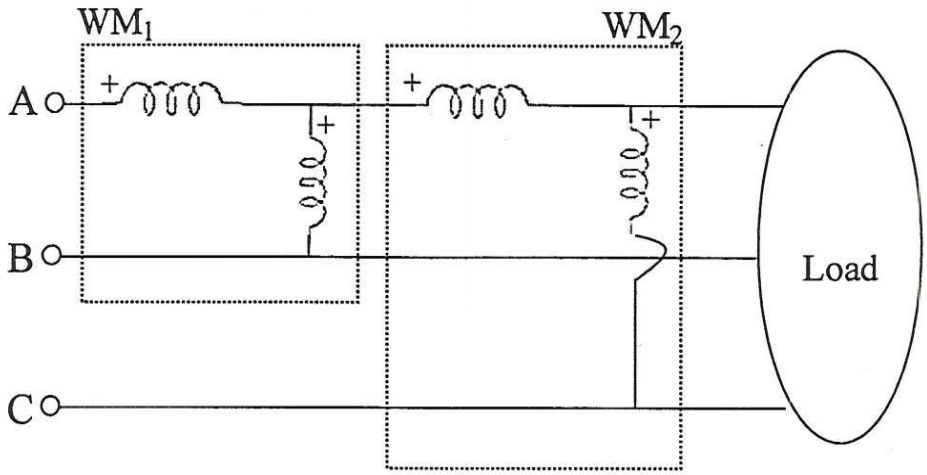
Question 4 (17 pts) Given the following balanced 3-phase system with positive phase sequence, express the readings of wattmeters WM_1 and WM_2 in terms of the total real power P and the total reactive power Q of the load.



$V_{An} = |V_m| \angle 0°$ V_{rms} $\quad I_A = |I_m| \angle -\theta$ A_{rms}

$V_{Bn} = |V_m| \angle -120°$ V_{rms}

$V_{Cn} = |V_m| \angle +120°$ V_{rms}

$P = 3|V_m||I_m|\cos\theta$ & $Q = 3|V_m||I_m|\sin\theta$

$WM_1 = Re\{V_{AB} I_A^*\}$

$= Re\{\sqrt{3}|V_m| \angle 30° \; |I_m| \angle \theta\}$

$= \sqrt{3}|V_m||I_m|\cos(\theta + 30°)$

$= \sqrt{3}|V_m||I_m|\{\cos\theta\cos 30° - \sin\theta\sin 30°\}$

$= \frac{3}{2}|V_m||I_m|\cos\theta - \frac{\sqrt{3}}{2}|V_m||I_m|\sin\theta$

$= \frac{1}{2}P - \frac{1}{2\sqrt{3}}Q$

$WM_2 = Re\{V_{AC} I_A^*\}$

$= Re\{\sqrt{3}|V_m| \angle -30° \; |I_m| \angle \theta\}$

$= \sqrt{3}|V_m||I_m|\cos(\theta - 30°)$

$= \sqrt{3}|V_m||I_m|\{\cos\theta\cos 30° + \sin\theta\sin 30°\}$

$= \frac{3}{2}|V_m||I_m|\cos\theta + \frac{\sqrt{3}}{2}|V_m||I_m|\sin\theta$

$= \frac{1}{2}P + \frac{1}{2\sqrt{3}}Q$