METU/EEED

EE 202/ VPS VI

1) Solve the differential equation given below by Laplace Transformation.

$$(D^3+D^2+4D+4)\,x(t)=(2D+4)\,u_s(t)$$

$$x(0^-)=5,\; Dx(0^-)=1,\; D^2x(0^-)=-1.$$

$u_s(t)$ is

(a) $3e^{2t}$, (b) $3e^{-2t}$, (c) $4e^{-t}$, (d) $5\cos(t+20^\circ)$, (e) $5e^{-2t}\cos(t+20^\circ)$,

(f) $\delta(t)$, (g) $u(t)$.

2) Solve the state equation using Laplace Transformation.

(a) $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-1 & \alpha\\ -1 & \beta\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1\\ -2\end{bmatrix}(2\cos(2t)),\quad \begin{bmatrix}x_1(0^-)\\ x_2(0^-)\end{bmatrix}=\begin{bmatrix}4\\ -6\end{bmatrix}$$

(i) $\alpha=1$, $\beta=-3$, (ii) $\alpha=4$, $\beta=-1$, (iii) $\alpha=1$, $\beta=1$, (iv) $\alpha=4$, $\beta=3$.

(b) $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-4 & 3\\ -2 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}6e^{t}\\ 10\cos(2t+60^\circ)\end{bmatrix},\quad \begin{bmatrix}x_1(0^-)\\ x_2(0^-)\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$$

(c) $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}-1 & -2 & 0\\ 1 & 1 & -1\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}+\begin{bmatrix}1\\ 1\\ 1\end{bmatrix},\quad \begin{bmatrix}x_1(0^-)\\ x_2(0^-)\\ x_3(0^-)\end{bmatrix}=\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$$

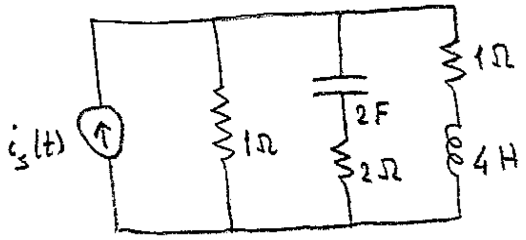
3) (i) Formulate the circuit by the node or the mesh formulation method.

(ii) Laplace Transform the formulation equation.

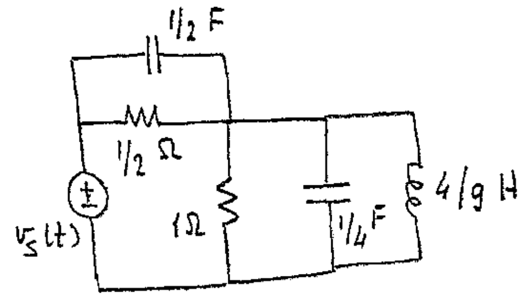
(iii) Transform the circuit to the s-domain and formulate by the method of Part (i).

(iv) Express the Laplace Transforms of the formulation variables in terms of the initial conditions and the Laplace Transform of the input.

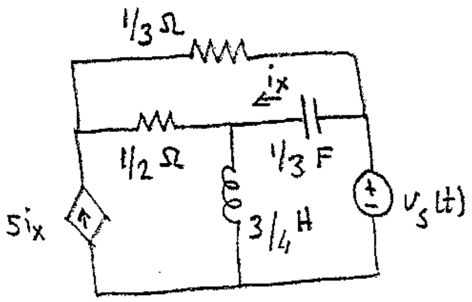
(v) Find the zero-input and impulse responses.



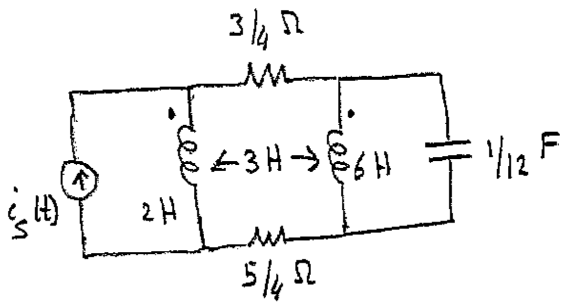
(a)



(b)

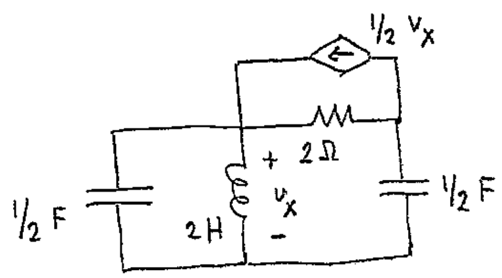


(c)



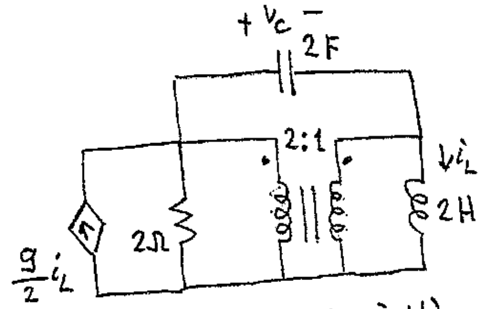
(d)

4) The initial conditions are specified at $t=0$.
Find the indicated variables for $t \geq 0$.



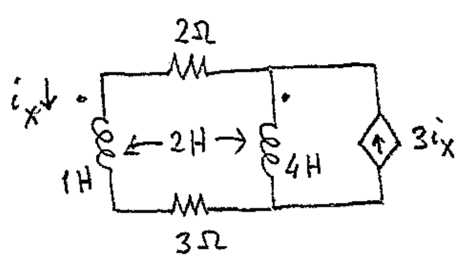
$v_x(t)$

(a)



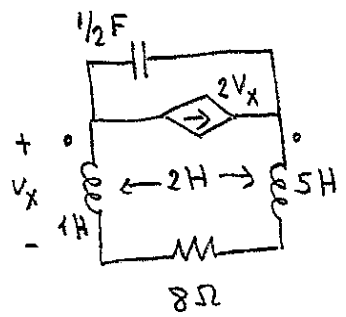
$v_c(t)$, $i_L(t)$

(b)



$i_x(t)$

(c)



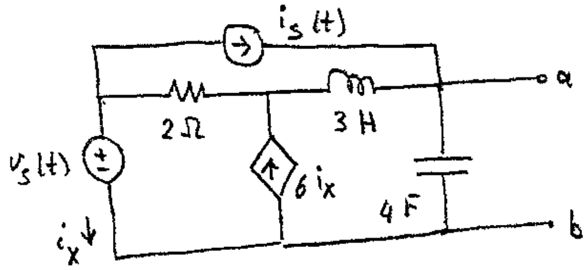
$v_x(t)$

(d)

5) The initial conditions are zero at $t=0^-$.

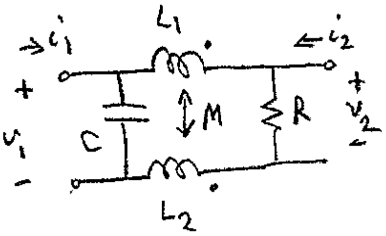
Transform the one-port to the s-domain.

Obtain the Thevenin and Norton equivalents.



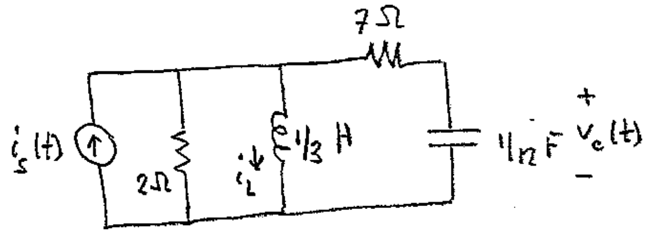
6) The initial conditions are zero at $t=0^-$.

Obtain the impedance functions $Z_{11}(s)$, $Z_{12}(s)$, $Z_{21}(s)$, $Z_{22}(s)$.



$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

7)



$v_c(0^-) = V_0$

$i_L(0^-) = I_0$

(a) Transform the circuit to the s-domain.

(b) Express $V_c(s)$ and $I_L(s)$ in terms of $I_s(s)$, V_0, I_0.

(c) Find the differential equations satisfied by $v_c(t)$ and $i_L(t)$.

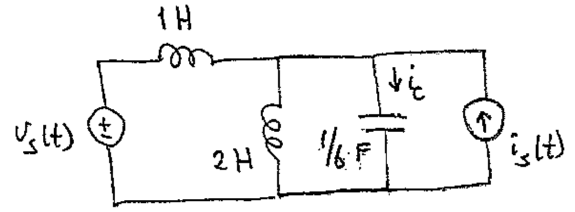
(d) For $V_0 = 12\,V$ and $I_0 = 6\,A$ find the zero-input solutions for $v_c(t)$ and $i_L(t)$.

(e) Find $v_c(0+)$, $i_L(0+)$, $v_c(+\infty)$, $i_L(+\infty)$ directly from the circuit and also using the initial and final value theorems for

(i) $i_s(t) = 10\,\delta(t)$ A, (ii) $i_s(t) = 10\,u(t)$ A.

(f) Find the impulse and step responses for $v_c(t)$ and $i_L(t)$.

(g) For $i_s(t) = 10\cos(4t+45^\circ)$ A, find the zero-state responses for $v_c(t)$ and $i_L(t)$.

8) 

The initial conditions are zero at $t=0^-$.

(a) Transform the circuit to the s-domain.

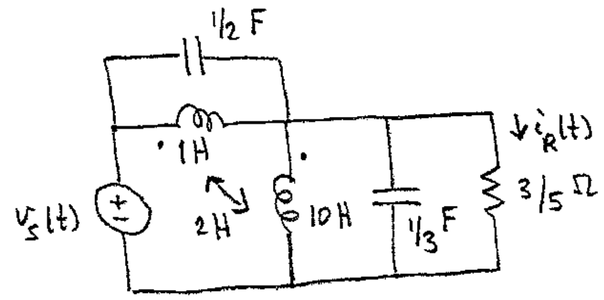
(b) Express $I_c(s)$ in terms of $V_s(s)$ and $I_s(s)$.

(c) Find the differential equation satisfied by $i_c(t)$.

(d) For $v_s(t)=6\,\delta(t)$ V and $i_s(t)=4u(t)$ A, find $i_c(0^+)$ directly from the circuit and also using the initial value theorem.

(e) For $v_s(t)=6\,\delta(t)$ V and $i_s(t)=4u(t)$ A, find $i_c(t)$ for $t\geq 0$.

(f) For $v_s(t)=6\cos(3t)$ V and $i_s(t)=4\cos(4t+30°)$ A, find $i_c(t)$ for $t\geq 0$.

9) 

The initial conditions are zero at $t=0^-$.

(a) Transform the circuit to the s-domain.

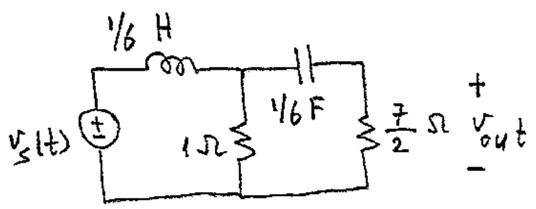
(b) Obtain the transfer function $Y_T(s)=I_R(s)/V_s(s)$.

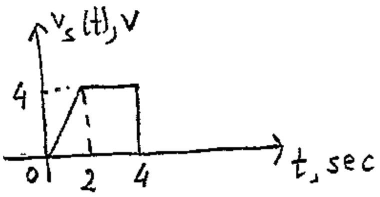
(c) Plot the pole/zero diagram.

(d) Find $i_R(0^+)$ and $i_R(+\infty)$ directly from the circuit and also using the initial and final value theorems for

(i) $v_s(t)=\delta(t)$ V, (ii) $v_s(t)=u(t)$ V.

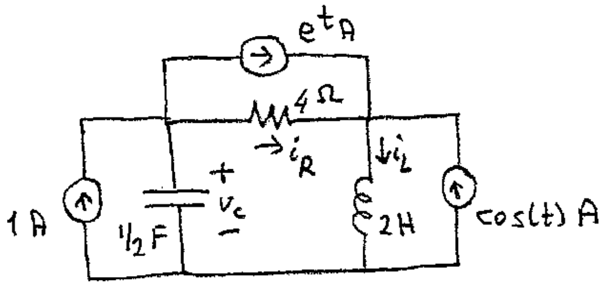
(e) Find the impulse and step responses for $i_R(t)$.

10) 



Find the zero-state response.

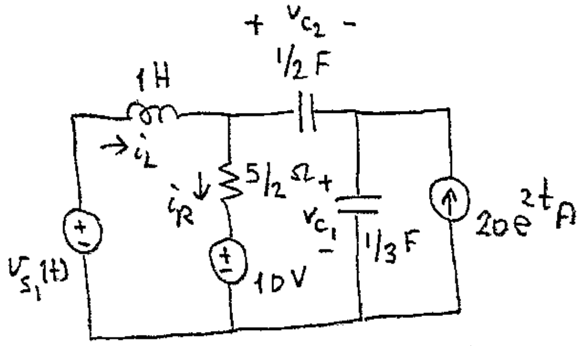
11) Find the indicated variables for $t \geq 0$.



$v_c(0^-) = 2V$, $i_L(0^-) = -3$ A

v_c, i_L, i_R

(a)

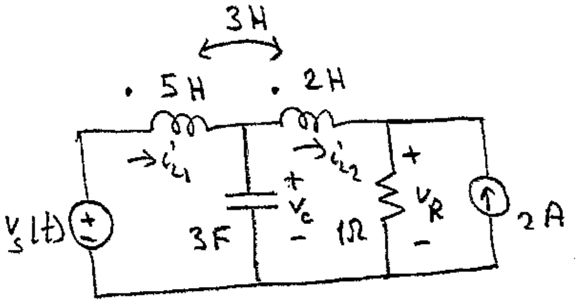


$v_{s_1}(t) = 10\cos(2t + 45°)$ V

$v_{c_1}(0^-) = 3$ V, $v_{c_2}(0^-) = 5$ V, $i_L(0^-) = 2$ A

v_{c_1}, i_R

(b)

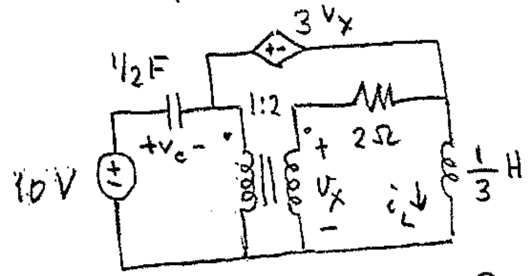


$v_s(t) = 4\sin(3t)$ V

$v_c(0^-) = 2$ V, $i_{L_1}(0^-) = 1$ A, $i_{L_2}(0^-) = 3$ A

v_R

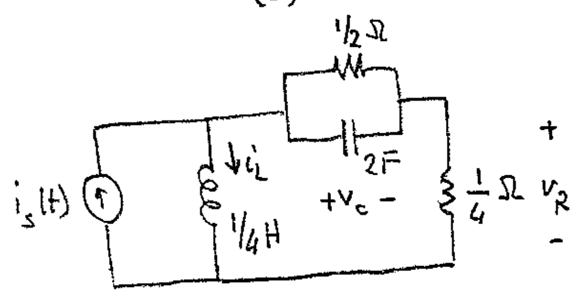
(c)



$v_c(0^-) = 4$V, $i_L(0^-) = 3$ A

v_c, i_L

(d)

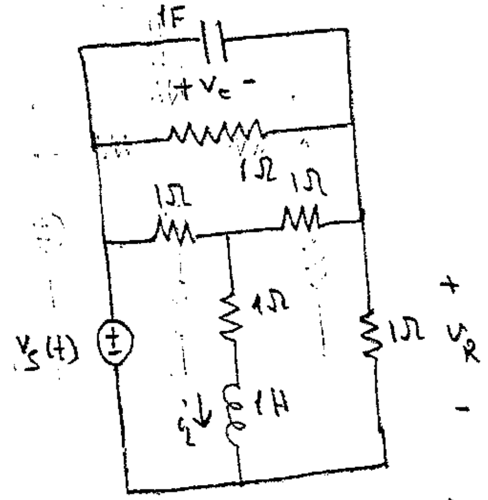


$i_s(t) = 4u(t) + 8\delta(t)$ A

$v_c(0^-) = 2$V, $i_L(0^-) = 4$ A

v_R

(e)



$v_c(0^-) = 1$V, $i_L(0^-) = 1$ A

(i) $v_s(t) = e^{-3t}$ V, (ii) $v_s(t) = e^{-2t}$ V

v_R

(f)