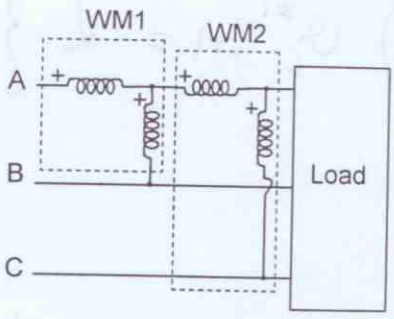
Question 4 (10 pts)

In a balanced three-phase load with a positive phase sequence, the complex power is:

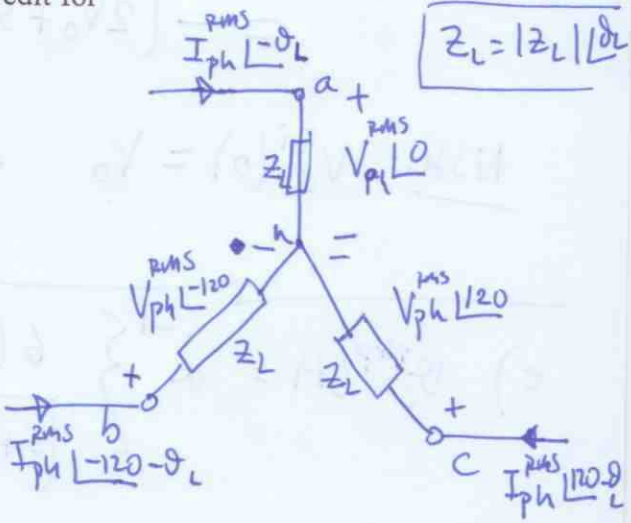
$$S = 1500 + j500\sqrt{3}\ \text{VA}$$

What are the wattmeter readings?

NOTE: You have to show your derivations to obtain credit for this question.



$Z_L = |Z_L| \angle \theta_L$



$WM_1 \Rightarrow Re\{V_{ab} I_a^*\}$

$WM_2 \Rightarrow Re\{V_{ac} I_a^*\}$

$$WM_1 \Rightarrow Re\left\{\sqrt{3}\, V_{ph}^{RMS} \angle 30\ I_{ph}^{RMS} \angle +\theta_L\right\}$$

$$= \sqrt{3}\, V_{ph}^{RMS} I_{ph}^{RMS} \cos(30 + \theta_L)$$

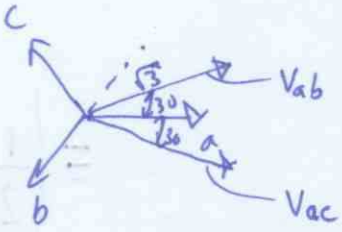
$$= \sqrt{3}\, V_{ph}^{RMS} I_{ph}^{RMS} \left[\cos 30 \cos\theta_L - \sin 30 \sin\theta_L\right]$$

$$= \sqrt{3}\, V_{ph}^{RMS} I_{ph}^{RMS} \frac{\sqrt{3}}{2} \cos\theta_L - \sqrt{3}\, V_{ph}^{RMS} I_{ph}^{RMS} \frac{1}{2} \sin\theta_L$$

$$= \frac{3\, V_{ph}^{RMS} I_{ph}^{RMS} \cos\theta_L}{2} - \frac{3\, V_{ph}^{RMS} I_{ph}^{RMS} \sin\theta_L}{\sqrt{3} \cdot 2}$$

$$= \frac{1500}{2} - \frac{500\sqrt{3}}{\sqrt{3} \cdot 2}$$

$$= 500 \text{ Watt.}$$



$$WM_2 \Rightarrow Re\left\{\sqrt{3}\, V_{ph}^{RMS} \angle -30\ I_{ph}^{RMS} \angle \theta_L\right\}$$

$$= \sqrt{3}\, V_{ph}^{RMS} I_{ph}^{RMS} \cos(-30 + \theta_L)$$

$$= \frac{1500}{2} + \frac{500\sqrt{3}}{\sqrt{3} \cdot 2} = 1000 \text{ Watt.}$$