, (b) phase plot.

5. **Evaluate.** Although we could use *MATLAB* to validate the solution, we will use a more straightforward approach. First, we must realize that the denominator assumes that $\zeta = 0$ for the approximation, so we will use the following equation to check our answer:

$$\mathbf{H}(s) \simeq \frac{s + 1}{s^2 + 10^2}$$

We also note that we need to actually solve for H_{dB} and the corresponding phase angle ϕ. First, let $\omega = 0$.

$$H_{\text{dB}} = 20 \log_{10}(1/100) = -40 \qquad \text{and} \qquad \phi = 0°$$

Now try $\omega = 1$.

$$H_{\text{dB}} = 20 \log_{10}(1.4142/99) = -36.9 \text{ dB}$$

which is the expected 3 dB up from the corner frequency.

$$\phi = 45° \qquad \text{from} \qquad \mathbf{H}(j) = \frac{j + 1}{-1 + 100}$$

Now try $\omega = 100$.

$$H_{\text{dB}} = 20 \log_{10}(100) - 20 \log_{10}(9900) = 39.91 \text{ dB}$$

ϕ is 90° from the numerator minus 180°, which gives −90°. We now have checked three different points and got close agreement, and, since this is an approximation, we can feel confident that we have worked the problem successfully.

You can reasonably ask why did we not check at $\omega = 10$? If we just use the approximate value we used above, we end up with an infinite value, which is to be expected from $\zeta = 0$ (see Fig. 14.12a). If we used the actual value of $\mathbf{H}(j10)$ we will still end up being far from the approximate values, since $\zeta = 0.6$ and Fig. 14.12a shows a significant deviation from the approximation. We could have reworked the problem with $\zeta = 0.707$, which would have gotten us closer to the approximation. However, we really have enough points without doing this.

6. **Satisfactory?** We are satisfied the problem has been worked successfully and we can present the results as a solution to the problem.

Practice Problem 14.5

Construct the Bode plots for

$$H(s) = \frac{10}{s(s^2 + 80s + 400)}$$

Answer: See Fig. 14.18.

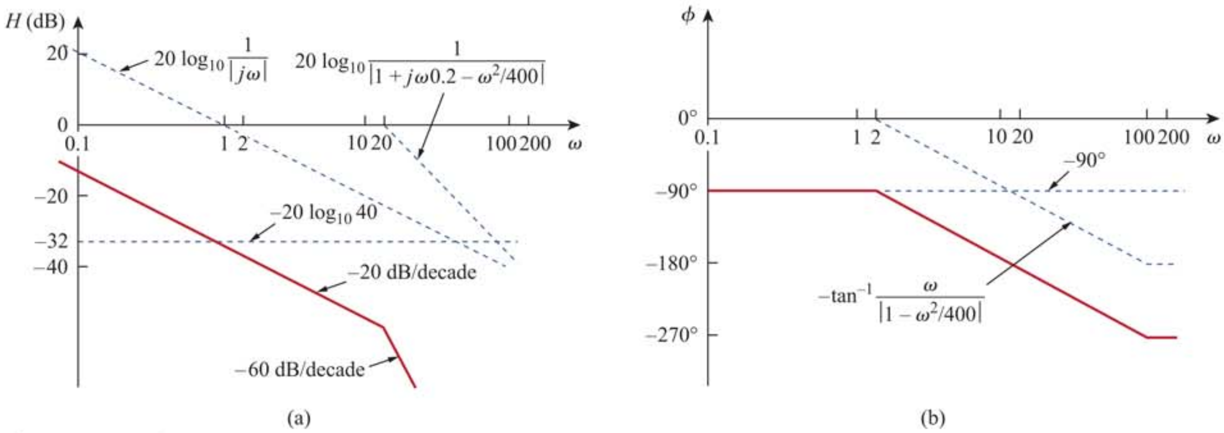


Figure 14.18
For Practice Prob. 14.5: (a) magnitude plot, (b) phase plot.

Example 14.6

Given the Bode plot in Fig. 14.19, obtain the transfer function $\mathbf{H}(\omega)$.

Solution:

To obtain $\mathbf{H}(\omega)$ from the Bode plot, we keep in mind that a zero always causes an upward turn at a corner frequency, while a pole causes a downward turn. We notice from Fig. 14.19 that there is a zero $j\omega$ at the origin which should have intersected the frequency axis at $\omega = 1$. This is indicated by the straight line with slope +20 dB/decade. The fact that this straight line is shifted by 40 dB indicates that there is a 40-dB gain; that is,

$$40 = 20 \log_{10} K \quad \Rightarrow \quad \log_{10} K = 2$$

or

$$K = 10^2 = 100$$

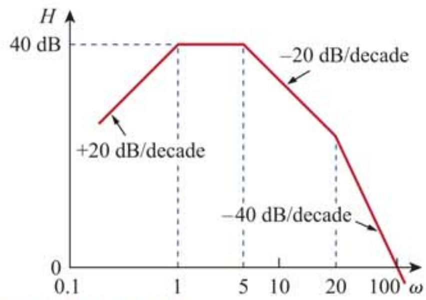


Figure 14.19
For Example 14.6.

In addition to the zero $j\omega$ at the origin, we notice that there are three factors with corner frequencies at $\omega = 1$, 5, and 20 rad/s. Thus, we have:

1. A pole at $p = 1$ with slope −20 dB/decade to cause a downward turn and counteract the zero at the origin. The pole at $p = 1$ is determined as $1/(1 + j\omega/1)$.
2. Another pole at $p = 5$ with slope −20 dB/decade causing a downward turn. The pole is $1/(1 + j\omega/5)$.
3. A third pole at $p = 20$ with slope −20 dB/decade causing a further downward turn. The pole is $1/(1 + j\omega/20)$.

Putting all these together gives the corresponding transfer function as

$$\mathbf{H}(\omega) = \frac{100 j\omega}{(1 + j\omega/1)(1 + j\omega/5)(1 + j\omega/20)} = \frac{j\omega 10^4}{(j\omega + 1)(j\omega + 5)(j\omega + 20)}$$

or

$$\mathbf{H}(s) = \frac{10^4 s}{(s + 1)(s + 5)(s + 20)}, \qquad s = j\omega$$

Practice Problem 14.6

Obtain the transfer function $\mathbf{H}(\omega)$ corresponding to the Bode plot in Fig. 14.20.

Answer: $\mathbf{H}(\omega) = \dfrac{4{,}000(s + 5)}{(s + 10)(s + 100)^2}$.

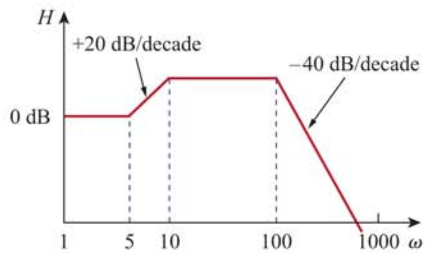


Figure 14.20
For Practice Prob. 14.6.

To see how to use *MATLAB* to produce Bode plots, refer to Section 14.11.

14.5 Series Resonance

The most prominent feature of the frequency response of a circuit may be the sharp peak (or *resonant peak*) exhibited in its amplitude characteristic. The concept of resonance applies in several areas of science and engineering. Resonance occurs in any system that has a complex conjugate pair of poles; it is the cause of oscillations of stored energy from one form to another. It is the phenomenon that allows frequency discrimination in communications networks. Resonance occurs in any circuit that has at least one inductor and one capacitor.

> **Resonance** is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

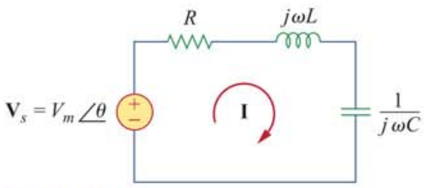


Figure 14.21
The series resonant circuit.

Consider the series *RLC* circuit shown in Fig. 14.21 in the frequency domain. The input impedance is

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C} \tag{14.22}$$

or

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{14.23}$$

Resonance results when the imaginary part of the transfer function is zero, or

$$\text{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0 \tag{14.24}$$

The value of ω that satisfies this condition is called the *resonant frequency* ω_0. Thus, the resonance condition is

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{14.25}$$

or

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}} \tag{14.26}$$

Since $\omega_0 = 2\pi f_0$,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \tag{14.27}$$

Note that at resonance:

1. The impedance is purely resistive, thus, $\mathbf{Z} = R$. In other words, the *LC* series combination acts like a short circuit, and the entire voltage is across *R*.
2. The voltage $\mathbf{V}_s$ and the current $\mathbf{I}$ are in phase, so that the power factor is unity.
3. The magnitude of the transfer function $\mathbf{H}(\omega) = \mathbf{Z}(\omega)$ is minimum.
4. The inductor voltage and capacitor voltage can be much more than the source voltage.

> Note No. 4 becomes evident from the fact that
>
> $$|\mathbf{V}_L| = \frac{V_m}{R}\omega_0 L = QV_m$$
>
> $$|\mathbf{V}_C| = \frac{V_m}{R}\frac{1}{\omega_0 C} = QV_m$$
>
> where Q is the quality factor, defined in Eq. (14.38).

The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \tag{14.28}$$

is shown in Fig. 14.22; the plot only shows the symmetry illustrated in this graph when the frequency axis is a logarithm. The average power dissipated by the *RLC* circuit is

$$P(\omega) = \frac{1}{2} I^2 R \tag{14.29}$$

The highest power dissipated occurs at resonance, when $I = V_m/R$, so that

$$P(\omega_0) = \frac{1}{2} \frac{V_m^2}{R} \tag{14.30}$$

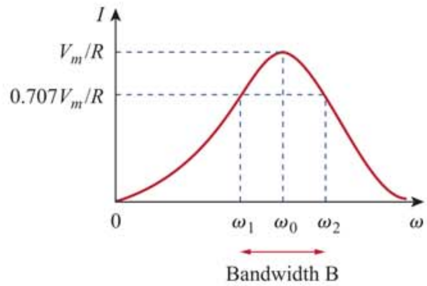


Figure 14.22
The current amplitude versus frequency for the series resonant circuit of Fig. 14.21.

At certain frequencies $\omega = \omega_1, \omega_2$, the dissipated power is half the maximum value; that is,

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R} \tag{14.31}$$

Hence, ω_1 and ω_2 are called the *half-power frequencies.*

The half-power frequencies are obtained by setting Z equal to $\sqrt{2}R$, and writing

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \tag{14.32}$$

Solving for ω, we obtain

$$\boxed{\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}} \tag{14.33}$$

We can relate the half-power frequencies with the resonant frequency. From Eqs. (14.26) and (14.33),

$$\omega_0 = \sqrt{\omega_1 \omega_2} \tag{14.34}$$

showing that the resonant frequency is the geometric mean of the half-power frequencies. Notice that ω_1 and ω_2 are in general not symmetrical around the resonant frequency ω_0, because the frequency response is not generally symmetrical. However, as will be explained shortly, symmetry of the half-power frequencies around the resonant frequency is often a reasonable approximation.

Although the height of the curve in Fig. 14.22 is determined by R, the width of the curve depends on other factors. The width of the response curve depends on the *bandwidth* B, which is defined as the difference between the two half-power frequencies,

$$B = \omega_2 - \omega_1 \tag{14.35}$$

This definition of bandwidth is just one of several that are commonly used. Strictly speaking, B in Eq. (14.35) is a half-power bandwidth, because it is the width of the frequency band between the half-power frequencies.

The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the *quality factor* Q. At resonance, the reactive energy

Although the same symbol Q is used for the reactive power, the two are not equal and should not be confused. Q here is dimensionless, whereas reactive power Q is in VAR. This may help distinguish between the two.

in the circuit oscillates between the inductor and the capacitor. The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \tag{14.36}$$

It is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property. In the series RLC circuit, the peak energy stored is $\frac{1}{2}LI^2$, while the energy dissipated in one period is $\frac{1}{2}(I^2R)(1/f_0)$. Hence,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R} \tag{14.37}$$

or

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \tag{14.38}$$

Notice that the quality factor is dimensionless. The relationship between the bandwidth B and the quality factor Q is obtained by substituting Eq. (14.33) into Eq. (14.35) and utilizing Eq. (14.38).

$$B = \frac{R}{L} = \frac{\omega_0}{Q} \tag{14.39}$$

or $B = \omega_0^2 CR$. Thus

The **quality factor** of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

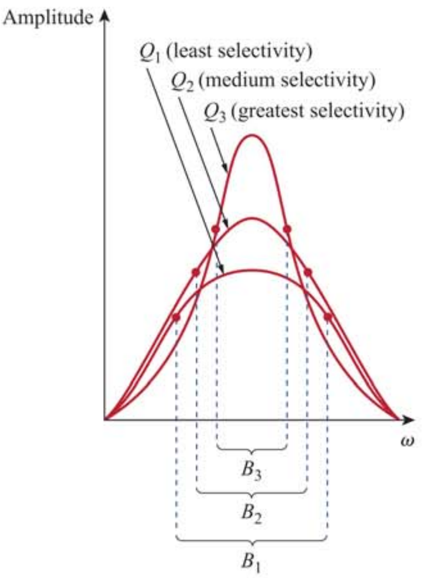


Figure 14.23
The higher the circuit Q, the smaller the bandwidth.

The quality factor is a measure of the selectivity (or "sharpness" of resonance) of the circuit.

Keep in mind that Eqs. (14.33), (14.38), and (14.39) only apply to a series RLC circuit.

As illustrated in Fig. 14.23, the higher the value of Q, the more selective the circuit is but the smaller the bandwidth. The *selectivity* of an RLC circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high. If the band of frequencies is wide, the quality factor must be low.

A resonant circuit is designed to operate at or near its resonant frequency. It is said to be a *high-Q circuit* when its quality factor is equal to or greater than 10. For high-Q circuits ($Q \geq 10$), the half-power frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2} \tag{14.40}$$

High-Q circuits are used often in communications networks.

We see that a resonant circuit is characterized by five related parameters: the two half-power frequencies ω_1 and ω_2, the resonant frequency ω_0, the bandwidth B, and the quality factor Q.

Example 14.7

In the circuit of Fig. 14.24, $R = 2\ \Omega$, $L = 1$ mH, and $C = 0.4\ \mu$F. (a) Find the resonant frequency and the half-power frequencies. (b) Calculate the quality factor and bandwidth. (c) Determine the amplitude of the current at ω_0, ω_1, and ω_2.

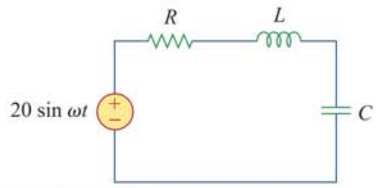


Figure 14.24
For Example 14.7.

Solution:

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/s}$$

■ **METHOD 1** The lower half-power frequency is

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$= -\frac{2}{2 \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2}$$
$$= -1 + \sqrt{1 + 2500} \text{ krad/s} = 49 \text{ krad/s}$$

Similarly, the upper half-power frequency is

$$\omega_2 = 1 + \sqrt{1 + 2500} \text{ krad/s} = 51 \text{ krad/s}$$

(b) The bandwidth is

$$B = \omega_2 - \omega_1 = 2 \text{ krad/s}$$

or

$$B = \frac{R}{L} = \frac{2}{10^{-3}} = 2 \text{ krad/s}$$

The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

■ **METHOD 2** Alternatively, we could find

$$Q = \frac{\omega_0 L}{R} = \frac{50 \times 10^3 \times 10^{-3}}{2} = 25$$

From Q, we find

$$B = \frac{\omega_0}{Q} = \frac{50 \times 10^3}{25} = 2 \text{ krad/s}$$

Since $Q > 10$, this is a high-Q circuit and we can obtain the half-power frequencies as

$$\omega_1 = \omega_0 - \frac{B}{2} = 50 - 1 = 49 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 50 + 1 = 51 \text{ krad/s}$$

as obtained earlier.

(c) At $\omega = \omega_0$,

$$I = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

At $\omega = \omega_1, \omega_2$,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{10}{\sqrt{2}} = 7.071 \text{ A}$$

Practice Problem 14.7

A series-connected circuit has $R = 4\ \Omega$ and $L = 25$ mH. (a) Calculate the value of C that will produce a quality factor of 50. (b) Find ω_1, ω_2, and B. (c) Determine the average power dissipated at $\omega = \omega_0, \omega_1, \omega_2$. Take $V_m = 100$ V.

Answer: (a) 0.625 μF, (b) 7920 rad/s, 8080 rad/s, 160 rad/s, (c) 1.25 kW, 0.625 kW, 0.625 kW.

14.6 Parallel Resonance

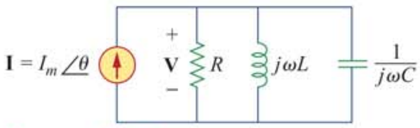


Figure 14.25
The parallel resonant circuit.

The parallel *RLC* circuit in Fig. 14.25 is the dual of the series *RLC* circuit. So we will avoid needless repetition. The admittance is

$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \tag{14.41}$$

or

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \tag{14.42}$$

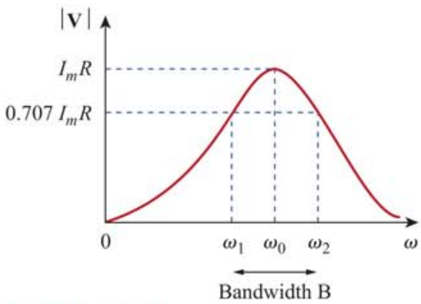


Figure 14.26
The current amplitude versus frequency for the series resonant circuit of Fig. 14.25.

Resonance occurs when the imaginary part of **Y** is zero,

$$\omega C - \frac{1}{\omega L} = 0 \tag{14.43}$$

or

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}} \tag{14.44}$$

which is the same as Eq. (14.26) for the series resonant circuit. The voltage $|\mathbf{V}|$ is sketched in Fig. 14.26 as a function of frequency. Notice that at resonance, the parallel *LC* combination acts like an open circuit, so that the entire current flows through *R*. Also, the inductor and capacitor current can be much more than the source current at resonance.

We can see this from the fact that

$$|\mathbf{I}_L| = \frac{I_m R}{\omega_0 L} = QI_m$$

$$|\mathbf{I}_C| = \omega_0 C I_m R = QI_m$$

where Q is the quality factor, defined in Eq. (14.47).

We exploit the duality between Figs. 14.21 and 14.25 by comparing Eq. (14.42) with Eq. (14.23). By replacing *R*, *L*, and *C* in the expressions for the series circuit with $1/R$, C, and L respectively, we obtain for the parallel circuit

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$
$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad \textbf{(14.45)}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC} \qquad \textbf{(14.46)}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L} \qquad \textbf{(14.47)}$$

It should be noted that Eqs. (14.45) to (14.47) apply only to a parallel *RLC* circuit. Using Eqs. (14.45) and (14.47), we can express the half-power frequencies in terms of the quality factor. The result is

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \qquad \omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q} \qquad \textbf{(14.48)}$$

Again, for high-Q circuits ($Q \geq 10$)

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2} \qquad \textbf{(14.49)}$$

Table 14.4 presents a summary of the characteristics of the series and parallel resonant circuits. Besides the series and parallel *RLC* considered here, other resonant circuits exist. Example 14.9 treats a typical example.

TABLE 14.4

Summary of the characteristics of resonant *RLC* circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

Example 14.8

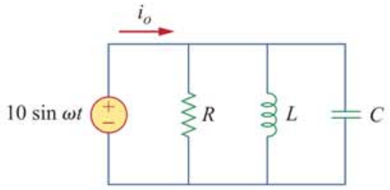


Figure 14.27
For Example 14.8.

In the parallel *RLC* circuit of Fig. 14.27, let $R = 8\ \text{k}\Omega$, $L = 0.2$ mH, and $C = 8\ \mu$F. (a) Calculate ω_0, Q, and B. (b) Find ω_1 and ω_2. (c) Determine the power dissipated at ω_0, ω_1, and ω_2.

Solution:

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = \frac{10^5}{4} = 25 \text{ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{8 \times 10^3}{25 \times 10^3 \times 0.2 \times 10^{-3}} = 1{,}600$$

$$B = \frac{\omega_0}{Q} = 15.625 \text{ rad/s}$$

(b) Due to the high value of Q, we can regard this as a high-Q circuit, Hence,

$$\omega_1 = \omega_0 - \frac{B}{2} = 25{,}000 - 7.812 = 24{,}992 \text{ rad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 25{,}000 + 7.812 = 25{,}008 \text{ rad/s}$$

(c) At $\omega = \omega_0$, $\mathbf{Y} = 1/R$ or $\mathbf{Z} = R = 8\ \text{k}\Omega$. Then

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\underline{/-90^\circ}}{8{,}000} = 1.25\underline{/-90^\circ} \text{ mA}$$

Since the entire current flows through R at resonance, the average power dissipated at $\omega = \omega_0$ is

$$P = \frac{1}{2}|\mathbf{I}_o|^2 R = \frac{1}{2}(1.25 \times 10^{-3})^2 (8 \times 10^3) = 6.25 \text{ mW}$$

or

$$P = \frac{V_m^2}{2R} = \frac{100}{2 \times 8 \times 10^3} = 6.25 \text{ mW}$$

At $\omega = \omega_1, \omega_2$,

$$P = \frac{V_m^2}{4R} = 3.125 \text{ mW}$$

Practice Problem 14.8

A parallel resonant circuit has $R = 100\ \text{k}\Omega$, $L = 20$ mH, and $C = 5$ nF. Calculate ω_0, ω_1, ω_2, Q, and B.

Answer: 100 krad/s, 99 krad/s, 101 krad/s, 50, 2 krad/s.

Example 14.9

Determine the resonant frequency of the circuit in Fig. 14.28.

Solution:
The input admittance is

$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$ and

$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \quad \Rightarrow \quad \omega_0 = 2 \text{ rad/s}$$

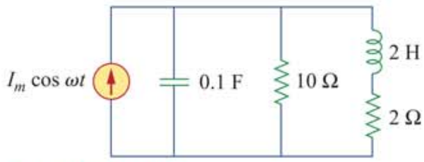


Figure 14.28
For Example 14.9.

Practice Problem 14.9

Calculate the resonant frequency of the circuit in Fig. 14.29.

Answer: 100 rad/s.

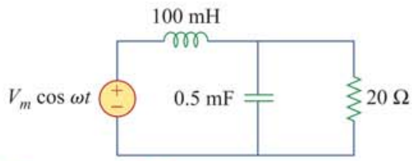


Figure 14.29
For Practice Prob. 14.9

14.7 Passive Filters

The concept of filters has been an integral part of the evolution of electrical engineering from the beginning. Several technological achievements would not have been possible without electrical filters. Because of this prominent role of filters, much effort has been expended on the theory, design, and construction of filters and many articles and books have been written on them. Our discussion in this chapter should be considered introductory.

> A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment.

A filter is a *passive filter* if it consists of only passive elements *R*, *L*, and *C*. It is said to be an *active filter* if it consists of active elements (such as transistors and op amps) in addition to passive elements *R*, *L*, and *C*. We consider passive filters in this section and active filters in the next section. *LC* filters have been used in practical applications for more than eight decades. *LC* filter technology feeds related areas such as equalizers, impedance-matching networks, transformers, shaping networks, power dividers, attenuators, and directional couplers, and is continuously providing practicing engineers with oppurtunities to innovate and experiment. Besides the *LC* filters we study in these sections, there are other kinds of filters—such as digital filters, electromechanical filters, and microwave filters—which are beyond the level of this text.

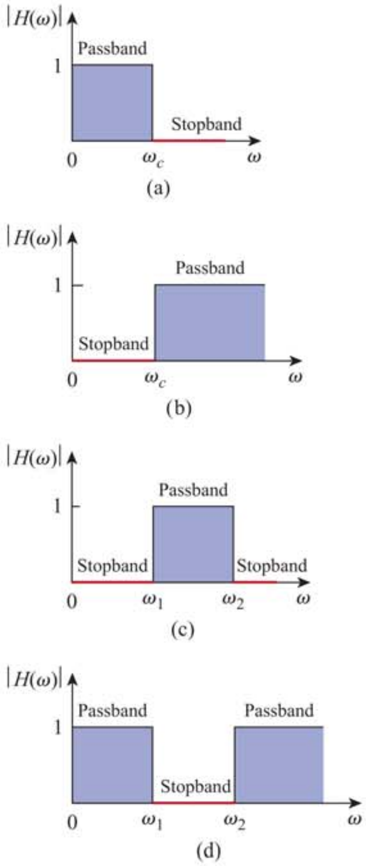


Figure 14.30
Ideal frequency response of four types of filter: (a) lowpass filter, (b) highpass filter, (c) bandpass filter, (d) bandstop filter.

As shown in Fig. 14.30, there are four types of filters whether passive or active:

1. A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig. 14.30(a).
2. A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig. 14.30(b).
3. A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. 14.30(c).
4. A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. 14.30(d).

Table 14.5 presents a summary of the characteristics of these filters. Be aware that the characteristics in Table 14.5 are only valid for first- or second-order filters—but one should not have the impression that only these kinds of filter exist. We now consider typical circuits for realizing the filters shown in Table 14.5.

TABLE 14.5

Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

14.7.1 Lowpass Filter

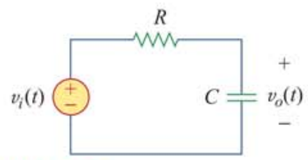


Figure 14.31
A lowpass filter.

A typical lowpass filter is formed when the output of an *RC* circuit is taken off the capacitor as shown in Fig. 14.31. The transfer function (see also Example 14.1) is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC} \tag{14.50}$$

Note that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 0$. Figure 14.32 shows the plot of $|H(\omega)|$, along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the *cutoff frequency* ω_c, is obtained by setting the magnitude of $\mathbf{H}(\omega)$ equal to $1/\sqrt{2}$, thus,

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

or

$$\omega_c = \frac{1}{RC} \tag{14.51}$$

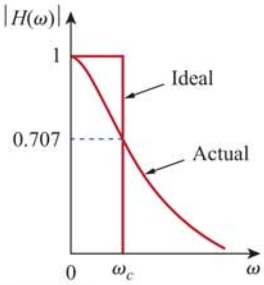


Figure 14.32
Ideal and actual frequency response of a lowpass filter.

The cutoff frequency is also called the *rolloff frequency.*

> A **lowpass filter** is designed to pass only frequencies from dc up to the cutoff frequency ω_c.

The cutoff frequency is the frequency at which the transfer function **H** drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters.

14.7.2. Highpass Filter

A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig. 14.33. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} \tag{14.52}$$

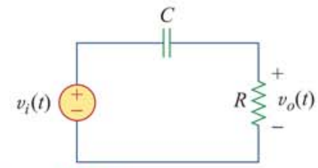


Figure 14.33
A highpass filter.

Note that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 1$. Figure 14.34 shows the plot of $|H(\omega)|$. Again, the corner or cutoff frequency is

$$\omega_c = \frac{1}{RC} \tag{14.53}$$

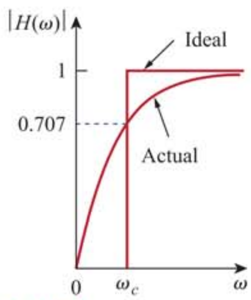


Figure 14.34
Ideal and actual frequency response of a highpass filter.

> A **highpass filter** is designed to pass all frequencies above its cutoff frequency ω_c.

A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.

14.7.3 Bandpass Filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. 14.35. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)} \tag{14.54}$$

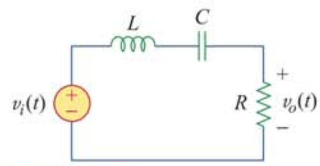


Figure 14.35
A bandpass filter.

We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. Figure 14.36 shows the plot of $|H(\omega)|$. The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_0, the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{14.55}$$

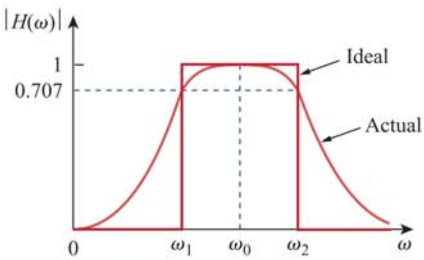


Figure 14.36
Ideal and actual frequency response of a bandpass filter.

> A **bandpass filter** is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

Since the bandpass filter in Fig. 14.35 is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined as in Section 14.5. A bandpass filter can also be formed by cascading the lowpass filter (where $\omega_2 = \omega_c$) in Fig. 14.31 with the highpass filter (where $\omega_1 = \omega_c$) in Fig. 14.33. However, the result would not be the same as just adding the output of the lowpass filter to the input of the highpass filter, because one circuit loads the other and alters the desired transfer function.

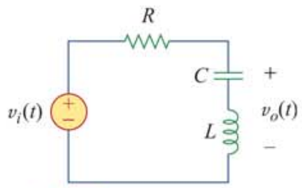


Figure 14.37
A bandstop filter.

14.7.4 Bandstop Filter

A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *bandstop, band-reject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. 14.37. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \qquad \textbf{(14.56)}$$

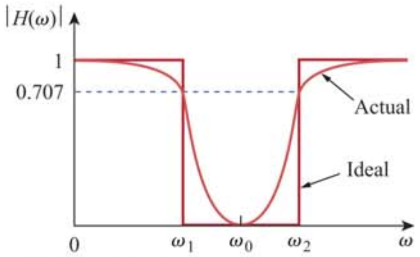


Figure 14.38
Ideal and actual frequency response of a bandstop filter.

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 14.38 shows the plot of $|H(\omega)|$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \textbf{(14.57)}$$

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section 14.5 for a series resonant circuit. Here, ω_0 is called the *frequency of rejection*, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the *bandwidth of rejection*. Thus,

> A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

Notice that adding the transfer functions of the bandpass and the bandstop gives unity at any frequency for the same values of *R*, *L*, and *C*. Of course, this is not true in general but true for the circuits treated here. This is due to the fact that the characteristic of one is the inverse of the other.

In concluding this section, we should note that:

1. From Eqs. (14.50), (14.52), (14.54), and (14.56), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter as the next section shows.
2. There are other ways to get the types of filters treated in this section.
3. The filters treated here are the simple types. Many other filters have sharper and complex frequency responses.

Example 14.10

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2\ \text{k}\Omega$, $L = 2$ H, and $C = 2\ \mu$F.

Solution:
The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \,\|\, 1/sC}{sL + R \,\|\, 1/sC}, \qquad s = j\omega \qquad \textbf{(14.10.1)}$$

But

$$R \left\| \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}\right.$$

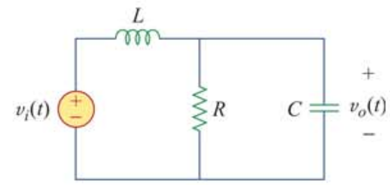


Figure 14.39
For Example 14.10.

Substituting this into Eq. (14.10.1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R} \tag{14.10.2}$$

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}} \tag{14.10.3}$$

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c\, 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

Practice Problem 14.10

For the circuit in Fig. 14.40, obtain the transfer function $\mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$. Identify the type of filter the circuit represents and determine the corner frequency. Take $R_1 = 100\ \Omega = R_2$, $L = 2$ mH.

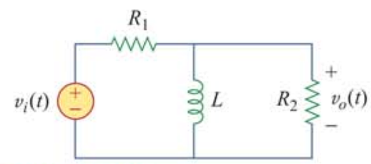


Figure 14.40
For Practice Prob. 14.10.

Answer: $\dfrac{R_2}{R_1 + R_2}\left(\dfrac{j\omega}{j\omega + \omega_c}\right)$, highpass filter

$$\omega_c = \frac{R_1 R_2}{(R_1 + R_2)L} = 25 \text{ krad/s}.$$

Example 14.11

If the bandstop filter in Fig. 14.37 is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C. Take $R = 150\ \Omega$ and the bandwidth as 100 Hz.

Solution:
We use the formulas for a series resonant circuit in Section 14.5.

$$B = 2\pi(100) = 200\pi \text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387 \text{ H}$$

Rejection of the 200-Hz sinusoid means that f_0 is 200 Hz, so that ω_0 in Fig. 14.38 is

$$\omega_0 = 2\pi f_0 = 2\pi(200) = 400\pi$$

Since $\omega_0 = 1/\sqrt{LC}$,

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.653\ \mu\text{F}$$

Practice Problem 14.11

Design a bandpass filter of the form in Fig. 14.35 with a lower cutoff frequency of 20.1 kHz and an upper cutoff frequency of 20.3 kHz. Take $R = 20\ \text{k}\Omega$. Calculate L, C, and Q.

Answer: 15.92 H, 3.9 pF, 101.

14.8 Active Filters

There are three major limitations to the passive filters considered in the previous section. First, they cannot generate gain greater than 1; passive elements cannot add energy to the network. Second, they may require bulky and expensive inductors. Third, they perform poorly at frequencies below the audio frequency range (300 Hz $< f <$ 3,000 Hz). Nevertheless, passive filters are useful at high frequencies.

Active filters consist of combinations of resistors, capacitors, and op amps. They offer some advantages over passive *RLC* filters. First, they are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters. Second, they can provide amplifier gain in addition to providing the same frequency response as *RLC* filters. Third, active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function. (Bode plots, being logarithmic, may be added when transfer functions are cascaded.) However, active filters are less reliable and less stable. The practical limit of most active filters is about 100 kHz—most active filters operate well below that frequency.

Filters are often classified according to their order (or number of poles) or their specific design type.

14.8.1 First-Order Lowpass Filter

One type of first-order filter is shown in Fig. 14.41. The components selected for Z_i and Z_f determine whether the filter is lowpass or highpass, but one of the components must be reactive.

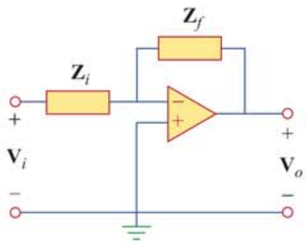


Figure 14.41
A general first-order active filter.

Figure 14.42 shows a typical active lowpass filter. For this filter, the transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \tag{14.58}$$

where $\mathbf{Z}_i = R_i$ and

$$\mathbf{Z}_f = R_f \left\| \frac{1}{j\omega C_f} \right. = \frac{R_f/j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f} \tag{14.59}$$

Therefore,

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f} \tag{14.60}$$

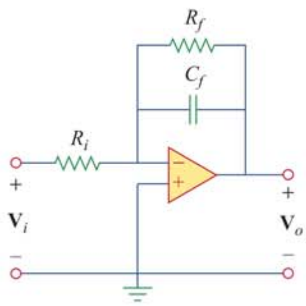


Figure 14.42
Active first-order lowpass filter.

We notice that Eq. (14.60) is similar to Eq. (14.50), except that there is a low frequency ($\omega \to 0$) gain or dc gain of $-R_f/R_i$. Also, the corner frequency is

$$\omega_c = \frac{1}{R_f C_f} \tag{14.61}$$

which does not depend on R_i. This means that several inputs with different R_i could be summed if required, and the corner frequency would remain the same for each input.

14.8.2 First-Order Highpass Filter

Figure 14.43 shows a typical highpass filter. As before,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \tag{14.62}$$

where $\mathbf{Z}_i = R_i + 1/j\omega C_i$ and $\mathbf{Z}_f = R_f$ so that

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i} \tag{14.63}$$

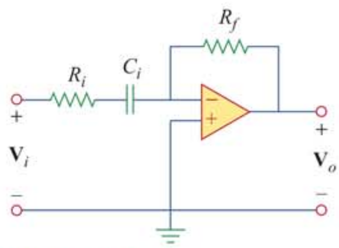


Figure 14.43
Active first-order highpass filter.

This is similar to Eq. (14.52), except that at very high frequencies ($\omega \to \infty$), the gain tends to $-R_f/R_i$. The corner frequency is

$$\omega_c = \frac{1}{R_i C_i} \tag{14.64}$$

14.8.3 Bandpass Filter

The circuit in Fig. 14.42 may be combined with that in Fig. 14.43 to form a bandpass filter that will have a gain K over the required range of frequencies. By cascading a unity-gain lowpass filter, a unity-gain

This way of creating a bandpass filter, not necessarily the best, is perhaps the easiest to understand.

highpass filter, and an inverter with gain $-R_f/R_i$, as shown in the block diagram of Fig. 14.44(a), we can construct a bandpass filter whose frequency response is that in Fig. 14.44(b). The actual construction of the bandpass filter is shown in Fig. 14.45.

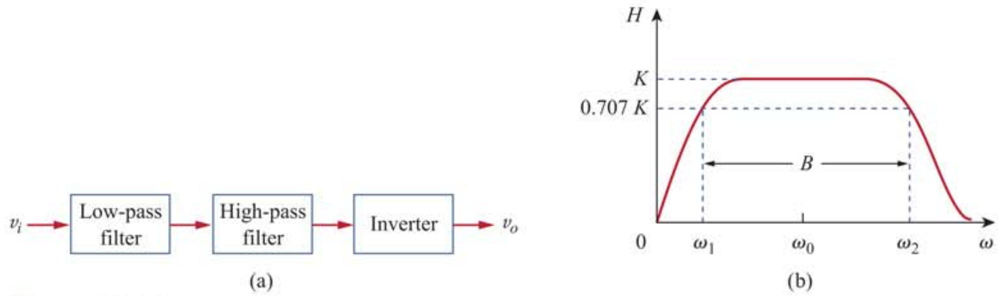


Figure 14.44
Active bandpass filter: (a) block diagram, (b) frequency response.

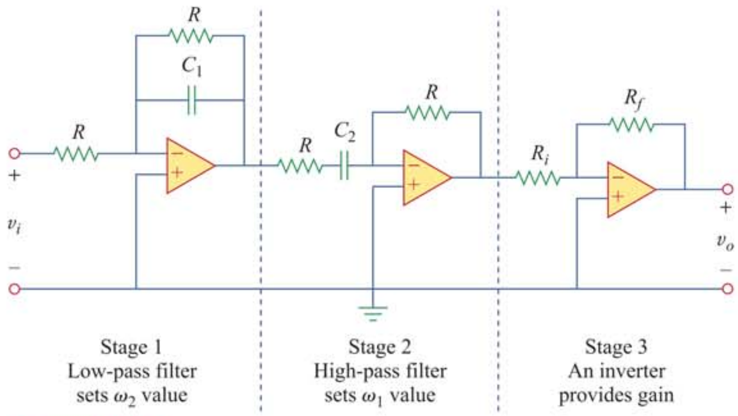


Figure 14.45
Active bandpass filter.

The analysis of the bandpass filter is relatively simple. Its transfer function is obtained by multiplying Eqs. (14.60) and (14.63) with the gain of the inverter; that is,

$$\begin{aligned} \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} &= \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right) \\ &= -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R} \end{aligned} \tag{14.65}$$

The lowpass section sets the upper corner frequency as

$$\omega_2 = \frac{1}{RC_1} \tag{14.66}$$

while the highpass section sets the lower corner frequency as

$$\omega_1 = \frac{1}{RC_2} \tag{14.67}$$

With these values of ω_1 and ω_2, the center frequency, bandwidth, and quality factor are found as follows:

$$\omega_0 = \sqrt{\omega_1 \omega_2} \tag{14.68}$$

$$B = \omega_2 - \omega_1 \tag{14.69}$$

$$Q = \frac{\omega_0}{B} \tag{14.70}$$

To find the passband gain K, we write Eq. (14.65) in the standard form of Eq. (14.15),

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)} = -\frac{R_f}{R_i}\frac{j\omega\omega_2}{(\omega_1 + j\omega)(\omega_2 + j\omega)} \tag{14.71}$$

At the center frequency $\omega_0 = \sqrt{\omega_1\omega_2}$, the magnitude of the transfer function is

$$|\mathbf{H}(\omega_0)| = \left|\frac{R_f}{R_i}\frac{j\omega_0\omega_2}{(\omega_1 + j\omega_0)(\omega_2 + j\omega_0)}\right| = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2} \tag{14.72}$$

Thus, the passband gain is

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2} \tag{14.73}$$

14.8.4 Bandreject (or Notch) Filter

A bandreject filter may be constructed by parallel combination of a lowpass filter and a highpass filter and a summing amplifier, as shown in the block diagram of Fig. 14.46(a). The circuit is designed such that the lower cutoff frequency ω_1 is set by the lowpass filter while the upper cutoff frequency ω_2 is set by the highpass filter. The gap between ω_1 and ω_2 is the bandwidth of the filter. As shown in Fig. 14.46(b), the filter passes frequencies below ω_1 and above ω_2. The block diagram in Fig. 14.46(a) is actually constructed as shown in Fig. 14.47. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}\right) \tag{14.74}$$

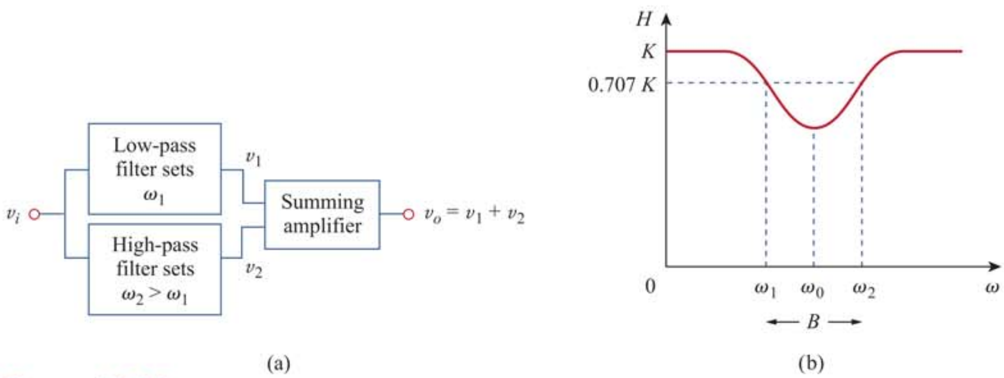


Figure 14.46
Active bandreject filter: (a) block diagram, (b) frequency response.

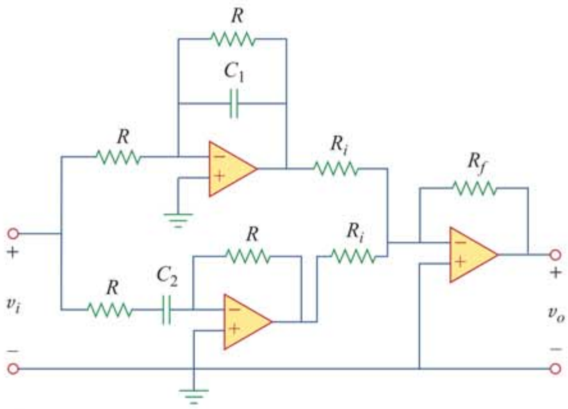


Figure 14.47
Active bandreject filter.

The formulas for calculating the values of ω_1, ω_2, the center frequency, bandwidth, and quality factor are the same as in Eqs. (14.66) to (14.70).

To determine the passband gain K of the filter, we can write Eq. (14.74) in terms of the upper and lower corner frequencies as

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{R_f}{R_i}\left(\frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1}\right) \\ &= \frac{R_f}{R_i}\frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)}\end{aligned} \quad \textbf{(14.75)}$$

Comparing this with the standard form in Eq. (14.15) indicates that in the two passbands ($\omega \to 0$ and $\omega \to \infty$) the gain is

$$K = \frac{R_f}{R_i} \quad \textbf{(14.76)}$$

We can also find the gain at the center frequency by finding the magnitude of the transfer function at $\omega_0 = \sqrt{\omega_1\omega_2}$, writing

$$\begin{aligned}H(\omega_0) &= \left|\frac{R_f}{R_i}\frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)}\right| \\ &= \frac{R_f}{R_i}\frac{2\omega_1}{\omega_1 + \omega_2}\end{aligned} \quad \textbf{(14.77)}$$

Again, the filters treated in this section are only typical. There are many other active filters that are more complex.

Example 14.12

Design a lowpass active filter with a dc gain of 4 and a corner frequency of 500 Hz.

Solution:

From Eq. (14.61), we find

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f} \quad \textbf{(14.12.1)}$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4 \tag{14.12.2}$$

We have two equations and three unknowns. If we select $C_f = 0.2\ \mu$F, then

$$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59\text{ k}\Omega$$

and

$$R_i = \frac{R_f}{4} = 397.5\ \Omega$$

We use a 1.6-kΩ resistor for R_f and a 400-Ω resistor for R_i. Figure 14.42 shows the filter.

Practice Problem 14.12

Design a highpass filter with a high-frequency gain of 5 and a corner frequency of 2 kHz. Use a 0.1-μF capacitor in your design.

Answer: $R_i = 800\ \Omega$ and $R_f = 4\text{ k}\Omega$.

Example 14.13

Design a bandpass filter in the form of Fig. 14.45 to pass frequencies between 250 Hz and 3,000 Hz and with $K = 10$. Select $R = 20\text{ k}\Omega$.

Solution:

1. **Define.** The problem is clearly stated and the circuit to be used in the design is specified.
2. **Present.** We are asked to use the op amp circuit specified in Fig. 14.45 to design a bandpass filter. We are given the value of R to use (20 kΩ). In addition, the frequency range of the signals to be passed is 250 Hz to 3 kHz.
3. **Alternative.** We will use the equations developed in Section 14.8.3 to obtain a solution. We will then use the resulting transfer function to validate the answer.
4. **Attempt.** Since $\omega_1 = 1/RC_2$, we obtain

$$C_2 = \frac{1}{R\omega_1} = \frac{1}{2\pi f_1 R} = \frac{1}{2\pi \times 250 \times 20 \times 10^3} = \mathbf{31.83\ nF}$$

Similarly, since $\omega_2 = 1/RC_1$,

$$C_1 = \frac{1}{R\omega_2} = \frac{1}{2\pi f_2 R} = \frac{1}{2\pi \times 3{,}000 \times 20 \times 10^3} = \mathbf{2.65\ nF}$$

From Eq. (14.73),

$$\frac{R_f}{R_i} = K\frac{\omega_1 + \omega_2}{\omega_2} = K\frac{f_1 + f_2}{f_2} = \frac{10(3{,}250)}{3{,}000} = 10.83$$

If we select $R_i = \mathbf{10\ k\Omega}$, then $R_f = 10.83R_i \simeq \mathbf{108.3\ k\Omega}$.

5. **Evaluate.** The output of the first op amp is given by

$$\frac{V_i - 0}{20\ \text{k}\Omega} + \frac{V_1 - 0}{20\ \text{k}\Omega} + \frac{s2.65 \times 10^{-9}(V_1 - 0)}{1}$$

$$= 0 \rightarrow V_1 = -\frac{V_i}{1 + 5.3 \times 10^{-5}s}$$

The output of the second op amp is given by

$$\frac{V_1 - 0}{20\ \text{k}\Omega + \dfrac{1}{s31.83\ \text{nF}}} + \frac{V_2 - 0}{20\ \text{k}\Omega} = 0 \rightarrow$$

$$V_2 = -\frac{6.366 \times 10^{-4}sV_1}{1 + 6.366 \times 10^{-4}s}$$

$$= \frac{6.366 \times 10^{-4}sV_i}{(1 + 6.366 \times 10^{-4}s)(1 + 5.3 \times 10^{-5}s)}$$

The output of the third op amp is given by

$$\frac{V_2 - 0}{10\ \text{k}\Omega} + \frac{V_o - 0}{108.3\ \text{k}\Omega} = 0 \rightarrow V_o = 10.83V_2 \rightarrow j2\pi \times 25°$$

$$V_o = -\frac{6.894 \times 10^{-3}sV_i}{(1 + 6.366 \times 10^{-4}s)(1 + 5.3 \times 10^{-5}s)}$$

Let $j2\pi \times 25°$ and solve for the magnitude of V_o/V_i.

$$\frac{V_o}{V_i} = \frac{-j10.829}{(1 + j1)(1)}$$

$|V_o/V_i| = \mathbf{(0.7071)10.829}$, which is the lower corner frequency point.

Let $s = j2\pi \times 3000 = j18.849\ \text{k}\Omega$. We then get

$$\frac{V_o}{V_i} = \frac{-j129.94}{(1 + j12)(1 + j1)}$$

$$= \frac{129.94\underline{/-90°}}{(12.042\underline{/85.24°})(1.4142\underline{/45°})} = \mathbf{(0.7071)10.791\underline{/-18.61°}}$$

Clearly this is the upper corner frequency and the answer checks.

6. **Satisfactory?** We have satisfactorily designed the circuit and can present the results as a solution to the problem.

Practice Problem 14.13

Design a notch filter based on Fig. 14.47 for $\omega_0 = 20$ krad/s, $K = 5$, and $Q = 10$. Use $R = R_i = 10\ \text{k}\Omega$.

Answer: $C_1 = 4.762$ nF, $C_2 = 5.263$ nF, and $R_f = 50\ \text{k}\Omega$.

14.9 Scaling

In designing and analyzing filters and resonant circuits or in circuit analysis in general, it is sometimes convenient to work with element values of 1 Ω, 1 H, or 1 F, and then transform the values to realistic values by *scaling*. We have taken advantage of this idea by not using realistic element values in most of our examples and problems; mastering circuit analysis is made easy by using convenient component values. We have thus eased calculations, knowing that we could use scaling to then make the values realistic.

There are two ways of scaling a circuit: *magnitude* or *impedance scaling*, and *frequency scaling*. Both are useful in scaling responses and circuit elements to values within the practical ranges. While magnitude scaling leaves the frequency response of a circuit unaltered, frequency scaling shifts the frequency response up or down the frequency spectrum.

14.9.1 Magnitude Scaling

> **Magnitude scaling** is the process of increasing all impedances in a network by a factor, the frequency response remaining unchanged.

Recall that impedances of individual elements R, L, and C are given by

$$\mathbf{Z}_R = R, \qquad \mathbf{Z}_L = j\omega L, \qquad \mathbf{Z}_C = \frac{1}{j\omega C} \tag{14.78}$$

In magnitude scaling, we multiply the impedance of each circuit element by a factor K_m and let the frequency remain constant. This gives the new impedances as

$$\begin{gathered}\mathbf{Z}'_R = K_m\mathbf{Z}_R = K_mR, \qquad \mathbf{Z}'_L = K_m\mathbf{Z}_L = j\omega K_mL \\ \mathbf{Z}'_C = K_m\mathbf{Z}_C = \frac{1}{j\omega C/K_m}\end{gathered} \tag{14.79}$$

Comparing Eq. (14.79) with Eq. (14.78), we notice the following changes in the element values: $R \rightarrow K_mR$, $L \rightarrow K_mL$, and $C \rightarrow C/K_m$. Thus, in magnitude scaling, the new values of the elements and frequency are

$$\boxed{\begin{aligned} R' &= K_mR, \qquad & L' &= K_mL \\ C' &= \frac{C}{K_m}, \qquad & \omega' &= \omega \end{aligned}} \tag{14.80}$$

The primed variables are the new values and the unprimed variables are the old values. Consider the series or parallel RLC circuit. We now have

$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_mLC/K_m}} = \frac{1}{\sqrt{LC}} = \omega_0 \tag{14.81}$$

showing that the resonant frequency, as expected, has not changed. Similarly, the quality factor and the bandwidth are not affected by magnitude scaling. Also, magnitude scaling does not affect transfer functions in the forms of Eqs. (14.2a) and (14.2b), which are dimensionless quantities.

14.9.2 Frequency Scaling

Frequency scaling is equivalent to relabeling the frequency axis of a frequency response plot. It is needed when translating frequencies such as a resonant frequency, a corner frequency, a bandwidth, etc., to a realistic level. It can be used to bring capacitance and inductance values into a range that is convenient to work with.

Frequency scaling is the process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

We achieve frequency scaling by multiplying the frequency by a factor K_f while keeping the impedance the same.

From Eq. (14.78), we see that the impedances of L and C are frequency-dependent. If we apply frequency scaling to $\mathbf{Z}_L(\omega)$ and $\mathbf{Z}_C(\omega)$ in Eq. (14.78), we obtain

$$\mathbf{Z}_L = j(\omega K_f)L' = j\omega L \quad \Rightarrow \quad L' = \frac{L}{K_f} \tag{14.82a}$$

$$\mathbf{Z}_C = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C} \quad \Rightarrow \quad C' = \frac{C}{K_f} \tag{14.82b}$$

since the impedance of the inductor and capacitor must remain the same after frequency scaling. We notice the following changes in the element values: $L \rightarrow L/K_f$ and $C \rightarrow C/K_f$. The value of R is not affected, since its impedance does not depend on frequency. Thus, in frequency scaling, the new values of the elements and frequency are

$$\boxed{\begin{array}{ll} R' = R, & L' = \dfrac{L}{K_f} \\ C' = \dfrac{C}{K_f}, & \omega' = K_f\omega \end{array}} \tag{14.83}$$

Again, if we consider the series or parallel RLC circuit, for the resonant frequency

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(L/K_f)(C/K_f)}} = \frac{K_f}{\sqrt{LC}} = K_f\omega_0 \tag{14.84}$$

and for the bandwidth

$$B' = K_f B \tag{14.85}$$

but the quality factor remains the same ($Q' = Q$).

14.9.3 Magnitude and Frequency Scaling

If a circuit is scaled in magnitude and frequency at the same time, then

$$\boxed{\begin{array}{ll} R' = K_m R, & L' = \dfrac{K_m}{K_f}L \\ C' = \dfrac{1}{K_m K_f}C, & \omega' = K_f\omega \end{array}} \tag{14.86}$$

These are more general formulas than those in Eqs. (14.80) and (14.83). We set $K_m = 1$ in Eq. (14.86) when there is no magnitude scaling or $K_f = 1$ when there is no frequency scaling.

Example 14.14

A fourth-order Butterworth lowpass filter is shown in Fig. 14.48(a). The filter is designed such that the cutoff frequency $\omega_c = 1$ rad/s. Scale the circuit for a cutoff frequency of 50 kHz using 10- kΩ resistors.

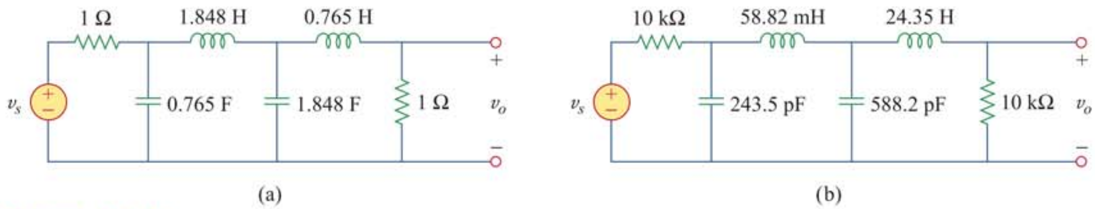


Figure 14.48
For Example 14.14: (a) Normalized Butterworth lowpass filter, (b) scaled version of the same lowpass filter.

Solution:
If the cutoff frequency is to shift from $\omega_c = 1$ rad/s to $\omega_c' = 2\pi(50)$ krad/s, then the frequency scale factor is

$$K_f = \frac{\omega_c'}{\omega_c} = \frac{100\pi \times 10^3}{1} = \pi \times 10^5$$

Also, if each 1-Ω resistor is to be replaced by a 10- kΩ resistor, then the magnitude scale factor must be

$$K_m = \frac{R'}{R} = \frac{10 \times 10^3}{1} = 10^4$$

Using Eq. (14.86),

$$L_1' = \frac{K_m}{K_f}L_1 = \frac{10^4}{\pi \times 10^5}(1.848) = 58.82 \text{ mH}$$

$$L_2' = \frac{K_m}{K_f}L_2 = \frac{10^4}{\pi \times 10^5}(0.765) = 24.35 \text{ mH}$$

$$C_1' = \frac{C_1}{K_m K_f} = \frac{0.765}{\pi \times 10^9} = 243.5 \text{ pF}$$

$$C_2' = \frac{C_2}{K_m K_f} = \frac{1.848}{\pi \times 10^9} = 588.2 \text{ pF}$$

The scaled circuit is shown in Fig. 14.48(b). This circuit uses practical values and will provide the same transfer function as the prototype in Fig. 14.48(a), but shifted in frequency.

Practice Problem 14.14

A third-order Butterworth filter normalized to $\omega_c = 1$ rad/s is shown in Fig. 14.49. Scale the circuit to a cutoff frequency of 10 kHz. Use 15-nF capacitors.

Answer: $R_1' = R_2' = 1.061$ kΩ, $C_1' = C_2' = 15$ nF, $L' = 33.77$ mH.

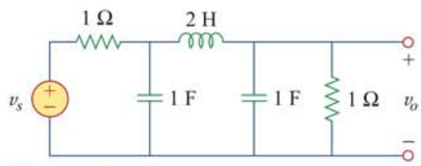


Figure 14.49
For Practice Prob. 14.14.

14.10 Frequency Response Using *PSpice*

PSpice is a useful tool in the hands of the modern circuit designer for obtaining the frequency response of circuits. The frequency response is obtained using the AC Sweep as discussed in Section D.5 (Appendix D). This requires that we specify in the AC Sweep dialog box *Total Pts, Start Freq*, *End Freq*, and the sweep type. *Total Pts* is the number of points in the frequency sweep, and *Start Freq* and *End Freq* are, respectively, the starting and final frequencies, in hertz. In order to know what frequencies to select for *Start Freq* and *End Freq*, one must have an idea of the frequency range of interest by making a rough sketch of the frequency response. In a complex circuit where this may not be possible, one may use a trial-and-error approach.

There are three types of sweeps:

Linear: The frequency is varied linearly from *Start Freq* to *End Freq* with *Total Pts* equally spaced points (or responses).

Octave: The frequency is swept logarithmically by octaves from *Start Freq* to *End Freq* with *Total Pts* per octave. An octave is a factor of 2 (e.g., 2 to 4, 4 to 8, 8 to 16).

Decade: The frequency is varied logarithmically by decades from *Start Freq* to *End Freq* with *Total Pts* per decade. A decade is a factor of 10 (e.g., from 2 Hz to 20 Hz, 20 Hz to 200 Hz, 200 Hz to 2 kHz).

It is best to use a linear sweep when displaying a narrow frequency range of interest, as a linear sweep displays the frequency range well in a narrow range. Conversely, it is best to use a logarithmic (octave or decade) sweep for displaying a wide frequency range of interest—if a linear sweep is used for a wide range, all the data will be crowded at the high- or low-frequency end and insufficient data at the other end.

With the above specifications, *PSpice* performs a steady-state sinusoidal analysis of the circuit as the frequency of all the independent sources is varied (or swept) from *Start Freq* to *End Freq*.

The *PSpice* A/D program produces a graphical output. The output data type may be specified in the *Trace Command Box* by adding one of the following suffixes to V or I:

M Amplitude of the sinusoid.

P Phase of the sinusoid.

dB Amplitude of the sinusoid in decibels, i.e., $20 \log_{10}$ (amplitude).

Example 14.15

Determine the frequency response of the circuit shown in Fig. 14.50.

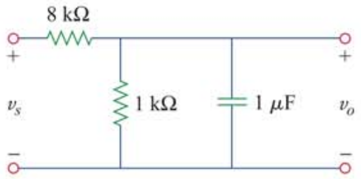


Figure 14.50
For Example 14.15.

Solution:

We let the input voltage v_s be a sinusoid of amplitude 1 V and phase 0°. Figure 14.51 is the schematic for the circuit. The capacitor is rotated 270° counterclockwise to ensure that pin 1 (the positive terminal) is on top. The voltage marker is inserted to the output voltage across the capacitor. To perform a linear sweep for $1 < f < 1000$ Hz with 50 points, we select **Analysis/Setup/AC Sweep**, **DCLICK** *Linear*, type 50 in the *Total Pts* box, type 1 in the *Start Freq* box, and type 1000 in the *End Freq* box. After saving the file, we select **Analysis/Simulate** to simulate the circuit. If there are no errors, the *PSpice* A/D window will

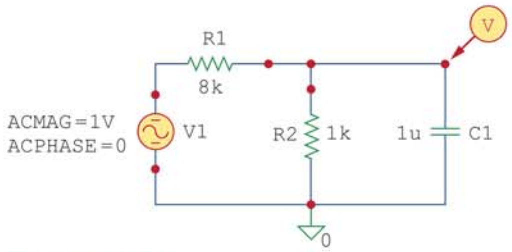


Figure 14.51
The schematic for the circuit in Fig. 14.50.

display the plot of V(C1:1), which is the same as V_o or $H(\omega) = V_o/1$, as shown in Fig. 14.52(a). This is the magnitude plot, since V(C1:1) is the same as VM(C1:1). To obtain the phase plot, select **Trace/Add** in the *PSpice* A/D menu and type VP(C1:1) in the **Trace Command** box. Figure 14.52(b) shows the result. By hand, the transfer function is

$$H(\omega) = \frac{V_o}{V_s} = \frac{1{,}000}{9{,}000 + j\omega 8}$$

or

$$H(\omega) = \frac{1}{9 + j16\pi \times 10^{-3}}$$

showing that the circuit is a lowpass filter as demonstrated in Fig. 14.52. Notice that the plots in Fig. 14.52 are similar to those in Fig. 14.3 (note that the horizontal axis in Fig. 14.52 is logrithic while the horizontal axis in Fig. 14.3 is linear.)

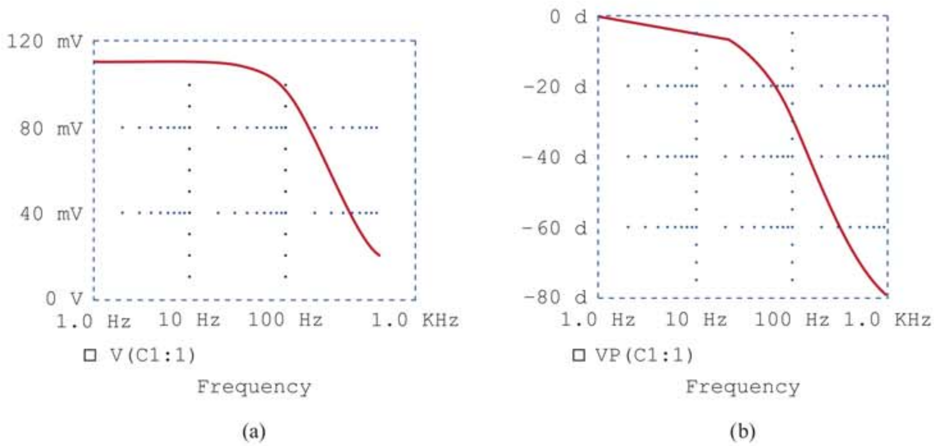


Figure 14.52
For Example 14.15: (a) magnitude plot, (b) phase plot of the frequency response.

Practice Problem 14.15

Obtain the frequency response of the circuit in Fig. 14.53 using *PSpice*. Use a linear frequency sweep and consider $1 < f < 1000$ Hz with 100 points.

Answer: See Fig. 14.54.

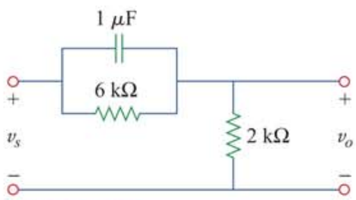
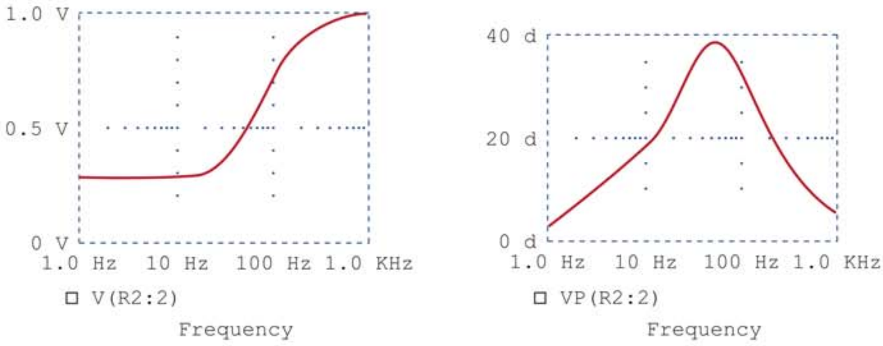


Figure 14.53
For Practice Prob. 14.15.



(a) (b)

Figure 14.54
For Practice Problem 14.15: (a) magnitude plot, (b) phase plot of the frequency response.

Example 14.16

Use *PSpice* to generate the gain and phase Bode plots of V in the circuit of Fig. 14.55.

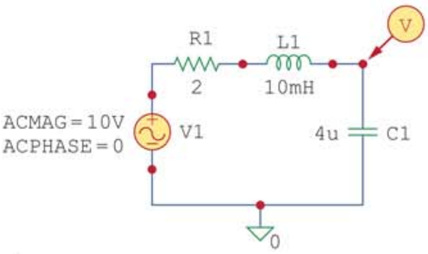


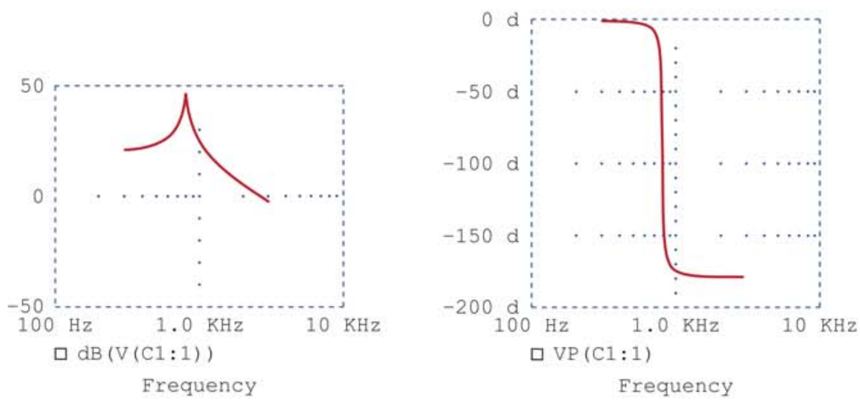
Figure 14.55
For Example 14.16.

Solution:

The circuit treated in Example 14.15 is first-order while the one in this example is second-order. Since we are interested in Bode plots, we use decade frequency sweep for $300 < f < 3{,}000$ Hz with 50 points per decade. We select this range because we know that the resonant frequency of the circuit is within the range. Recall that

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5 \text{ krad/s} \qquad \text{or} \qquad f_0 = \frac{\omega}{2\pi} = 795.8 \text{ Hz}$$

After drawing the circuit as in Fig. 14.55, we select **Analysis/Setup/AC Sweep, DCLICK** *Decade*, enter 50 in the *Total Pts* box, 300 as the *Start Freq*, and 3,000 in the *End Freq* box. Upon saving the file, we simulate it by selecting **Analysis/Simulate**. This will automatically bring up the *PSpice* A/D window and display V(C1:1) if there are no errors. Since we are interested in the Bode plot, we select **Trace/Add** in the *PSpice* A/D menu and type dB(V(C1:1)) in the **Trace Command** box. The result is the Bode magnitude plot in Fig. 14.56(a). For the



(a) (b)

Figure 14.56
For Example 14.16: (a) Bode plot, (b) phase plot of the response.

phase plot, we select **Trace/Add** in the *PSpice* A/D menu and type VP(C1:1) in the **Trace Command** box. The result is the Bode phase plot of Fig. 14.56(b). Notice that the plots confirm the resonant frequency of 795.8 Hz.

Practice Problem 14.16

Consider the network in Fig. 14.57. Use *PSpice* to obtain the Bode plots for V_o over a frequency from 1 kHz to 100 kHz using 20 points per decade.

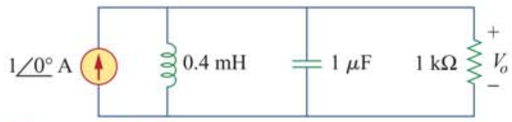


Figure 14.57
For Practice Prob. 14.16.

Answer: See Fig. 14.58.

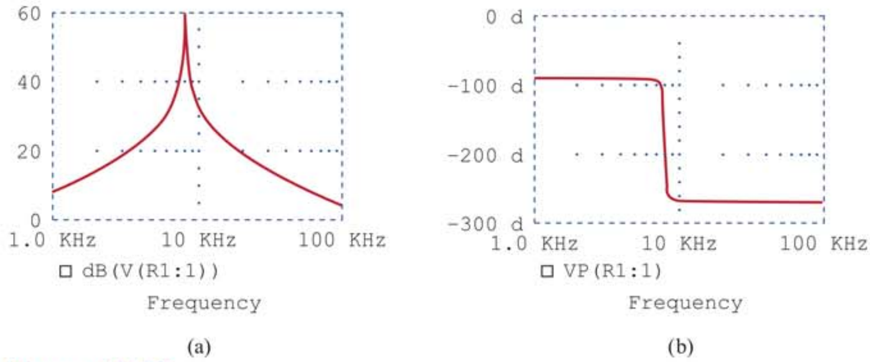


Figure 14.58
For Practice Prob. 14.16: Bode (a) magnitude plot, (b) phase plot.

14.11 Computation Using *MATLAB*

MATLAB is a software package that is widely used for engineering computation and simulation. A review of *MATLAB* is provided in Appendix E for the beginner. This section shows how to use the software to numerically perform most of the operations presented in this chapter and Chapter 15. The key to describing a system in *MATLAB* is to specify the numerator (num) and denominator (den) of the transfer function of the system. Once this is done, we can use several *MATLAB* commands to obtain the system's Bode plots (frequency response) and the system's response to a given input.

The command **bode** produces the Bode plots (both magnitude and phase) of a given transfer function $H(s)$. The format of the command is **bode** (num, den), where num is the numerator of $H(s)$ and den is its denominator. The frequency range and number of points are automatically selected. For example, consider the transfer function in Example 14.3. It is better to first write the numerator and denominator in polynomial forms.

Thus,

$$H(s) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)} = \frac{200s}{s^2 + 12s + 20}, \qquad s = j\omega$$

Using the following commands, the Bode plots are generated as shown in Fig. 14.59. If necessary, the command **logspace** can be included to generate a logarithmically spaced frequency and the command **semilogx** can be used to produce a semilog scale.

```
>> num = [200 0];     % specify the numerator of H(s)
>> den = [1 12 20];   % specify the denominator of H(s)
>> bode(num, den);    % determine and draw Bode plots
```

The step response $y(t)$ of a system is the output when the input $x(t)$ is the unit step function. The command **step** plots the step response of a system given the numerator and denominator of the transfer function of that system. The time range and number of points are automatically selected. For example, consider a second-order system with the transfer function

$$H(s) = \frac{12}{s^2 + 3s + 12}$$

We obtain the step response of the system shown in Fig. 14.60 by using the following commands.

```
>> n = 12;
>> d = [1 3 12];
>> step(n,d);
```

We can verify the plot in Fig. 14.60 by obtaining $y(t) = x(t) * u(t)$ or $Y(s) = X(s)H(s)$.

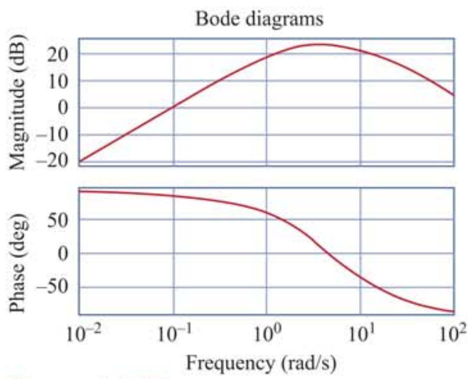


Figure 14.59
Magnitude and phase plots.

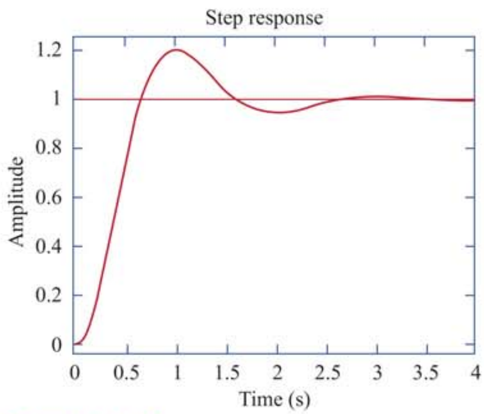


Figure 14.60
The Step response of $H(s) = 12/(s^2 + 3s + 12)$.

The command **lsim** is a more general command than **step**. It calculates the time response of a system to any arbitrary input signal. The format of the command is $y =$ **lsim** (num, den, x, t), where $x(t)$ is the input signal, t is the time vector, and $y(t)$ is the output generated. For example, assume a system is described by the transfer function

$$H(s) = \frac{s + 4}{s^3 + 2s^2 + 5s + 10}$$

To find the response $y(t)$ of the system to input $x(t) = 10e^{-t}u(t)$, we use the following *MATLAB* commands. Both the response $y(t)$ and the input $x(t)$ are plotted in Fig. 14.61.

```
>> t = 0:0.02:5; % time vector 0 < t < 5 with increment
      0.02
>> x = 10*exp(-t);
>> num = [1  4];
>> den = [1  2  5  10];
>> y = lsim(num,den,x,t);
>> plot(t,x,t,y)
```

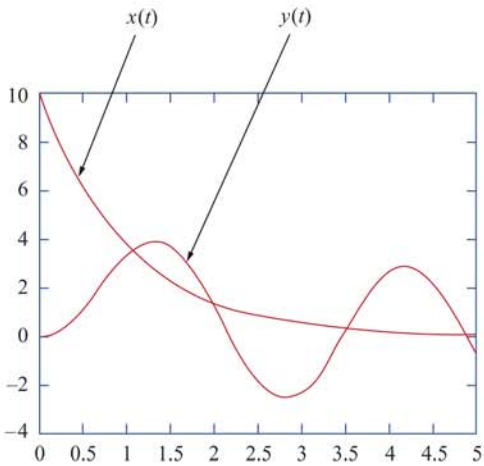


Figure 14.61
The response of the system described by $H(s) = (s + 4)/(s^2 + 2s^2 + 5s + 10)$ to an exponential input.

14.12 †Applications

Resonant circuits and filters are widely used, particularly in electronics, power systems, and communications systems. For example, a Notch filter with a cutoff frequency at 60 Hz may be used to eliminate the 60-Hz power line noise in various communications electronics. Filtering of signals in communications systems is necessary in order to select the desired signal from a host of others in the same range (as in the case of radio receivers discussed next) and also to minimize the effects of noise and interference on the desired signal. In this section, we consider one practical application of resonant circuits and two applications of filters. The focus of each application is not to understand the details of how each device works but to see how the circuits considered in this chapter are applied in the practical devices.

14.12.1 Radio Receiver

Series and parallel resonant circuits are commonly used in radio and TV receivers to tune in stations and to separate the audio signal from the radio-frequency carrier wave. As an example, consider the block diagram of an AM radio receiver shown in Fig. 14.62. Incoming amplitude-modulated radio waves (thousands of them at different frequencies from different broadcasting stations) are received by the antenna. A resonant circuit (or a bandpass filter) is needed to select just one of the incoming waves. The selected signal is very weak and is amplified in stages in order to generate an audible audio-frequency wave. Thus, we have the radio frequency (RF) amplifier to amplify the selected broadcast signal, the intermediate frequency (IF) amplifier to amplify an internally generated signal based on the RF signal, and the audio amplifier to amplify the audio signal just before it reaches the loudspeaker. It is much easier to amplify the signal at three stages than to build an amplifier to provide the same amplification for the entire band.

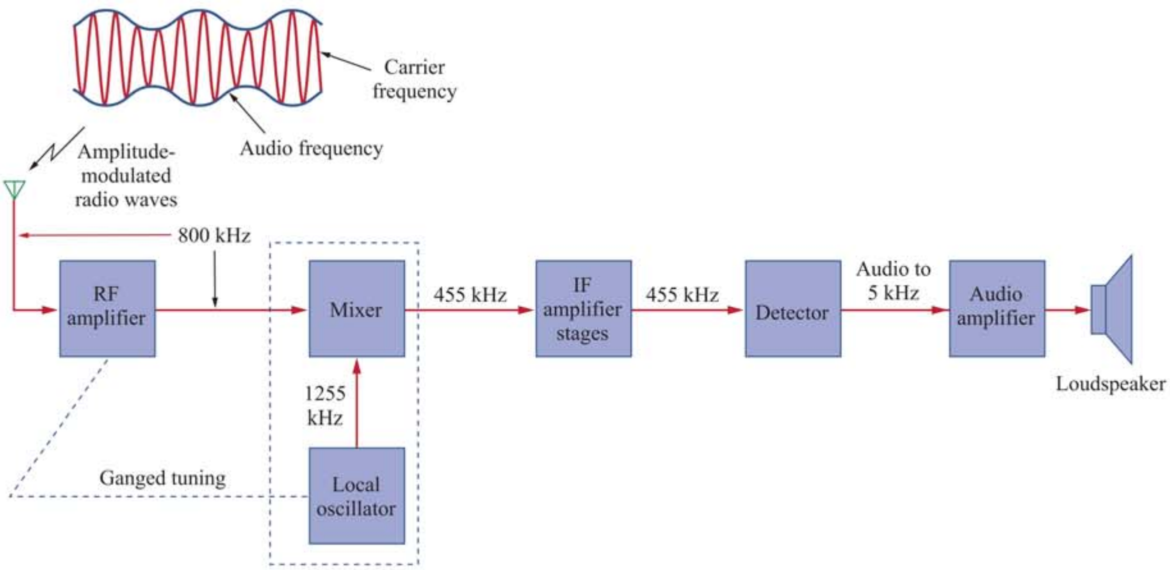


Figure 14.62
A simplified block diagram of a superheterodyne AM radio receiver.

The type of AM receiver shown in Fig. 14.62 is known as the *super-heterodyne receiver*. In the early development of radio, each amplification stage had to be tuned to the frequency of the incoming signal. This way, each stage must have several tuned circuits to cover the entire AM band (540 to 1600 kHz). To avoid the problem of having several resonant circuits, modern receivers use a *frequency mixer* or *heterodyne* circuit, which always produces the same IF signal (445 kHz) but retains the audio frequencies carried on the incoming signal. To produce the constant IF frequency, the rotors of two separate variable capacitors are mechanically coupled with one another so that they can be rotated simultaneously with a single control; this is called *ganged tuning*. A *local oscillator* ganged with the RF amplifier produces an RF signal that is combined with the incoming wave by the frequency mixer to produce an output signal that contains the sum and the difference frequencies of the two signals. For example, if the resonant circuit is tuned to receive an 800-kHz incoming signal, the local oscillator must produce a 1,255-kHz signal, so that the sum ($1,255 + 800 = 2,055$ kHz) and the difference ($1,255 - 800 = 455$ kHz) of frequencies are available at the output of the mixer. However, only the difference, 455 kHz, is used in practice. This is the only frequency to which all the IF amplifier stages are tuned, regardless of the station dialed. The original audio signal (containing the "intelligence") is extracted in the detector stage. The detector basically removes the IF signal, leaving the audio signal. The audio signal is amplified to drive the loudspeaker, which acts as a transducer converting the electrical signal to sound.

Our major concern here is the tuning circuit for the AM radio receiver. The operation of the FM radio receiver is different from that of the AM receiver discussed here, and in a much different range of frequencies, but the tuning is similar.

Example 14.17

The resonant or tuner circuit of an AM radio is portrayed in Fig. 14.63. Given that $L = 1\ \mu$H, what must be the range of C to have the resonant frequency adjustable from one end of the AM band to another?

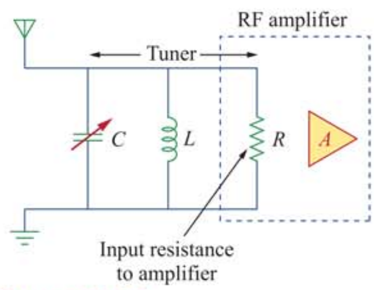


Figure 14.63
The tuner circuit for Example 14.17.

Solution:
The frequency range for AM broadcasting is 540 to 1,600 kHz. We consider the low and high ends of the band. Since the resonant circuit in Fig. 14.63 is a parallel type, we apply the ideas in Section 14.6. From Eq. (14.44),

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

or

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

For the high end of the AM band, $f_0 = 1{,}600$ kHz, and the corresponding C is

$$C_1 = \frac{1}{4\pi^2 \times 1{,}600^2 \times 10^6 \times 10^{-6}} = 9.9 \text{ nF}$$

For the low end of the AM band, $f_0 = 540$ kHz, and the corresponding C is

$$C_2 = \frac{1}{4\pi^2 \times 540^2 \times 10^6 \times 10^{-6}} = 86.9 \text{ nF}$$

Thus, C must be an adjustable (gang) capacitor varying from 9.9 nF to 86.9 nF.

Practice Problem 14.17

For an FM radio receiver, the incoming wave is in the frequency range from 88 to 108 MHz. The tuner circuit is a parallel RLC circuit with a 4-μH coil. Calculate the range of the variable capacitor necessary to cover the entire band.

Answer: From 0.543 pF to 0.818 pF.

14.12.2 Touch-Tone Telephone

A typical application of filtering is the touch-tone telephone set shown in Fig. 14.64. The keypad has 12 buttons arranged in four rows and three columns. The arrangement provides 12 distinct signals by using seven tones divided into two groups: the low-frequency group (697 to 941 Hz) and the high-frequency group (1,209 to 1,477 Hz). Pressing a button generates a sum of two sinusoids corresponding to its unique pair of frequencies. For example, pressing the number 6 button generates sinusoidal tones with frequencies 770 Hz and 1,477 Hz.

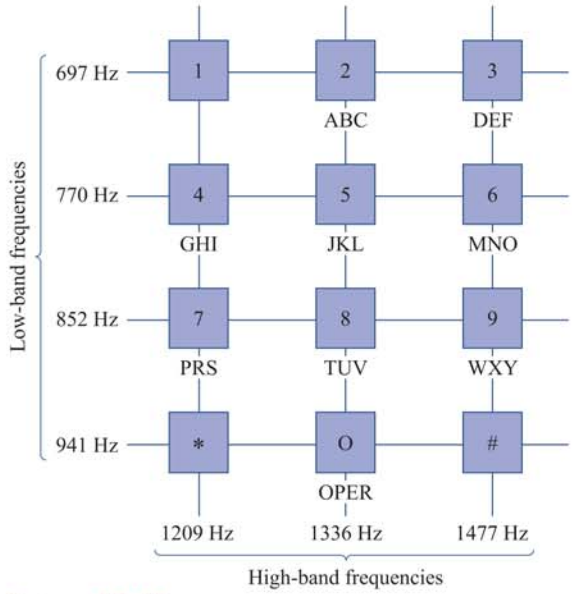


Figure 14.64
Frequency assignments for touch-tone dialing.
Adapted from G. Daryanani, *Principles of Active Network Synthesis and Design* [New York: John Wiley & Sons], 1976, p. 79.

When a caller dials a telephone number, a set of signals is transmitted to the telephone office, where the touch-tone signals are decoded by detecting the frequencies they contain. Figure 14.65 shows the block diagram for the detection scheme. The signals are first amplified and separated into their respective groups by the lowpass (LP) and highpass (HP) filters. The limiters (L) are used to convert the separated tones into square waves. The individual tones are identified using seven bandpass (BP) filters, each filter passing one tone and rejecting other tones. Each filter is followed by a detector (D), which is energized when its input voltage exceeds a certain level. The outputs of the detectors provide the required dc signals needed by the switching system to connect the caller to the party being called.

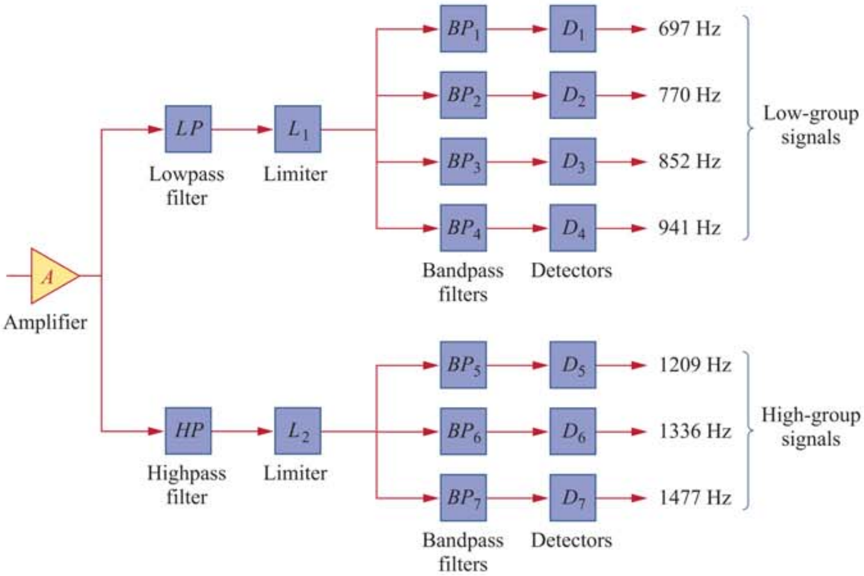


Figure 14.65
Block diagram of detection scheme.
G. Daryanani, *Principles of Active Network Synthesis and Design* [New York: John Wiley & Sons], 1976, p. 79.

Example 14.18

Using the standard 600-Ω resistor used in telephone circuits and a series *RLC* circuit, design the bandpass filter BP_2 in Fig. 14.65.

Solution:
The bandpass filter is the series *RLC* circuit in Fig. 14.35. Since BP_2 passes frequencies 697 Hz to 852 Hz and is centered at $f_0 = 770$ Hz, its bandwidth is

$$B = 2\pi(f_2 - f_1) = 2\pi(852 - 697) = 973.89 \text{ rad/s}$$

From Eq. (14.39),

$$L = \frac{R}{B} = \frac{600}{973.89} = 0.616 \text{ H}$$

From Eq. (14.27) or (14.55),

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times 770^2 \times 0.616} = 69.36 \text{ nF}$$

Practice Problem 14.18

Repeat Example 14.18 for bandpass filter BP_6.

Answer: 0.356 H, 39.83 nF.

14.12.3 Crossover Network

Another typical application of filters is the *crossover network* that couples an audio amplifier to woofer and tweeter speakers, as shown in Fig. 14.66(a). The network basically consists of one highpass *RC*

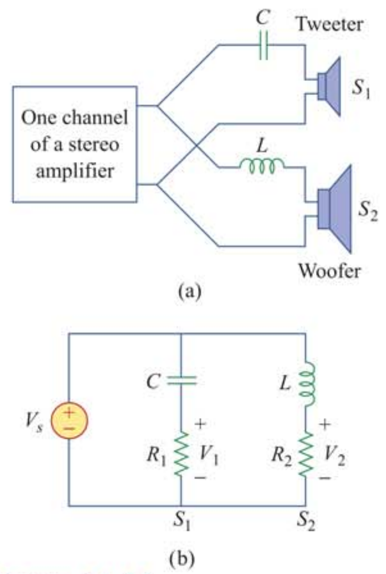


Figure 14.66
(a) A crossover network for two loudspeakers, (b) equivalent model.

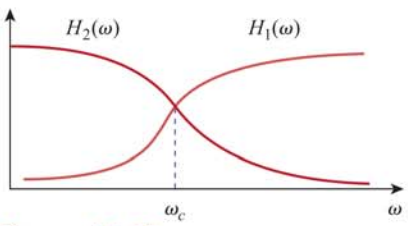


Figure 14.67
Frequency responses of the crossover network in Fig. 14.66.

filter and one lowpass *RL* filter. It routes frequencies higher than a prescribed crossover frequency f_c to the tweeter (high-frequency loudspeaker) and frequencies below f_c into the woofer (low-frequency loudspeaker). These loudspeakers have been designed to accommodate certain frequency responses. A woofer is a low-frequency loudspeaker designed to reproduce the lower part of the frequency range, up to about 3 kHz. A tweeter can reproduce audio frequencies from about 3 kHz to about 20 kHz. The two speaker types can be combined to reproduce the entire audio range of interest and provide the optimum in frequency response.

By replacing the amplifier with a voltage source, the approximate equivalent circuit of the crossover network is shown in Fig. 14.66(b), where the loudspeakers are modeled by resistors. As a highpass filter, the transfer function V_1/V_s is given by

$$H_1(\omega) = \frac{V_1}{V_s} = \frac{j\omega R_1 C}{1 + j\omega R_1 C} \tag{14.87}$$

Similarly, the transfer function of the lowpass filter is given by

$$H_2(\omega) = \frac{V_2}{V_s} = \frac{R_2}{R_2 + j\omega L} \tag{14.88}$$

The values of R_1, R_2, L, and C may be selected such that the two filters have the same cutoff frequency, known as the *crossover frequency*, as shown in Fig. 14.67.

The principle behind the crossover network is also used in the resonant circuit for a TV receiver, where it is necessary to separate the video and audio bands of RF carrier frequencies. The lower-frequency band (picture information in the range from about 30 Hz to about 4 MHz) is channeled into the receiver's video amplifier, while the high-frequency band (sound information around 4.5 MHz) is channeled to the receiver's sound amplifier.

Example 14.19

In the crossover network of Fig. 14.66, suppose each speaker acts as a 6-Ω resistance. Find C and L if the crossover frequency is 2.5 kHz.

Solution:
For the highpass filter,

$$\omega_c = 2\pi f_c = \frac{1}{R_1 C}$$

or

$$C = \frac{1}{2\pi f_c R_1} = \frac{1}{2\pi \times 2.5 \times 10^3 \times 6} = 10.61\ \mu\text{F}$$

For the lowpass filter,

$$\omega_c = 2\pi f_c = \frac{R_2}{L}$$

or

$$L = \frac{R_2}{2\pi f_c} = \frac{6}{2\pi \times 2.5 \times 10^3} = 382\ \mu\text{H}$$

Practice Problem 14.19

If each speaker in Fig. 14.66 has an 8-Ω resistance and $C = 10\ \mu F$, find L and the crossover frequency.

Answer: 0.64 mH, 1.989 kHz.

14.13 Summary

1. The transfer function $\mathbf{H}(\omega)$ is the ratio of the output response $\mathbf{Y}(\omega)$ to the input excitation $\mathbf{X}(\omega)$; that is, $\mathbf{H}(\omega) = \mathbf{Y}(\omega)/\mathbf{X}(\omega)$.
2. The frequency response is the variation of the transfer function with frequency.
3. Zeros of a transfer function $\mathbf{H}(s)$ are the values of $s = j\omega$ that make $H(s) = 0$, while poles are the values of s that make $H(s) \rightarrow \infty$.
4. The decibel is the unit of logarithmic gain. For a voltage or current gain G, its decibel equivalent is $G_{\text{dB}} = 20 \log_{10} G$.
5. Bode plots are semilog plots of the magnitude and phase of the transfer function as it varies with frequency. The straight-line approximations of H (in dB) and ϕ (in degrees) are constructed using the corner frequencies defined by the poles and zeros of $\mathbf{H}(\omega)$.
6. The resonant frequency is that frequency at which the imaginary part of a transfer function vanishes. For series and parallel RLC circuits.
$$\omega_0 = \frac{1}{\sqrt{LC}}$$
7. The half-power frequencies (ω_1, ω_2) are those frequencies at which the power dissipated is one-half of that dissipated at the resonant frequency. The geometric mean between the half-power frequencies is the resonant frequency, or
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$
8. The bandwidth is the frequency band between half-power frequencies:
$$B = \omega_2 - \omega_1$$
9. The quality factor is a measure of the sharpness of the resonance peak. It is the ratio of the resonant (angular) frequency to the bandwidth,
$$Q = \frac{\omega_0}{B}$$
10. A filter is a circuit designed to pass a band of frequencies and reject others. Passive filters are constructed with resistors, capacitors, and inductors. Active filters are constructed with resistors, capacitors, and an active device, usually an op amp.
11. Four common types of filters are lowpass, highpass, bandpass, and bandstop. A lowpass filter passes only signals whose frequencies are below the cutoff frequency ω_c. A highpass filter passes only signals whose frequencies are above the cutoff frequency ω_c. A bandpass filter passes only signals whose frequencies are within a prescribed

range ($\omega_1 < \omega < \omega_2$). A bandstop filter passes only signals whose frequencies are outside a prescribed range ($\omega_1 > \omega > \omega_2$).

12. Scaling is the process whereby unrealistic element values are magnitude-scaled by a factor K_m and/or frequency-scaled by a factor K_f to produce realistic values.

$$R' = K_m R, \qquad L' = \frac{K_m}{K_f}L, \qquad C' = \frac{1}{K_m K_f}C$$

13. *PSpice* can be used to obtain the frequency response of a circuit if a frequency range for the response and the desired number of points within the range are specified in the AC Sweep.
14. The radio receiver—one practical application of resonant circuits—employs a bandpass resonant circuit to tune in one frequency among all the broadcast signals picked up by the antenna.
15. The touch-tone telephone and the crossover network are two typical applications of filters. The touch-tone telephone system employs filters to separate tones of different frequencies to activate electronic switches. The crossover network separates signals in different frequency ranges so that they can be delivered to different devices such as tweeters and woofers in a loudspeaker system.

Review Questions

14.1 A zero of the transfer function

$$H(s) = \frac{10(s+1)}{(s+2)(s+3)}$$

is at

(a) 10 (b) −1 (c) −2 (d) −3

14.2 On the Bode magnitude plot, the slope of $1/(5 + j\omega)^2$ for large values of ω is

(a) 20 dB/decade (b) 40 dB/decade
(c) −40 dB/decade (d) −20 dB/decade

14.3 On the Bode phase plot for $0.5 < \omega < 50$, the slope of $[1 + j10\omega - \omega^2/25]^2$ is

(a) 45°/decade (b) 90°/decade
(c) 135°/decade (d) 180°/decade

14.4 How much inductance is needed to resonate at 5 kHz with a capacitance of 12 nF?

(a) 2,652 H (b) 11.844 H
(c) 3.333 H (d) 84.43 mH

14.5 The difference between the half-power frequencies is called the:

(a) quality factor (b) resonant frequency
(c) bandwidth (d) cutoff frequency

14.6 In a series *RLC* circuit, which of these quality factors has the steepest magnitude response curve near resonance?

(a) $Q = 20$ (b) $Q = 12$
(c) $Q = 8$ (d) $Q = 4$

14.7 In a parallel *RLC* circuit, the bandwidth *B* is directly proportional to *R*.

(a) True (b) False

14.8 When the elements of an *RLC* circuit are both magnitude-scaled and frequency-scaled, which quality is unaffected?

(a) resistor (b) resonant frequency
(c) bandwidth (d) quality factor

14.9 What kind of filter can be used to select a signal of one particular radio station?

(a) lowpass (b) highpass
(c) bandpass (d) bandstop

14.10 A voltage source supplies a signal of constant amplitude, from 0 to 40 kHz, to an RC lowpass filter. A load resistor, connected in parallel across the capacitor, experiences the maximum voltage at:

(a) dc (b) 10 kHz
(c) 20 kHz (d) 40 kHz

Answers: 14.1b, 14.2c, 14.3d, 14.4d, 14.5c, 14.6a, 14.7b, 14.8d, 14.9c, 14.10a.

Problems

Section 14.2 Transfer Function

14.1 Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the RC circuit in Fig. 14.68. Express it using $\omega_0 = 1/RC$.

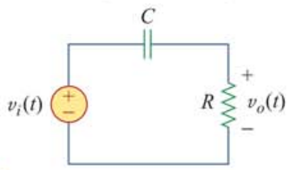


Figure 14.68
For Prob. 14.1.

14.2 Using Fig. 14.69, design a problem to help other students better understand how to determine transfer functions.

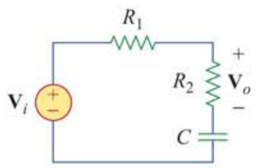


Figure 14.69
For Prob. 14.2.

14.3 Given the circuit in Fig. 14.70, $R_1 = 2\ \Omega$, $R_2 = 5\ \Omega$, $C_1 = 0.1$ F, and $C_2 = 0.2$ F, determine the transfer function $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_i(s)$.

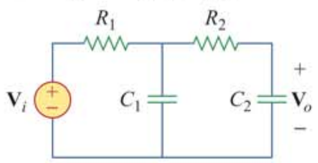


Figure 14.70
For Prob. 14.3.

14.4 Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ of the circuits shown in Fig. 14.71.

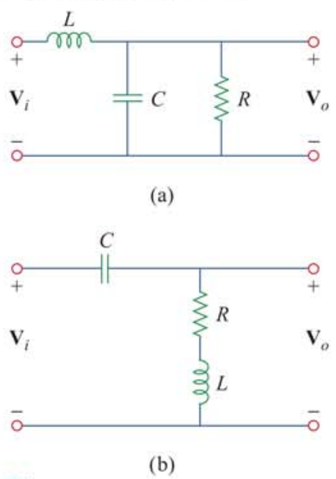


Figure 14.71
For Prob. 14.4.

14.5 For each of the circuits shown in Fig. 14.72, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s)$.

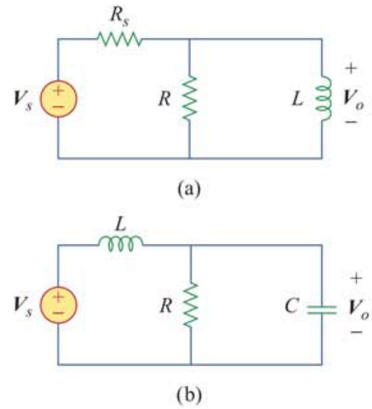


Figure 14.72
For Prob. 14.5.

14.6 For the circuit shown in Fig. 14.73, find $\mathbf{H}(s) = \mathbf{I}_o(s)/\mathbf{I}_s(s)$.

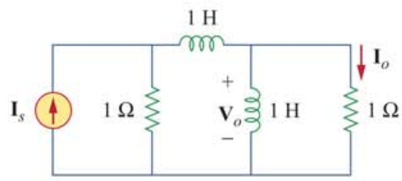


Figure 14.73
For Prob. 14.6.

Section 14.3 The Decibel Scale

14.7 Calculate $|\mathbf{H}(\omega)|$ if H_{dB} equals

(a) 0.05 dB (b) −6.2 dB (c) 104.7 dB

14.8 Design a problem to help other students calculate the magnitude in dB and phase in degrees of a variety of transfer functions at a single value of ω.

Section 14.4 Bode Plots

14.9 A ladder network has a voltage gain of

$$\mathbf{H}(\omega) = \frac{10}{(1 + j\omega)(10 + j\omega)}$$

Sketch the Bode plots for the gain.

14.10 Design a problem to help other students better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of $j\omega$.

14.11 Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{10 + j\omega}{j\omega(2 + j\omega)}$$

14.12 A transfer function is given by

$$T(s) = \frac{s + 1}{s(s + 10)}$$

Sketch the magnitude and phase Bode plots.

14.13 Construct the Bode plots for

$$G(s) = \frac{s + 1}{s^2(s + 10)}, \qquad s = j\omega$$

14.14 Draw the Bode plots for

$$\mathbf{H}(\omega) = \frac{50(j\omega + 1)}{j\omega(-\omega^2 + 10j\omega + 25)}$$

14.15 Construct the Bode magnitude and phase plots for

$$H(s) = \frac{40(s + 1)}{(s + 2)(s + 10)}, \qquad s = j\omega$$

14.16 Sketch Bode magnitude and phase plots for

$$H(s) = \frac{10}{s(s^2 + s + 16)}, \quad s = j\omega$$

14.17 Sketch the Bode plots for

$$G(s) = \frac{s}{(s + 2)^2(s + 1)}, \qquad s = j\omega$$

14.18 A linear network has this transfer function

$$H(s) = \frac{7s^2 + s + 4}{s^3 + 8s^2 + 14s + 5}, \qquad s = j\omega$$

Use *MATLAB* or equivalent to plot the magnitude and phase (in degrees) of the transfer function. Take $0.1 < \omega < 10$ rad/s.

14.19 Sketch the asymptotic Bode plots of the magnitude and phase for

$$H(s) = \frac{100s}{(s + 10)(s + 20)(s + 40)}, \qquad s = j\omega$$

14.20 Design a more complex problem than given in Prob. 14.10, to help other students better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of $j\omega$. Include at least a second order repeated root.

14.21 Sketch the magnitude Bode plot for

$$H(s) = \frac{s(s + 20)}{(s + 1)(s^2 + 60s + 400)}, \qquad s = j\omega$$

14.22 Find the transfer function $\mathbf{H}(\omega)$ with the Bode magnitude plot shown in Fig. 14.74.

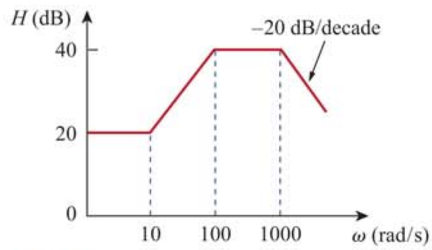


Figure 14.74
For Prob. 14.22.

14.23 The Bode magnitude plot of $\mathbf{H}(\omega)$ is shown in Fig. 14.75. Find $\mathbf{H}(\omega)$.

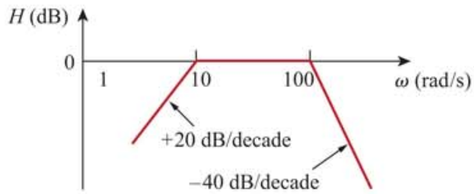


Figure 14.75
For Prob. 14.23.

14.24 The magnitude plot in Fig. 14.76 represents the transfer function of a preamplifier. Find $H(s)$.

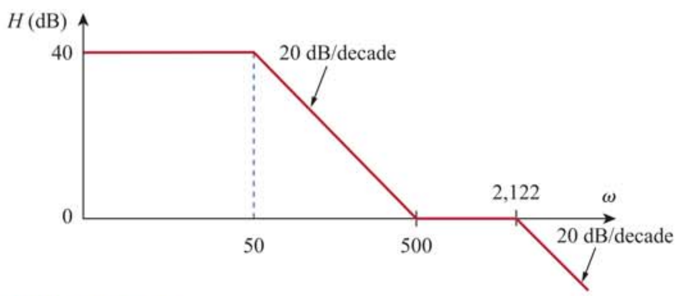


Figure 14.76
For Prob. 14.24.

Section 14.5 Series Resonance

14.25 A series RLC network has $R = 2$ kΩ, $L = 40$ mH, and $C = 1$ μF. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

14.26 Design a problem to help other students better understand ω_0, Q, and B at resonance in series RLC circuits.

14.27 Design a series RLC resonant circuit with $\omega_0 =$ 40 rad/s and $B = 10$ rad/s.

14.28 Design a series RLC circuit with $B = 20$ rad/s and $\omega_0 = 1{,}000$ rad/s. Find the circuit's Q. Let $R = 10\ \Omega$.

14.29 Let $v_s = 120\cos(at)$ V in the circuit of Fig. 14.77. Find ω_0, Q, and B, as seen by the capacitor.

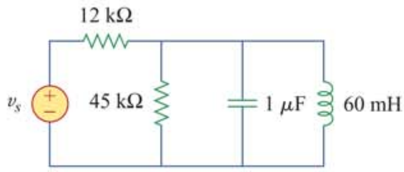


Figure 14.77
For Prob. 14.29.

14.30 A circuit consisting of a coil with inductance 10 mH and resistance 20 Ω is connected in series with a capacitor and a generator with an rms voltage of 120 V. Find:

(a) the value of the capacitance that will cause the circuit to be in resonance at 15 kHz

(b) the current through the coil at resonance

(c) the Q of the circuit

Section 14.6 Parallel Resonance

14.31 Design a parallel resonant RLC circuit with $\omega_0 = 10$ rad/s and $Q = 20$. Calculate the bandwidth of the circuit. Let $R = 10\ \Omega$.

14.32 Design a problem to help other students better understand the quality factor, the resonant frequency, and bandwidth of a parallel RLC circuit.

14.33 A parallel resonant circuit with quality factor 120 has a resonant frequency of 6×10^6 rad/s. Calculate the bandwidth and half-power frequencies.

14.34 A parallel RLC circuit is resonant at 5.6 MHz, has a Q of 80, and has a resistive branch of 40 kΩ. Determine the values of L and C in the other two branches.

14.35 A parallel RLC circuit has $R = 5$ kΩ, $L = 8$ mH, and $C = 60$ μF. Determine:

(a) the resonant frequency

(b) the bandwidth

(c) the quality factor

14.36 It is expected that a parallel RLC resonant circuit has a midband admittance of 25×10^{-3} S, quality factor of 80, and a resonant frequency of 200 krad/s. Calculate the values of R, L, and C. Find the bandwidth and the half-power frequencies.

14.37 Rework Prob. 14.25 if the elements are connected in parallel.

14.38 Find the resonant frequency of the circuit in Fig. 14.78.

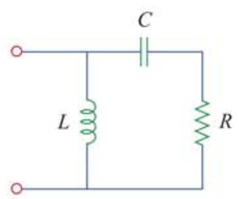


Figure 14.78
For Prob. 14.38.

14.39 For the "tank" circuit in Fig. 14.79, find the resonant frequency.

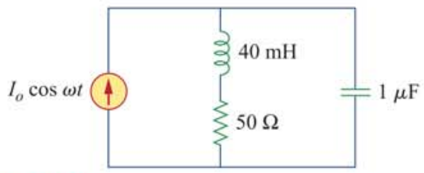


Figure 14.79
For Probs. 14.39 and 14.91.

14.40 A parallel resonance circuit has a resistance of 2 kΩ and half-power frequencies of 86 kHz and 90 kHz. Determine:

(a) the capacitance

(b) the inductance

(c) the resonant frequency

(d) the bandwidth

(e) the quality factor

14.41 Using Fig. 14.80, design a problem to help other students better understand the quality factor, the resonant frequency, and bandwidth of RLC circuits.

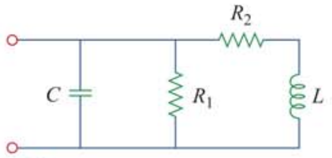


Figure 14.80
For Prob. 14.41.

14.42 For the circuits in Fig. 14.81, find the resonant frequency ω_0, the quality factor Q, and the bandwidth B.

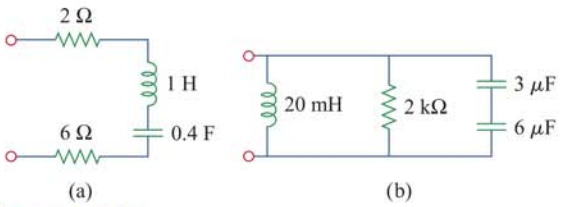


Figure 14.81
For Prob. 14.42.

14.43 Calculate the resonant frequency of each of the circuits in Fig. 14.82.

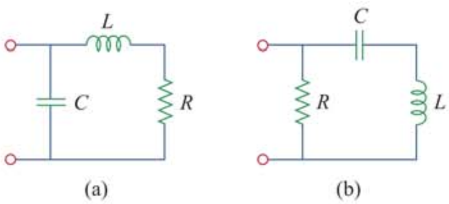


Figure 14.82
For Prob. 14.43.

***14.44** For the circuit in Fig. 14.83, find:

(a) the resonant frequency ω_0

(b) $\mathbf{Z}_{in}(\omega_0)$

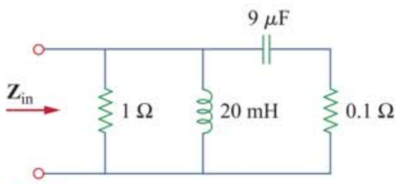


Figure 14.83
For Prob. 14.44.

14.45 For the circuit shown in Fig. 14.84, find ω_0, B, and Q, as seen by the voltage across the inductor.

* An asterisk indicates a challenging problem.

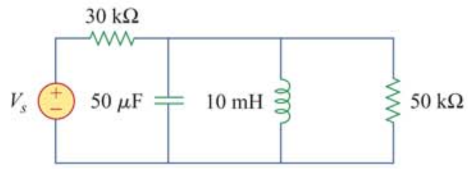


Figure 14.84
For Prob. 14.45.

14.46 For the network illustrated in Fig. 14.85, find

(a) the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{I}(\omega)$,

(b) the magnitude of $\mathbf{H}$ at $\omega_0 = 1$ rad/s.

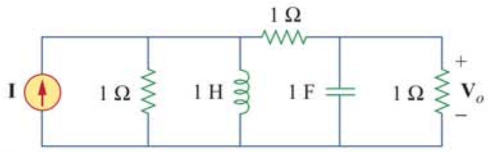


Figure 14.85
For Probs. 14.46, 14.78, and 14.92.

Section 14.7 Passive Filters

14.47 Show that a series LR circuit is a lowpass filter if the output is taken across the resistor. Calculate the corner frequency f_c if $L = 2$ mH and $R = 10$ kΩ.

14.48 Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the circuit in Fig. 14.86. Show that the circuit is a lowpass filter.

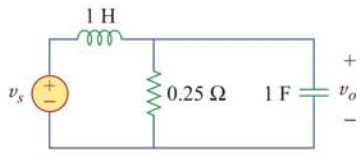


Figure 14.86
For Prob. 14.48.

14.49 **e2d** Design a problem to help other students better understand lowpass filters described by transfer functions.

14.50 Determine what type of filter is in Fig. 14.87. Calculate the corner frequency f_c.

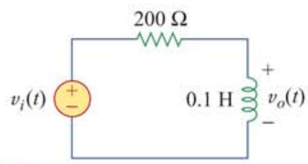


Figure 14.87
For Prob. 14.50.

14.51 Design an *RL* lowpass filter that uses a 40-mH coil and has a cutoff frequency of 5 kHz.

14.52 Design a problem to help other students better understand passive highpass filters.

14.53 Design a series *RLC* type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming $C = 80$ pF, find R, L, and Q.

14.54 Design a passive bandstop filter with $\omega_0 = 10$ rad/s and $Q = 20$.

14.55 Determine the range of frequencies that will be passed by a series *RLC* bandpass filter with $R = 10\ \Omega$, $L = 25$ mH, and $C = 0.4\ \mu$F. Find the quality factor.

14.56 (a) Show that for a bandpass filter,

$$\mathbf{H}(s) = \frac{sB}{s^2 + sB + \omega_0^2}, \qquad s = j\omega$$

where B = bandwidth of the filter and ω_0 is the center frequency.

(b) Similarly, show that for a bandstop filter,

$$\mathbf{H}(s) = \frac{s^2 + \omega_0^2}{s^2 + sB + \omega_0^2}, \qquad s = j\omega$$

14.57 Determine the center frequency and bandwidth of the bandpass filters in Fig. 14.88.

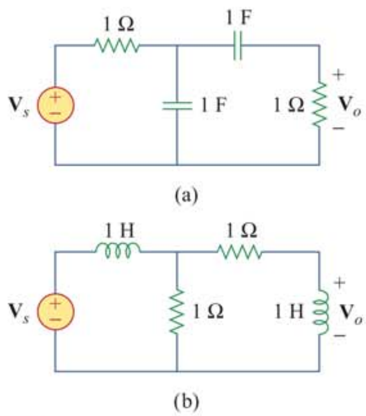


Figure 14.88
For Prob. 14.57.

14.58 The circuit parameters for a series *RLC* bandstop filter are $R = 2\ \text{k}\Omega$, $L = 0.1$ H, $C = 40$ pF. Calculate:

(a) the center frequency

(b) the half-power frequencies

(c) the quality factor

14.59 Find the bandwidth and center frequency of the bandstop filter of Fig. 14.89.

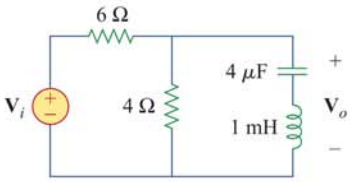


Figure 14.89
For Prob. 14.59.

Section 14.8 Active Filters

14.60 Obtain the transfer function of a highpass filter with a passband gain of 10 and a cutoff frequency of 50 rad/s.

14.61 Find the transfer function for each of the active filters in Fig. 14.90.

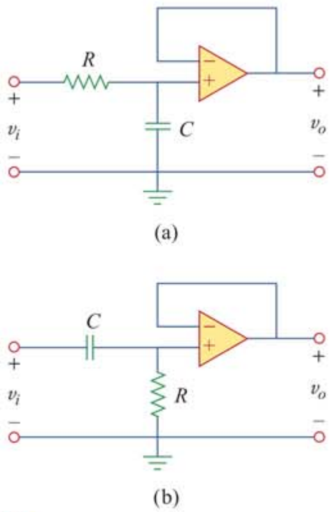


Figure 14.90
For Probs. 14.61 and 14.62.

14.62 The filter in Fig. 14.90(b) has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at:

(a) 200 Hz (b) 2 kHz (c) 10 kHz

14.63 Design an active first-order highpass filter with

$$\mathbf{H}(s) = -\frac{100s}{s + 10}, \qquad s = j\omega$$

Use a 1-μF capacitor.

14.64 Obtain the transfer function of the active filter in Fig. 14.91 on the next page. What kind of filter is it?

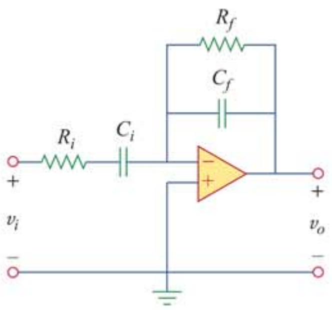


Figure 14.91
For Prob. 14.64.

14.65 A highpass filter is shown in Fig. 14.92. Show that the transfer function is

$$\mathbf{H}(\omega) = \left(1 + \frac{R_f}{R_i}\right)\frac{j\omega RC}{1 + j\omega RC}$$

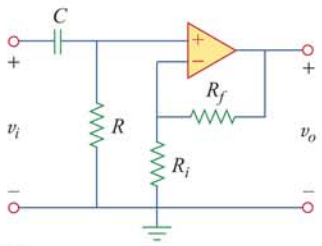


Figure 14.92
For Prob. 14.65.

14.66 A "general" first-order filter is shown in Fig. 14.93.

(a) Show that the transfer function is

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2C},$$

$s = j\omega$

(b) What condition must be satisfied for the circuit to operate as a highpass filter?

(c) What condition must be satisfied for the circuit to operate as a lowpass filter?

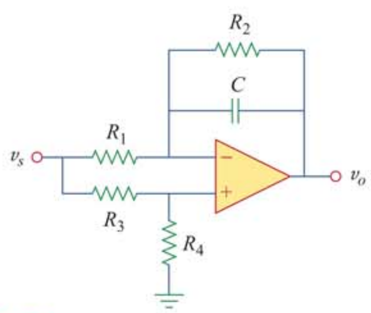


Figure 14.93
For Prob. 14.66.

14.67 Design an active lowpass filter with dc gain of 0.25 and a corner frequency of 500 Hz.

14.68 Design a problem to help other students better understand the design of active highpass filters when specifying a high-frequency gain and a corner frequency.

14.69 Design the filter in Fig. 14.94 to meet the following requirements:

(a) It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.

(b) It must provide a steady-state output of $v_o(t) = 10\sin(2\pi \times 10^8 t + 180°)$ V for an input $v_s(t) = 4\sin(2\pi \times 10^8 t)$ V.

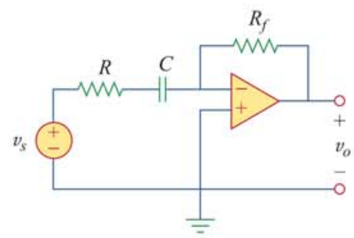


Figure 14.94
For Prob. 14.69.

***14.70** A second-order active filter known as a Butterworth filter is shown in Fig. 14.95.

(a) Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$.

(b) Show that it is a lowpass filter.

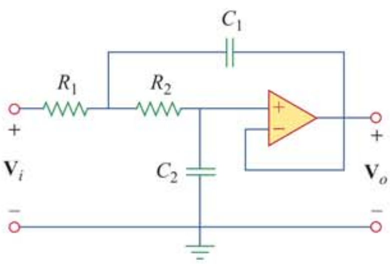


Figure 14.95
For Prob. 14.70.

Section 14.9 Scaling

14.71 Use magnitude and frequency scaling on the circuit of Fig. 14.76 to obtain an equivalent circuit in which the inductor and capacitor have magnitude 1 H and 1 F respectively.

14.72 Design a problem to help other students better understand magnitude and frequency scaling.

14.73 Calculate the values of R, L, and C that will result in $R = 12\ \text{k}\Omega$, $L = 40\ \mu\text{H}$, and $C = 300$ nF respectively when magnitude-scaled by 800 and frequency-scaled by 1000.

14.74 A circuit has $R_1 = 3\ \Omega$, $R_2 = 10\ \Omega$, $L = 2\text{H}$, and $C = 1/10$ F. After the circuit is magnitude-scaled by 100 and frequency-scaled by 10^6, find the new values of the circuit elements.

14.75 In an *RLC* circuit, $R = 20\ \Omega$, $L = 4$ H, and $C = 1$ F. The circuit is magnitude-scaled by 10 and frequency-scaled by 10^5. Calculate the new values of the elements.

14.76 Given a parallel *RLC* circuit with $R = 5$ kΩ, $L = 10$ mH, and $C = 20\ \mu$F, if the circuit is magnitude-scaled by $K_m = 500$ and frequency-scaled by $K_f = 10^5$, find the resulting values of R, L, and C.

14.77 A series *RLC* circuit has $R = 10\ \Omega$, $\omega_0 = 40$ rad/s, and $B = 5$ rad/s. Find L and C when the circuit is scaled:

(a) in magnitude by a factor of 600,

(b) in frequency by a factor of 1,000,

(c) in magnitude by a factor of 400 and in frequency by a factor of 10^5.

14.78 Redesign the circuit in Fig. 14.85 so that all resistive elements are scaled by a factor of 1,000 and all frequency-sensitive elements are frequency-scaled by a factor of 10^4.

***14.79** Refer to the network in Fig. 14.96.

(a) Find $\mathbf{Z}_{\text{in}}(s)$.

(b) Scale the elements by $K_m = 10$ and $K_f = 100$. Find $\mathbf{Z}_{\text{in}}(s)$ and ω_0.

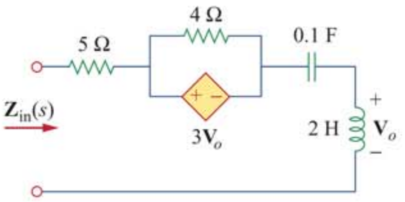


Figure 14.96
For Prob. 14.79.

14.80 (a) For the circuit in Fig. 14.97, draw the new circuit after it has been scaled by $K_m = 200$ and $K_f = 10^4$.

(b) Obtain the Thevenin equivalent impedance at terminals *a-b* of the scaled circuit at $\omega = 10^4$ rad/s.

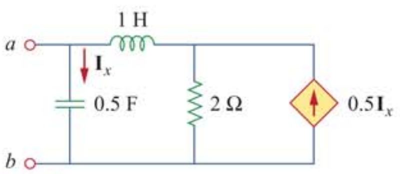


Figure 14.97
For Prob. 14.80.

14.81 The circuit shown in Fig. 14.98 has the impedance

$$Z(s) = \frac{1{,}000(s + 1)}{(s + 1 + j50)(s + 1 - j50)}, \qquad s = j\omega$$

Find:

(a) the values of R, L, C, and G

(b) the element values that will raise the resonant frequency by a factor of 10^3 by frequency scaling

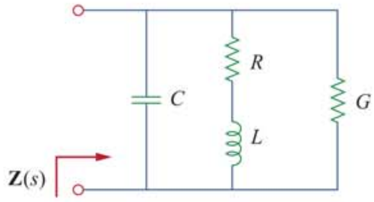


Figure 14.98
For Prob. 14.81.

14.82 Scale the lowpass active filter in Fig. 14.99 so that its corner frequency increases from 1 rad/s to 200 rad/s. Use a 1-μF capacitor.

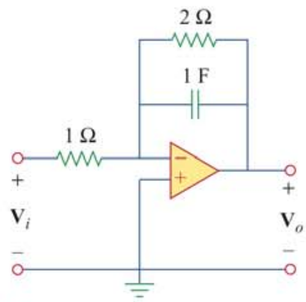


Figure 14.99
For Prob. 14.82.

14.83 The op amp circuit in Fig. 14.100 is to be magnitude-scaled by 100 and frequency-scaled by 10^5. Find the resulting element values.

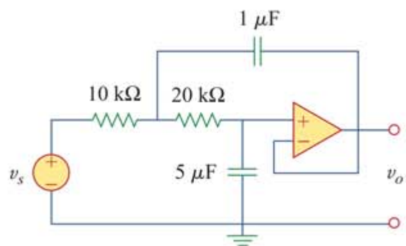


Figure 14.100
For Prob. 14.83.

Section 14.10 Frequency Response Using *PSpice*

14.84 Using *PSpice*, obtain the frequency response of the circuit in Fig. 14.101 on the next page.

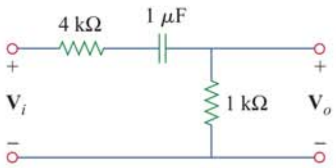


Figure 14.101
For Prob. 14.84.

14.85 Use *PSpice* to obtain the magnitude and phase plots of $\mathbf{V}_o/\mathbf{I}_s$ of the circuit in Fig. 14.102.

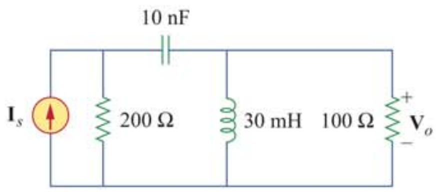


Figure 14.102
For Prob. 14.85.

14.86 Using Fig. 14.103, design a problem to help other students better understand how to use *PSpice* to obtain the frequency response (magnitude and phase of I) in electrical circuits.

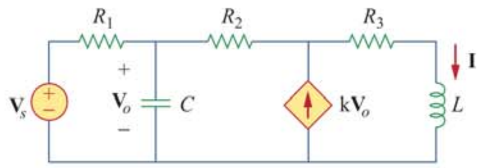


Figure 14.103
For Prob. 14.86.

14.87 In the interval $0.1 < f < 100$ Hz, plot the response of the network in Fig. 14.104. Classify this filter and obtain ω_0.

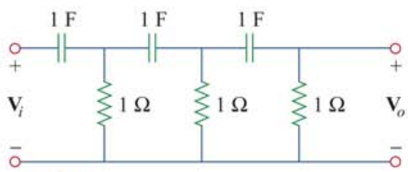


Figure 14.104
For Prob. 14.87.

14.88 Use *PSpice* to generate the magnitude and phase Bode plots of $\mathbf{V}_o$ in the circuit of Fig. 14.105.

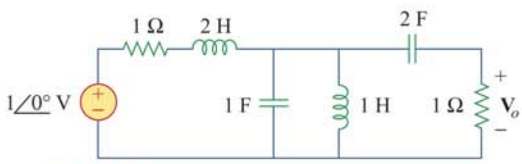


Figure 14.105
For Prob. 14.88.

14.89 Obtain the magnitude plot of the response $\mathbf{V}_o$ in the network of Fig. 14.106 for the frequency interval $100 < f < 1{,}000$ Hz.

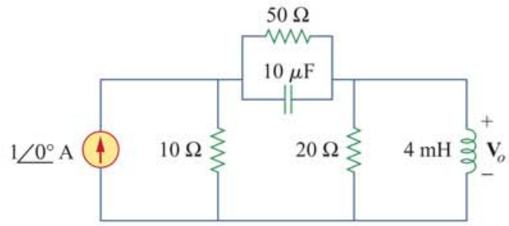


Figure 14.106
For Prob. 14.89.

14.90 Obtain the frequency response of the circuit in Fig. 14.40 (see Practice Problem 14.10). Take $R_1 = R_2 = 100\ \Omega$, $L = 2$ mH. Use $1 < f < 100{,}000$ Hz.

14.91 For the "tank" circuit of Fig. 14.79, obtain the frequency response (voltage across the capacitor) using *PSpice*. Determine the resonant frequency of the circuit.

14.92 Using *PSpice*, plot the magnitude of the frequency response of the circuit in Fig. 14.85.

Section 14.12 Applications

14.93 For the phase shifter circuit shown in Fig. 14.107, find $\mathbf{H} = V_o/V_s$.

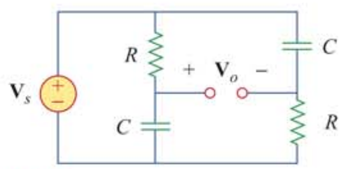


Figure 14.107
For Prob. 14.93.

14.94 For an emergency situation, an engineer needs to make an *RC* highpass filter. He has one 10-pF capacitor, one 30-pF capacitor, one 1.8-kΩ resistor, and one 3.3-kΩ resistor available. Find the greatest cutoff frequency possible using these elements.

14.95 A series-tuned antenna circuit consists of a variable capacitor (40 pF to 360 pF) and a 240-μH antenna coil that has a dc resistance of 12 Ω.

(a) Find the frequency range of radio signals to which the radio is tunable.

(b) Determine the value of Q at each end of the frequency range.

14.96 The crossover circuit in Fig. 14.108 is a lowpass filter that is connected to a woofer. Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

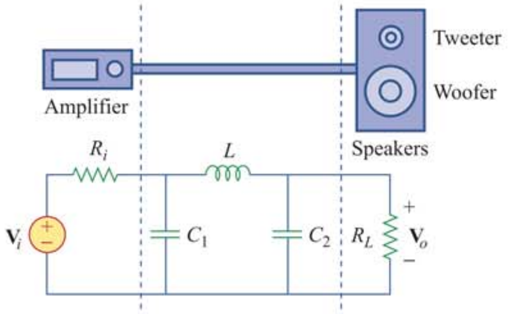


Figure 14.108
For Prob. 14.96.

14.97 The crossover circuit in Fig. 14.109 is a highpass filter that is connected to a tweeter. Determine the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

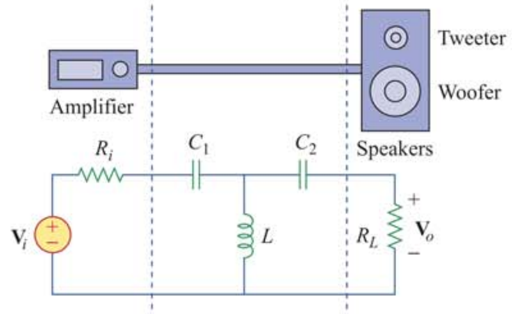


Figure 14.109
For Prob. 14.97.

Comprehensive Problems

14.98 A certain electronic test circuit produced a resonant curve with half-power points at 432 Hz and 454 Hz. If $Q = 20$, what is the resonant frequency of the circuit?

14.99 In an electronic device, a series circuit is employed that has a resistance of 100 Ω, a capacitive reactance of 5 kΩ, and an inductive reactance of 300 Ω when used at 2 MHz. Find the resonant frequency and bandwidth of the circuit.

14.100 In a certain application, a simple *RC* lowpass filter is designed to reduce high frequency noise. If the desired corner frequency is 20 kHz and $C = 0.5\ \mu\text{F}$, find the value of R.

14.101 In an amplifier circuit, a simple *RC* highpass filter is needed to block the dc component while passing the time-varying component. If the desired rolloff frequency is 15 Hz and $C = 10\ \mu\text{F}$, find the value of R.

14.102 Practical *RC* filter design should allow for source and load resistances as shown in Fig. 14.110. Let $R = 4\ \text{k}\Omega$ and $C = 40$-nF. Obtain the cutoff frequency when:

(a) $R_s = 0$, $R_L = \infty$,

(b) $R_s = 1\ \text{k}\Omega$, $R_L = 5\ \text{k}\Omega$.

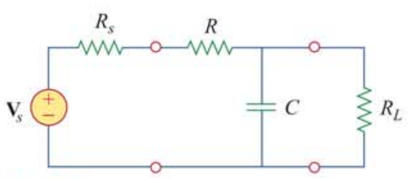


Figure 14.110
For Prob. 14.102.

14.103 The *RC* circuit in Fig. 14.111 is used for a lead compensator in a system design. Obtain the transfer function of the circuit.

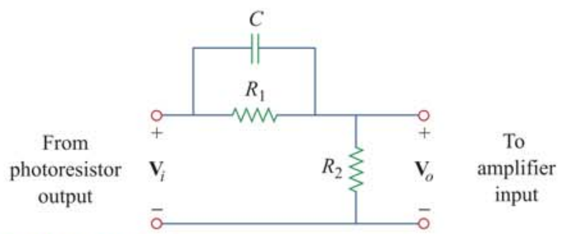


Figure 14.111
For Prob. 14.103.

14.104 A low-quality-factor, double-tuned bandpass filter is shown in Fig. 14.112. Use *PSpice* to generate the magnitude plot of $\mathbf{V}_o(\omega)$.

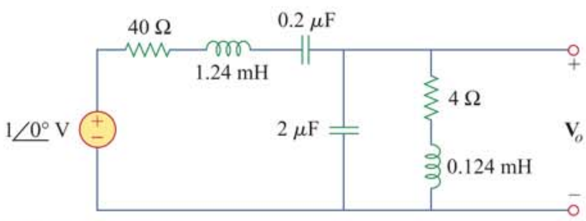


Figure 14.112
For Prob. 14.104.