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EE201;

Analysis: Given an electrical network → Analysis Method → Node analysis, Mesh analysis

→ Simplification Tools → Parallel - Series Combination; Δ-Y, Thevenin-Norton, Source Transformation

The analysis Methods are based on;

Conservation of energy (KVL)
Conservation of charge (KCL)
Components Equations

$W_{A\to B} = q\,\Delta V = q\,(V_B - V_A)$

Polar Coordinates:

$$\frac{1}{1+2i} = \frac{1 \angle 1}{\sqrt{5}\angle \tan^{-1}(2)} = \frac{1}{\sqrt{5}}\left(\angle -\tan^{-1}(2)\right)^{-1}$$

$e^{i\pi/3} = \cos\pi/3 + i\sin\pi/3$ $\quad 1^{1/2} = \{1, -1\}$ $\quad 1^{1/3} = \left(e^{i\pi J}\right)^{1/3}$; $\quad 1^{1/3} = \{120°, 1, -120°\}$

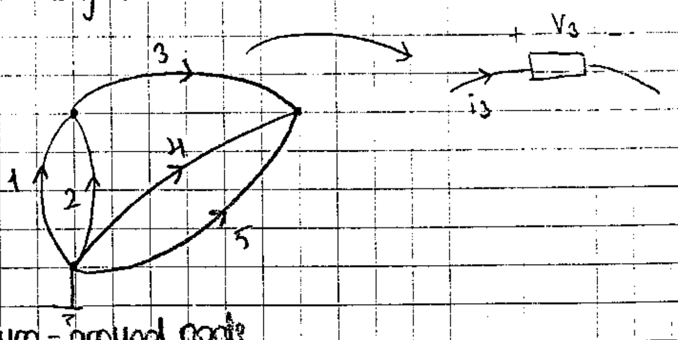
$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \; ; \; \cos(i2) = \frac{e^{-2} + e^{2}}{2} = \cosh(2)$$

Nth Order Dynamics Circuits

Graph Theoratical Approach and Regular Approach

Graph Theoratical Approach

Node Analysis:



Datum - ground node

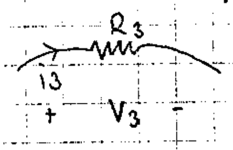
1 - A: Reduced Incidence Matrix (reduced: datum node is not included)

$\underline{A}\,\underline{J} = \underline{0}$ J: branch current matrix

* Schaum's Outline Series in Complex Variables

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{bmatrix}}_{\underline{J}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2- Branch Equations



$V_k = J_k \cdot R_k \; ; \; J_k = \frac{V_k}{R_k}$

$\underline{J} = \underline{\underline{G}}\,\underline{V} + \underline{J_s} \rightarrow$ source vector

3- $\underline{V} = \underline{\underline{A}}^T \underline{e} \rightarrow$ node voltages

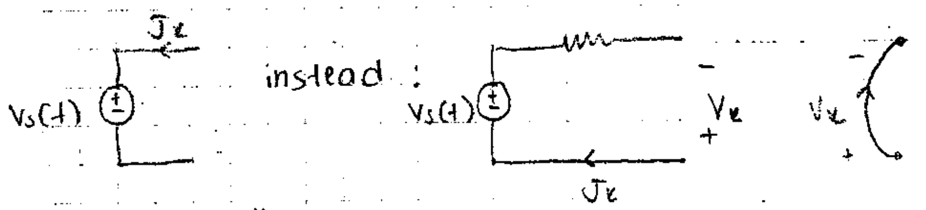
4- $\underline{\underline{A}}\,\underline{J} = 0 \quad (1)$

$\underline{\underline{A}}\,\underline{J} = \underline{\underline{A}}\,\underline{\underline{G}}\,\underline{V} + \underline{\underline{A}} J_s = 0$

$(\underline{\underline{A}}\,G\,\underline{\underline{A}}^T)\underline{e} = -\underline{\underline{A}}\,\underline{J_s}$

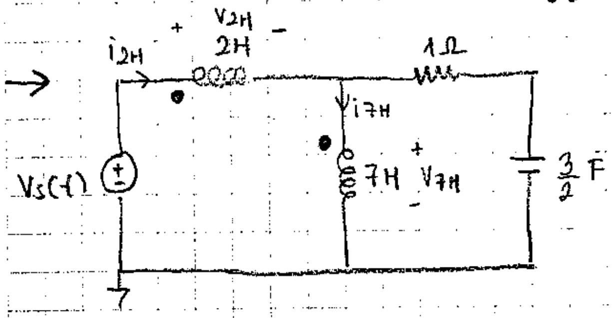
Generalized Branch

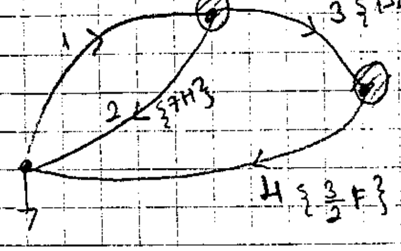
In node analysis, branch currents for all components are written, but if we have an independent source (V_s) ; [J_k for the V_s can be written]



$V_k + V_s(t) = J_k \cdot R$

→





$\underline{\underline{A}}\,\underline{J} = 0$

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \underline{\underline{A}}\,\underline{J}$$

2.) Branch Equations (3)

J_k : k^{th} branch current in terms of k^{th} branch voltages and possibly other branch voltages

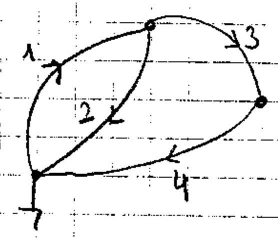
For mutual inductor:

$$\begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix}}_{M\text{(given)}} \frac{d}{dt} \begin{bmatrix} i_{2H}(t) \\ i_{7H}(t) \end{bmatrix} \qquad d/dt = D$$

Multiply D^{-1} both sides,

$$D^{-1} D \begin{bmatrix} i_{2H}(t) \\ i_{7H}(t) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix}^{-1} D^{-1} \begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 7 & -3 \\ -3 & 2 \end{bmatrix} D^{-1} \begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix}$$

$$= \begin{bmatrix} i_{2H}(t) - i_{2H}(0) \\ i_{7H}(t) - i_{7H}(0) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 7 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} \int_{0^-}^{t} V_{2H}(\tau)d\tau \\ \int_{0^-}^{t} V_{7H}(\tau)d\tau \end{bmatrix}$$



$$J_1 = i_{2H}(0) + \frac{7}{5}\int_{0^-}^{t} V_{2H}(\tau)d\tau - \frac{3}{5}\int_{0^-}^{t} V_{7H}(\tau)d\tau$$

$$J_2 = i_{7H}(0) - \frac{3}{5}\int_{0^-}^{t} V_{2H}(\tau)d\tau + \frac{2}{5}\int_{0^-}^{t} V_{7H}(\tau)d\tau$$

$$V_{2H} = V_1 + V_s \qquad V_{7H} = V_2 \qquad J_3 = \frac{V_3}{1} \qquad J_4 = C\frac{d}{dt}V_s(t) = \frac{3}{2}DV_4$$

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} 7/5\,D^{-1} & -3/5\,D^{-1} & 0 & 0 \\ -3/5\,D^{-1} & 2/5\,D^{-1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3/2\,D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} i_{2H}(0) \\ i_{7H}(0) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 7/5\,D^{-1} \\ -3/5\,D^{-1} \\ 0 \\ 0 \end{bmatrix} V_s(t)$$

$$\underline{J} = \underline{\underline{G}}\,\underline{V} + \underline{J_s}$$

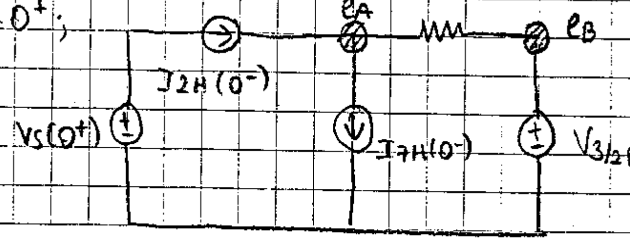
Node Equations : $(\underline{\underline{A}}\,\underline{\underline{G}}\,\underline{\underline{A}}^T)\,\underline{e} = -\underline{\underline{A}}\,\underline{J_s}$

$$\underline{\underline{A}} \to \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 3D^{-1}+1 & -1 \\ -1 & 1+3/2D \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} i_{2H}(0) - i_{7H}(0) + 2D^{-1}V_s(t) \\ 0 \end{bmatrix}$$

$\rightarrow$ integro differential equation

At $t = 0^+$;



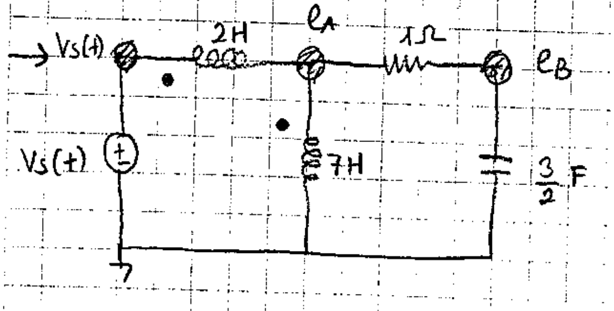
$e_A(0^+) = e_B(0^+) + J_{2H}(0^-) - J_{7H}(0^-)$

$e_B(0^+) = V_{3/2F}(0^-)$

$V_{3/2F}(0^-) \rightarrow$ I.C. for the capacitor

Sanırım $t = 0^+$'da $J_{2H}(0^-)$ vs. cont. olduğu için 0^- yazdık

Continued ;



KCL @ e_B ; $\frac{e_B - e_A}{1} + \frac{3}{2} D e_B = 0$

KCL @ e_A ; $\frac{e_A - e_B}{1} + i_{7H}(t) - i_{2H}(t) = 0$

$$\begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} d/dt\ I_{2H}(t) \\ d/dt\ I_{7H}(t) \end{bmatrix}$$

$$d/dt \begin{bmatrix} i_{2H}(t) \\ i_{7H}(t) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 7 & -3 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix}$$

$d/dt\ i_{2H}(t) = 7/5\, V_{2H}(t) - 3/5\, V_{7H}(t)$

$$\int_{0^-}^{t} (\_\_)\, d\tau \longrightarrow i_{2H}(t) - i_{2H}(0^-) = 7/5\, D^{-1} V_{2H}(t) - 3/5\, D^{-1} V_{7H}(t)$$

ltiply (-)→ ltiply (+)
$$\begin{bmatrix} i_{2H}(t) - i_{2H}(0^-) \\ i_{7H}(t) - i_{7H}(0^-) \end{bmatrix} = \begin{bmatrix} 7/5 & -3/5 \\ -3/5 & 2/5 \end{bmatrix} D^{-1} \begin{bmatrix} V_{2H}(t) \\ V_{7H}(t) \end{bmatrix}$$

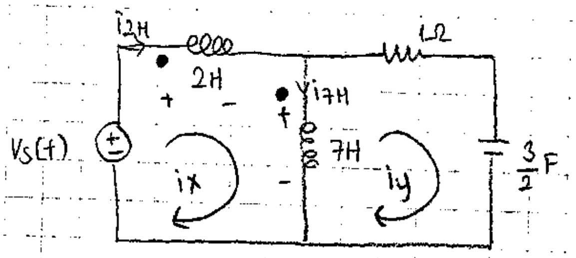
$$i_{7H}(t) - i_{2H}(t) = i_{7H}(0) - i_{2H}(0) + \begin{bmatrix} -2 & 1 \end{bmatrix} D^{-1} \begin{bmatrix} V_{7H}(t) - V_{2H}(t) \end{bmatrix}$$

$V_{7H}(t) = e_A(t)$

$V_{2H}(t) = V_s(t) - e_A(t)$

Also do not forget to find $e_A(0^+)$, $e_B(0^+)$ so that node equations are completed.

Mesh Analysis :



$$\begin{bmatrix} V_{2H} \\ V_{7H} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix} D \begin{bmatrix} i_{2H}(t) \\ i_{7H}(t) \end{bmatrix} \begin{matrix} \to ix \\ \to ix - iy \end{matrix}$$

Mesh ix ; $-V_s(t) + V_{2H}(t) + V_{7H}(t) = 0$

Mesh iy ; $iy + \frac{2}{3} D^{-1} iy(t) - V_{7H}(t) = 0$

Mesh i_y; $-V_{7H}(t) + i_y + V_c(t) = 0$

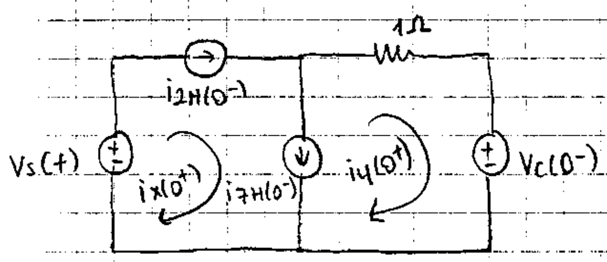
$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_y(\tau)\,d\tau$$

Mesh i_x; $-V_s(t) + 2Di_x + 3D(i_x - i_y) + 3Di_x + 7D(i_x - i_y) = 0$

$\rightarrow$ $-3Di_x - 7D(i_x - i_y) + i_y + V_c(0^-) + \frac{2}{3}D^{-1}i_y = 0$

$$\begin{bmatrix} 15D & -10D \\ -10D & 7D + \frac{2}{3}D^{-1} + 1 \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} V_s(t) \\ -V_c(0^-) \end{bmatrix} \qquad \begin{matrix} i_x(0^+) = ? \\ i_y(0^+) = ? \end{matrix}$$

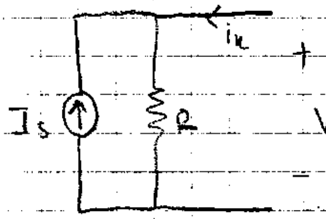
At $t = 0^+$;



$i_x(0^+) = I_{2H}(0^-)$

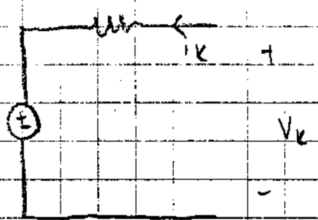
$i_y(0^+) = I_{2H}(0^-) - I_{7H}(0^-)$

If you want to use graph theoratical method for mesh analysis, then generalized branch for current sources can be required.



$V_k = R(I_s + i_k)$

Remember for the node analysis;



$\longrightarrow$ generalized branch

Modified Node Analysis

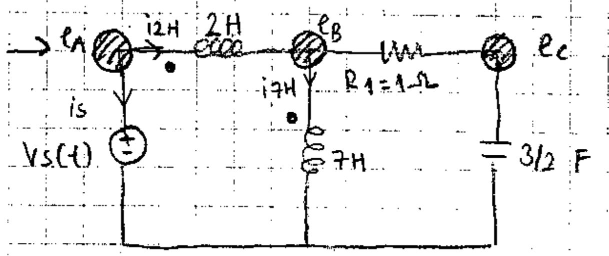
Previously, node analysis and mesh analysis equations are written
→ Integro differential equations (containing both D and D^{-1})
MNA: Only "D" terms, therefore differential equation set.

MNA: 1- Assign auxiliary variables for inductors and voltage sources in general to components whose current can not be written only with "D" operator and its voltage

2- Write node equations

3- Write equations for each auxiliary variables

Note: Do not forget initial condition.



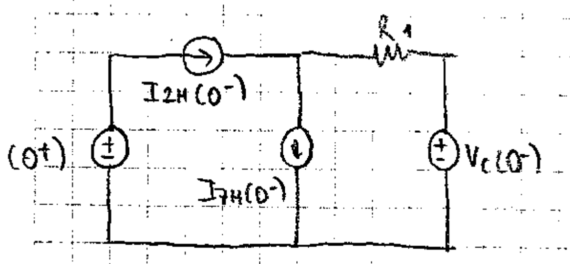
Unknowns: $\{e_A, e_B, e_C, i_s, i_{2H}, i_{7H}\}$: 6 unknowns

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1/R_1 & -1/R_1 & 0 & -1 & 1 \\ 0 & -1/R_1 & 1/R_1 + 3/2\,D & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & -2D & -3D \\ 0 & 1 & 0 & 0 & -3D & -7D \end{bmatrix} \begin{bmatrix} e_A \\ e_B \\ e_C \\ i_s \\ i_{2H} \\ i_{7H} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_s(t) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_{2H} \\ V_{7H} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix} D \begin{bmatrix} i_{2H} \\ i_{7H} \end{bmatrix}$$

$V_{2H} = e_A - e_B$

$V_{7H} = e_B$



$e_A(0^+) = V_s(0^+)$

$e_B(0^+) = V_c(0^-) + R_1 \left(I_{2H}(0^-) - I_{7H}(0^-) \right)$

$e_C(0^+) = V_C(0^-)$

$i_{2H}(0^+) = i_{2H}(0^-)$; $i_{7H}(0^+) = i_{7H}(0^-)$

- State equations -

Dynamics system containing N dynamic elements, there are many ways of characterizing the system

N^{th} order scalar differential equation ; $(D^N + 3D^{N-1} + 2D^{N-2} + \ldots + 12D + 25)\, x(t) = f_{ext}(t) + 2Df_{ext}(t)$

Mesh
Node] Matrix integro diff. equation

State eqn.
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_s(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} i_s(t)$$

1^{st} order matrix 2x2 differential equation

$$\underline{\dot{x}}(t) = \underline{A}\, \underline{x}(t) + \underline{B}.\underline{u}(t)$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_{\underline{B}} \underbrace{\begin{bmatrix} V_s(t) \\ I_s(t) \end{bmatrix}}_{\underline{V}(t)}$$

State variables = $\{V_C(t), I_L(t)\}$ (Cap voltages and inductor currents)

Solution with Laplace Transform (characteristic Eqn)

Writing State Equation

1.) Select a proper "tree" i) All voltage sources in tree, All current sources in co-tree

i) Put maximum number of capacitors in tree

ii) Put maximum number of inductors in co-tree

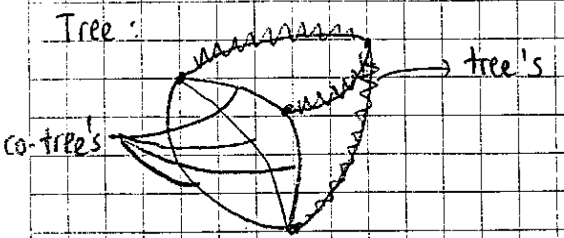
iii) If there is a transformer, put a branch, (only one branch) in tree, the other one in co-tree

2.) Write fundamental loop equations for each inductor in co-tree

3.) Write fundamental cutset eqn for each capacitor in tree

State Variables ; $\left\{ \begin{array}{l} \text{tree capacitors} \\ \text{co-tree inductors} \end{array} \right\}$

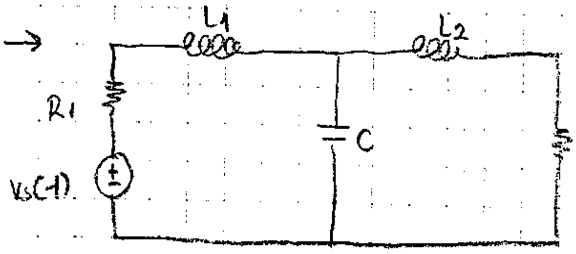
Tree :



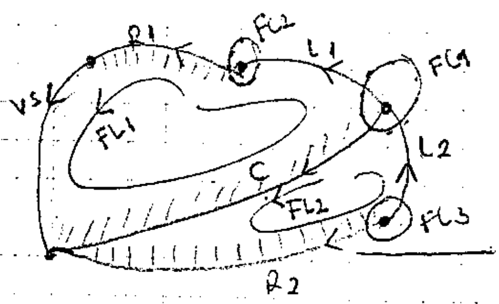
: Tree's : tüm node'lara ulaşacak
Loop oluşturmayacak

Fundamental loop : A loop which contains only one co-tree
: They're associated with a single co-tree branch

Fundamental cut-set : are associated with a single tree branch.



(Order at most tree)



[Inductorları tree yapmamak için R1 & R2 seçtik] Node'lara ulaştık böylece

- State equations : Do not use generalized branches

State variables : $\{V_c, I_{L1}, I_{L2}\}$

3 state variables $\longrightarrow$ 3^{rd} order system

$$\begin{bmatrix} \dot{V_c} \\ \dot{I_{L1}} \\ \dot{I_{L2}} \end{bmatrix} = \begin{bmatrix} 0 & -1/C & 1/C \\ 1/L_1 & -R_1/L_1 & 0 \\ -1/L_2 & 0 & +R_2/L_2 \end{bmatrix} \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ -1/L_1 \\ 0 \end{bmatrix} V_s + \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \dot{V_s}}$$

1) Write fundamental cut-set for capacitor current

FC1 $\longrightarrow$ $i_c + i_{L1} - i_{L2} = 0$ $\qquad \dot{V_c} = \dfrac{i_{L2} - i_{L1}}{C}$

$C \cdot \dot{V_c}$

(minimal)

2) Write fundamental loop for L1 voltage

$V_{L1} + V_{R1} + V_s - V_c = 0$

$L \dot{I_{L1}} + i_{R1} \cdot R + V_s - V_c = 0$

$\hookrightarrow FC_2 \longrightarrow i_{L1}$

$$\dot{I_{L1}} = \frac{1}{L_1}\left[-R_1 i_{L1} + V_c - V_s \right]$$

3.) Write fundamental loop for L_2 voltage

$FL_2 \rightarrow V_{L2} + V_C - V_{R2} = 0$

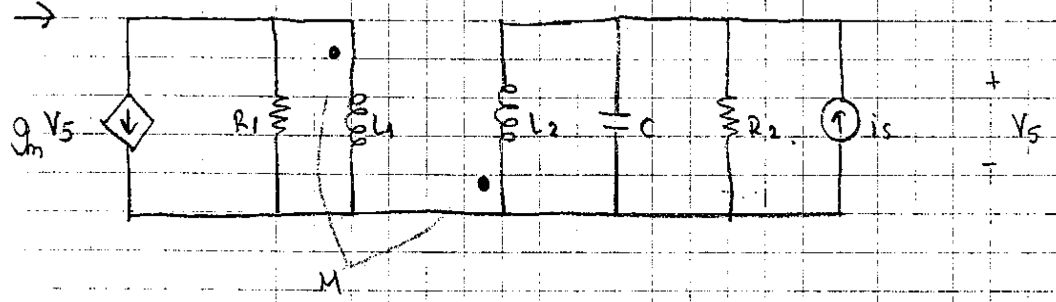
$L_2 I_{L2}^{\bullet} + V_C \underbrace{- R_2 \cdot i_{R2}}_{\rightarrow -R_2 \cdot i_{L2}} = 0$; $I_{L2}^{\bullet} = \frac{1}{L_2} \cdot (-V_C + R_2 i_{L2})$

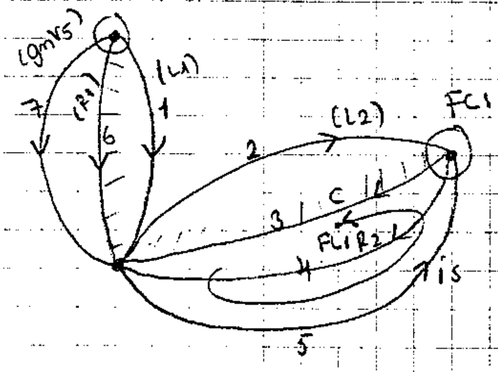
And we have ; $V_C^{\bullet}(t) = \frac{-i_{L1} + i_{L2}}{C}$

I.C.

$V_C(0^+) = V_C(0^-)$
$I_{L1}(0^+) = I_{L1}(0^-)$
$I_{L2}(0^+) = I_{L2}(0^-)$

→





(Çizgili yerler tree)

State variables ; $\{V_C, I_{L1}, I_{L2}\}$

- again 3rd order system

1.) Fundamental cut-set for $i_C = i_3$;

FC_1 ; $i_3 + i_4 - i_5 - i_{L2} = 0$

$C \cdot V_C^{\bullet} - i_S + \frac{V_4}{R_2} - i_{L2} = 0$

$\rightarrow V_4 = V_3 = V_C$;

$C \cdot V_C^{\bullet} - i_S + \frac{V_C}{R_2} - i_{L2} = 0$; $V_C^{\bullet} = \frac{1}{C}\left[i_S - \frac{V_C}{R_2} + i_{L2} \right]$

2.) 3.) State eqn for I_{L1} & I_{L2}

Write mutual inductor equation:

$$\begin{bmatrix} V_{L1} \\ V_{L2} \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} i_{L1}^{\bullet} \\ i_{L2}^{\bullet} \end{bmatrix}$$

GIPTA

V_{L_1} $\xrightarrow{\text{fund. loop}}$ $V_{L_1} = V_6$ $\xrightarrow{\text{terminal eqn.}}$ $i_6 R_1$ $\xrightarrow{\text{fundam. cutset}}$ $(-i_{L_1} - g_m V_5) R_1$ (10)

V_1

$\xrightarrow[\text{for } V_5]{\text{funda. loop}}$ $(-i_{L_1} - g_m V_C) R_1$ **

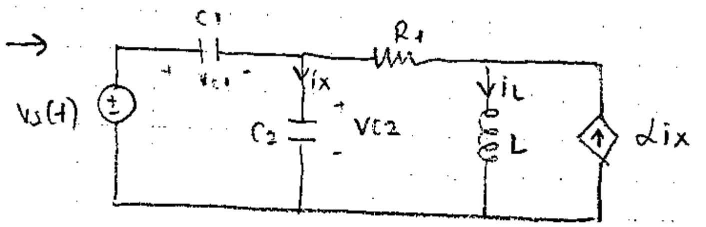
V_{L_2} $\xrightarrow{\text{fund. loop}}$ $-V_C = V_{L_2}$ ***

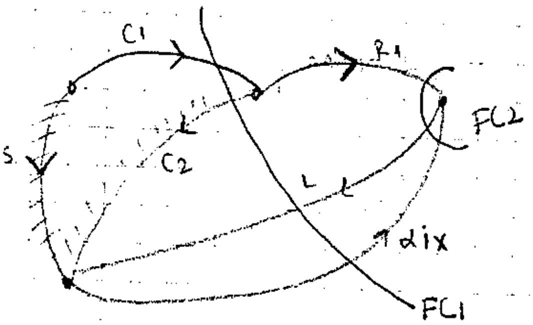
$$\begin{bmatrix} \dot{I}_{L_1} \\ \dot{I}_{L_2} \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}^{-1} \begin{bmatrix} V_{L_1} \\ V_{L_2} \end{bmatrix}$$

$V_{L_1} \to (-i_{L_1} - g_m V_C) R_1$

$V_{L_2} \to -V_C$

→





Voltage sourcelar treede olmak zorunda
Capacitorden seçsen de yeter.

State variables; $\{V_{C2}, I_L\}$ 2^{nd} order circuit

1) Fundamental cut-set for C_2 current

$FC_1 \longrightarrow -i_{C_1} + i_{C_2} + i_L - \alpha i_x = 0$

state variable

$i_x = C_2 \cdot \dot{V}_{C_2}$

$i_{C_1} = C_1 \cdot \dot{V}_{C_1}$, $\quad i_{C_2} = C_2 \cdot \dot{V}_{C_2}$

$C_1 \cdot (\dot{V}_S - \dot{V}_{C_2})$ fundamental loop

The Equation becomes; $-C_1\dot{V}_S + C_1\dot{V}_{C_2} + C_2\dot{V}_{C_2} + i_L - \alpha C_2 \dot{V}_{C_2} = 0$

$$(C_1 + C_2 - \alpha C_2)\dot{V}_{C_2} = -i_L + C_1 \dot{V}_S$$

$$\dot{V}_{C_2} = \frac{1}{C_1 + C_2 - \alpha C_2} \left(-i_L + C_1 \dot{V}_S\right)$$

2) Fundamental Loop for Inductor Voltage

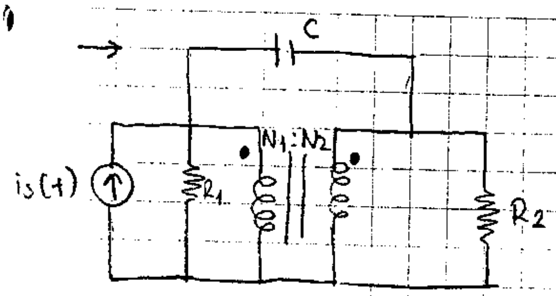
$\dot{V}_{C_2}$ yerine yaz

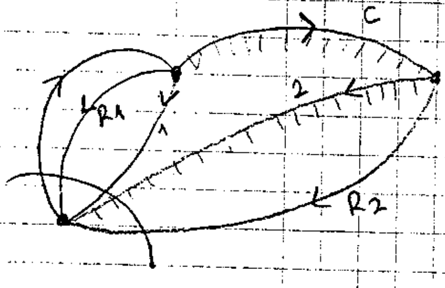
$V_{L_1} = -V_{R_1} + V_{C_2} = L_1 \dot{I}_L = -R_1 i_L + \alpha C_2 \dot{V}_2 R_1 + V_{C_2}$; $\dot{I}_L = \frac{1}{L}\left(-R_1 I_L + \alpha C_2 \dot{V}_{C_2} R_1 + V_{C_2}\right)$

$V_{R_1} = i_{R_1} \cdot R_1 = (i_L - \alpha i_x) R_1 = (i_L - \alpha C_2 \dot{V}_2) R_1$

fundamental cut-set 2

GIPTA





$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

1- Fundamental cut-set for $C.\dot{V_c}$ i.e. i_c current

$$C.\dot{V_c} = i_s - i_{R_1} - i_1$$

$$i_{R_1} \longrightarrow \frac{V_{R_1}}{R_1} \xrightarrow{\text{fund. loop}} \frac{(V_c + V_2)}{R_1}$$

$$V_2 = \frac{N_2 V_1}{N_1} \xrightarrow{\text{funda. loop}} \frac{N_2}{N_1}(V_c + V_2) \; ; \quad V_2 = \frac{\frac{N_2}{N_1} V_c}{\left(1 - \frac{N_2}{N_1}\right)} = \frac{N_2 V_c}{N_1 - N_2}$$

$$* \; i_{R_1} = \frac{1}{R_1} V_c \left(1 + \frac{N_2}{N_1 - N_2}\right)$$

$$* \; i_1 = -\frac{N_2}{N_1} i_2 \xrightarrow{\text{funda. cutset}} -\frac{N_2}{N_1}(i_s - i_{R_1} - i_1 - i_{R_2})$$

i_{R_2} ↓ Terminal eqn.

$$\frac{V_{R_2}}{R_2} \xrightarrow{\text{funda. loop}} \frac{V_2}{R_2}$$

$$i_1\left(1 - \frac{N_2}{N_1}\right) = -\frac{N_2}{N_1} i_s + \frac{N_2}{N_1} i_{R_1} + \frac{N_2}{N_1}\frac{V_2}{R_2} \rightarrow \text{bulmuştuk!}$$

$$i_1 = \frac{i_s - \frac{V_c}{N_1 - N_2}\left(\frac{N_1}{R_1} + \frac{N_2}{R_2}\right)}{1 - \frac{N_2}{N_1}}$$

$$C\dot{V_c} = i_s - i_{R_1} - i_1$$

<u>State eqn:</u>

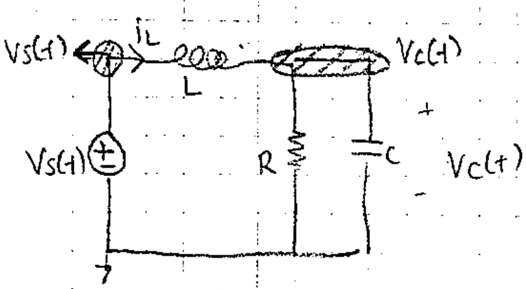
$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} V_s(t) + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \dot{V_s}(t)$$

HW 1 2PS - I : 1c, 1d, 2a, 2d due to : next week.

Solution of n^{th} order diff- equations

First, we'll examine zero-input case

i.e. Only initial conditions



Node equations: $\frac{V_c(t)}{R} + C\dot{V_c}(t) - i_L = 0$

$\hookrightarrow i_L(t) = i_L(0^-) + \frac{1}{L}\int_0^t \underbrace{V_L(\tau)d\tau}_{V_s - V_c(t)}$

$$\left(\frac{1}{R} + CD\right)V_c(t) - \left[i_L(0^-) + \frac{1}{L}D^{-1}\left(V_s(t) - V_c(t)\right)\right] \; 0$$

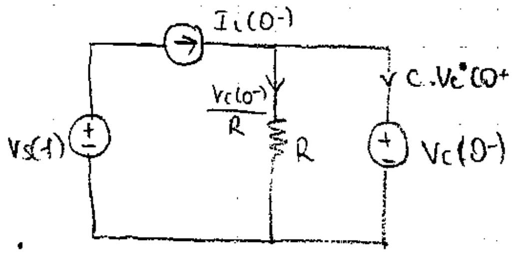
Apply $D = d/dt$ operator both sides ;

$$\left(\frac{D}{R} + CD^2\right)V_c(t) - \frac{1}{L}\left(V_s(t) - V_c(t)\right) = 0$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V_c(t) = \frac{1}{LC}V_s(t)$$

$V_c(0^+) = ?$

$\dot{V_c}(0^+) = ?$ At $t = 0^+$;



$C\dot{V_c}(0^+) = I_L(0^-) - \frac{V_c(0^-)}{R}$

Rewriting again;

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V_c(t) = \frac{1}{LC}V_s(t) \quad V_c(0^+),\ \dot{V_c}(0^+) \text{ given}$$

[Constant coefficient diff. equation]

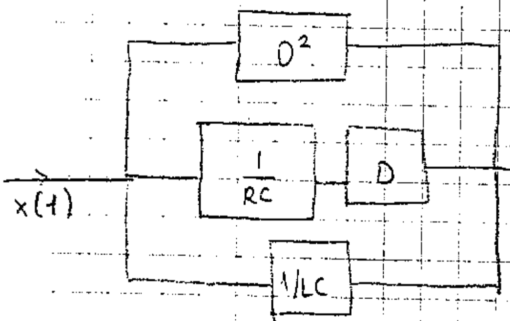
Solution
- particular
- homogenous

Solution
- zero input (only initial cond) sources are zero
- zero state (initial condit. are zero.) sources are active

For now, let's focus on zero-input then;

$\left(D^2+\frac{1}{RC}D+\frac{1}{LC}\right)V_c(t)=0$ $V_c(0^+)$, $\dot{V}_c(0^+)$ are given

For CCDE systems, the exponential functions and eigenfunctions of the system.



← definition of eigenfunction

$\left(D^2+\frac{1}{RC}D+\frac{1}{LC}\right)V_c(t)=0$

define $V_c(t)=\alpha e^{\lambda t}$

$\alpha\left(\lambda^2+\frac{1}{RC}\lambda+\frac{1}{LC}\right)e^{\lambda t}=0$

↳ $\alpha=0$ $V_c(t)=0$ ($V_c(0^+)\neq 0$ IC)

↳ $e^{\lambda t}\neq 0$

↳ $\lambda^2+\frac{1}{RC}\lambda+\frac{1}{LC}=0$ (set λ is equal to roots of the equation)

<u>Example</u>: Set $R=1/3$ $C=1$ $L=1/2$

$(D^2+3D+2)V_c(t)=0$

$V_c(0^+)=V_0$, $\dot{V}_c(0^+)=I_0-3V_0$ (hoca verdi)

$\lambda^2+3\lambda+2=0$ $\lambda=\{\underbrace{-1,-2}\}$ → natural frequencies (roots of charact. eq.)

$V_c(t)=\{\alpha e^{-t}, \beta e^{-2t}\}$

$\boxed{V_c(t)=\alpha e^{-t}+\beta e^{-2t}}$ Select α, β such that I.C. at 0^+ as satisfied.

Question: (Mode excitation)

$$(D^2+3D+2)V_c(t)=0 \quad V_c(0^+)=V_0 \quad \dot{V}_c(0^+)=J_0-3V_0$$

$V_c(t)=\alpha e^{-t}+\beta e^{-2t}$ (modes of the circuit)

Q: Excite only one mode by selecting I.C.'s (V_0, J_0) properly

$V_c(0^+)=\alpha+\beta$

$\dot{V}_c(0^+)=-\alpha-2\beta$

To have the mode $V_c(t)=\alpha e^{-t} \longrightarrow \beta=0 \longrightarrow V_c(0^+)=\alpha$

$\dot{V}_c(0^+)=-\alpha$

J_0-3V_0

$V_c(0^-)$

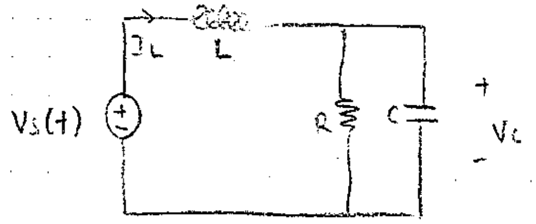
$$\begin{bmatrix} V_0 \\ J_0 \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

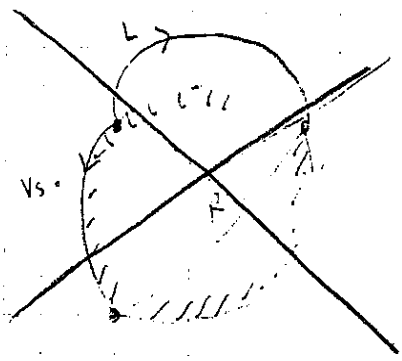
$J_L(0^-)$

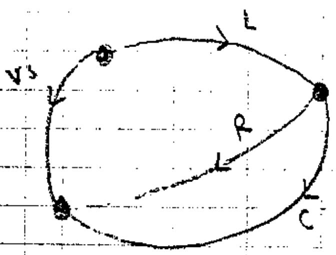
$V_c(t)=\beta e^{-2t} \longrightarrow \alpha=0 \longrightarrow V_c(0^+)=\beta \longrightarrow \begin{bmatrix} V_0 \\ J_0 \end{bmatrix} \propto \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\dot{V}_c(0^+)=-2\beta$

Example:







State variable: $\{V_c, J_L\}$

1- $C\dot{V}_c=-i_R+i_L=-\frac{V_c}{R}+i_L$

$\frac{V_R}{R}=\frac{V_c}{R}$

2- $L\dot{J}_L=V_s-V_R=V_s-V_c$

$$\begin{bmatrix} \dot{V_c}(t) \\ \dot{I_L}(t) \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} V_c(t) \\ I_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s(t)$$

$V_c(0^+) = V_c(0^-)$; $I_L(0^+) = I_L(0^-)$

To show that State eqn form ≡ Node Analysis form

Take derivative of $\dot{V_c}(t) =$ ______ equation

$$\ddot{V_c}(t) = -\frac{1}{RC}\dot{V_c}(t) + \frac{1}{C}\underbrace{\dot{I_L}(t)}_{-\frac{1}{L}V_c(t) + \frac{1}{L}V_s(t)}$$

$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V_c(t) = \frac{1}{LC}V_s(t)$ Same as node analysis eqn.

Example: $R = 1/3$, $C = 1$, $L = 1/2$

$$\begin{bmatrix} \dot{V_c}(t) \\ \dot{I_L}(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} V_c(t) \\ I_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} V_s(t)$$

$\dot{x}(t) = \underline{A}\,\underline{x}(t) + \underline{b}\,V_s(t)$

$\rightarrow$ zero input case

$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t)$

let $\underline{x}(t) = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} e^{\lambda t}$

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \lambda e^{\lambda t} = \underline{\underline{A}} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} e^{\lambda t} \xrightarrow{\text{trivial soln}} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ not suitable.}$$

$$\rightarrow \lambda \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \underline{\underline{A}} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$ λ : eigenvalues of $\underline{\underline{A}}$

$\underline{\underline{A}} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $\xrightarrow{|\lambda \underline{I} - \underline{A}| = 0}$ $\det\left(\begin{bmatrix} \lambda+3 & -1 \\ 2 & \lambda \end{bmatrix}\right) = 0$; $\lambda^2 + 3\lambda + 2 = 0$

$\lambda \{-1, -2\}$ (natural frequencies)

Mode excitation using State Formalism

$\underline{x}^{\circ}(t) = \underline{\underline{A}}\, \underline{x}(t)$ $\qquad A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$

$\underline{x}(t) = \left\{ \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{-t}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} e^{-2t} \right\}$

Only to have $x(t) = \underset{\lambda_1 = -1}{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda_1 t}} \longrightarrow x^{\circ}(t) = \underline{\underline{A}}\, x(t)$

$\lambda_1 \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \cancel{e^{\lambda_1 t}} = A \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \cancel{e^{\lambda_1 t}}$

$\downarrow$

$\underline{\underline{A}} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \lambda_1 \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$

$\downarrow$

eigenvector for $\lambda = \lambda_1$

Eigenvectors of A;

$\lambda = -1 \longrightarrow \underline{\underline{A}}\, \underline{e}_1 = \lambda \underline{e}_1 \Big|_{\lambda = -1} \longrightarrow (\lambda \underline{\underline{I}} - \underline{\underline{A}}) \underline{e}_1 = 0$

$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = 0 \qquad \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} V_c(t) \\ I_L(t) \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} \qquad x(t) = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda_1 t}$ is the solution

since $(\lambda_1 = -1)$

To have a solution in the form;

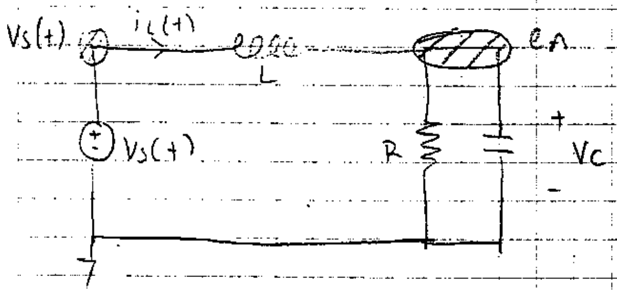
$x(t) = \underset{\lambda_2 = -2}{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} e^{\lambda_2 t}} \longrightarrow \begin{bmatrix} \lambda + 3 & -1 \\ 2 & \lambda \end{bmatrix} \underline{e}_2 \Big|_{\lambda = -2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \underline{e}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$e_2 \propto \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$ if $V_C(0^-) = 1V$
$I_L(0^-) = 1A$

Finding Natural Frequency

1. Scalar Diff Eqn
2. State Eqn
3. Modified Node Analysis



$i_L(t)$: auxiliary variables

$$\begin{bmatrix} 1/R + CD & -1 \\ 1 & LD \end{bmatrix} \begin{bmatrix} e_A(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} 0 \\ V_S(t) \end{bmatrix} \quad V_L(t) = L \frac{d}{dt} i_L(t)$$

RHS

Introduced (since MNA)

Zero-Input Solution RHS = $\underline{0}$

Guess

$$\begin{bmatrix} e_A(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t}$$ for zero input

$$\begin{bmatrix} 1/R + C\lambda & -1 \\ 1 & L\lambda \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For nontrivial Solution should have zero determinant

Let $R=1/3 \quad C=1 \quad L=1/2$

$$\left|\begin{bmatrix} \lambda+3 & -1 \\ 1 & \lambda/2 \end{bmatrix}\right| = 0$$

determinant

Characteristic Polynomial : $(\lambda+3)\lambda/2 + 1 = 0$

$\lambda^2+3\lambda+2=0 \cdot 2 = 0 \quad (\lambda+2)(\lambda+1)=0 \quad \lambda = \{-2,-1\}$

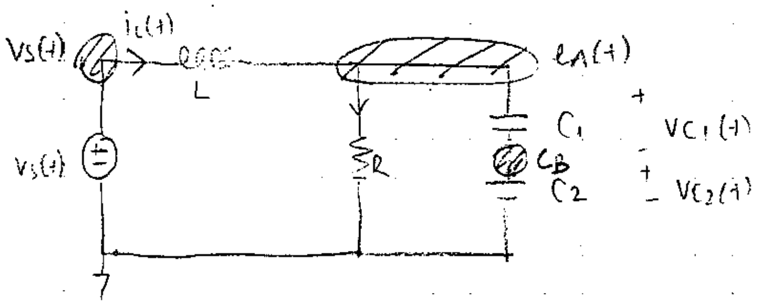
To only excite $\lambda=-1$ mode, what should 0^+ values of circuit variables.

$$\begin{bmatrix} e_A(t) \\ I_L(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{-t} \longleftarrow \text{desired single mode excitation}$$

$$\underbrace{\begin{bmatrix} \overset{2}{\lambda+3} & -1 \\ 1 & \overset{-1/2}{\lambda/2} \end{bmatrix}}_{\lambda=-1} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{-t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix} ; \quad \begin{bmatrix} e_A(t) \\ I_L(t) \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} \longrightarrow \begin{bmatrix} e_A(0^+) \\ I_L(0^+) \end{bmatrix} \propto \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

4- By node Analysis ($e_A, e_B, e_C \ldots$)



KCL @ e_A : $\dfrac{e_A}{R} + C_1(\dot{e}_A - \dot{e}_B) - \left[\left(i_L(0^-) + \dfrac{1}{L} \int_{0^-}^{t} (v_s(t) - e_A(t))\,dt \right) \right] = 0$

KCL @ e_B : $C_2 \cdot \dot{e}_B + C_1(\dot{e}_B - \dot{e}_A) = 0$

$$\begin{bmatrix} 1/R + C_1D + 1/L\, D^{-1} & -C_1D \\ -C_1D & C_1D + C_2D \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} i_L(0^-) + 1/L\, D^{-1} V_s(t) \\ 0 \end{bmatrix}$$

Zero input Solution: $\begin{bmatrix} e_A(t) \\ e_B(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t}$ (since CCDE)

In NA, we have integro diff. eqn (MNA diff. eqn).

$$\begin{bmatrix} D/R + C_1 D^2 + 1/L & -C_1 D^2 \\ -C_1 D & (C_1 + C_2) D \end{bmatrix} \begin{bmatrix} e_A(t) \\ e_B(t) \end{bmatrix} = \begin{bmatrix} 1/L \; V_s(t) \\ 0 \end{bmatrix}$$ $V_s(t) = 0$ zero-input

So take 1st derivative of KCL @ e_A eqn.

$R = 1/3\,\Omega$ $C_1 = 2$ F $L = 1/2$ H
$C_2 = 2$ F

$$\begin{bmatrix} e_A(t) \\ e_B(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t}$$

$$\begin{bmatrix} 3\lambda + 2\lambda^2 + 2 & -2\lambda^2 \\ -2\lambda & 4\lambda \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\det = 0;\; \longrightarrow (3\lambda + 2\lambda^2 + 2)\, 4\lambda - 4\lambda^3 = 0$

$4\lambda(\lambda^2 + 3\lambda + 2) = 0$

$\lambda = \{0, -1, -2\}$

To only excite $\lambda = 0$;

$$\begin{bmatrix} e_A(t) \\ e_B(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t}$$

$$\begin{bmatrix} 3\lambda + 2\lambda^2 + 2 & -2\lambda^2 \\ -2\lambda & 4\lambda \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = 0$

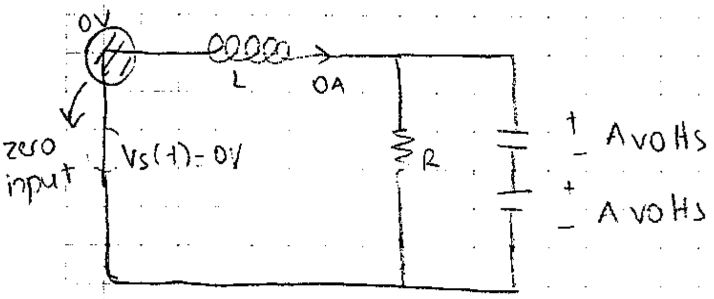
$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \propto \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\propto$ → proportional

$2\alpha_1 = 0$

$$\underbrace{\begin{bmatrix} e_A(t) \\ e_B(t) \end{bmatrix}}_{e_A(t) \to V_{C1}(t)+V_{C2}(t),\; e_B(t) \to V_{C2}(t)} \propto \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ A \end{bmatrix}$$

$$\begin{bmatrix} V_{C1}(t) \\ V_{C2}(t) \end{bmatrix} \propto \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -A \\ A \end{bmatrix}$$



<u>Note 1</u>: We preffered modified node analysis due to presence of D^{-1} operators in node analysis. We have cancelled D^{-1} operators by an additional derivative and this cancellation operation can make natural frequency at $\lambda=0$ invisible. (due to cancellations)

In this example, $\lambda=0$ mode is not cancelled (there was not any such cancellation leading to invisibility of a mode) but in general, this is possible.

- In general $\lambda=0$ mode can appear when you have series/parallel capacitors or inductors in the system.

<u>Note 2</u>: We prefer state eqns to the other method since.

1 Method for eqn writing

2 Method for natural freq. finding $\to$ eig(A) (matlab)

3 Mode excitations are also available $\longrightarrow$ eigenvectors of $\underline{A}$

<u>Stability</u>:

Let's focus on zero-input solution (no forcing term / external input)

Given initial condi. if initial cond. decay the zero,

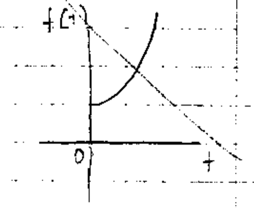
We call that system "stable" system (other def. are also possible)

$\begin{bmatrix} V_C(t) \\ I_L(t) \end{bmatrix} \longrightarrow \begin{array}{l} V_C(t) = \alpha_1 e^{\lambda_1 t} + \beta_1 e^{\lambda_2 t} \\ I_L(t) = \alpha_2 e^{\lambda_1 t} + \beta_2 e^{\lambda_2 t} \end{array}$ (21)

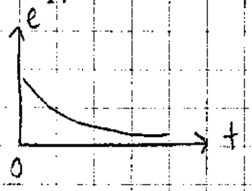
For stability $\longrightarrow$ $Re\{\lambda_1\} < 0$ natural frequencies should have negative real parts

$Re\{\lambda_2\} < 0$

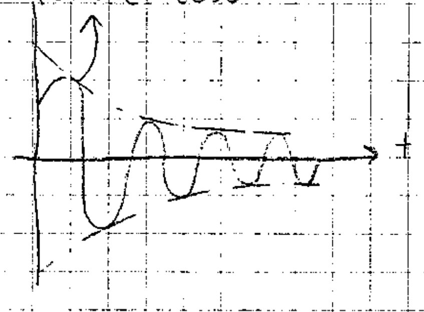
$f(t) = e^{2t}$



e^{-2t}



$f(t) = e^{-2t}\cos 5t$



$f(t) = e^{\lambda t}$ $\lambda = -2 + j3$

natarqument

complex valued function

$f(t) = e^{(-2+j3)t} = e^{-2t}(\cos 3t + j\sin 3t)$

$e^{j\phi} \triangleq \cos\phi + j\sin\phi$

$e^{z}_{z \in \mathbb{C}} = \sum_{k=0}^{\infty} \frac{z^k}{k!}$ k^{th} power of complex number

$\|f(t)\| \to 0$ as $t \to \infty$

then $f(t) \to 0 + j0$
$t \to \infty$

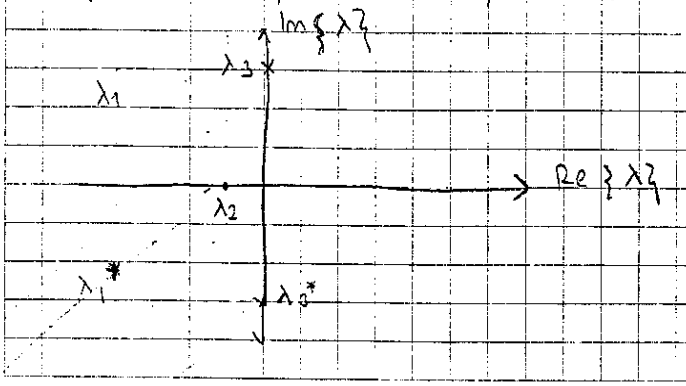
Further details:

$t \to \infty$

① All λ's have negative real parts $\longrightarrow$ asymtotically stable. $f(t) \to 0$

② All λ's have negative or zero real parts $\longrightarrow$ stable (only) $f(t) \to 5$ as $t \to \infty$

③ Any λ's have positive real part $\longrightarrow$ unstable. $f(t) \to \infty$



$\longrightarrow$ asymtotically stable

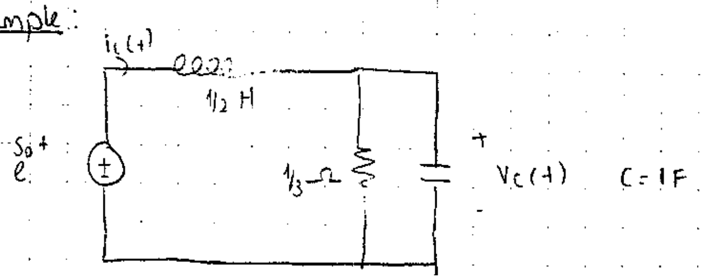
Solving N^{th} Order Circuits

Our focus is on inputs of type $f(t) = e^{s_0 t}$

s_0 is a complex number

In this part, we'll find only particular solution to $f(t) = e^{s_0 t}$

Example:



Scalar Diff. Eqn Case:

$(D^2+3D+2)V_C(t) = 2V_S(t) = 2e^{s_0 t}$

$V_C(t) = V_C^h(t) + V_C^p(t)$

$V_C^p(t) = Ae^{s_0 t}$

$\downarrow$ subst into diff eqn

$A(s_0^2+3s_0+2)\cancel{e^{s_0 t}} = 2\cancel{e^{s_0 t}}$

$A = \dfrac{2}{s_0^2+3s_0+2}$ provided that $s_0^2+3s_0+2 \neq 0$ or s_0 is not natural frequency

$\rightarrow (D^2+3D+2)V_C(t) = 2V_S(t)$

$V_C(0^+) = V_0$

$\dot{V}_C(0^+) = \dot{V}_0$

$V_C^{complete}(t) = \alpha e^{-t} + \beta e^{-2t} + V_C^{particular}$

Particular Solution

Let's $V_S(t) = e^{s_0 t}$ $\quad s_0 \in \mathbb{C} \rightarrow$ complex number

$\downarrow$

$(D^2+3D+2)V_C^p(t) = 2V_S(t)$

Guess $V_C^p(t) = Ae^{s_0 t} \longrightarrow A(s_0^2+3s_0+2)\cancel{e^{s_0 t}} = 2\cancel{e^{s_0 t}}$

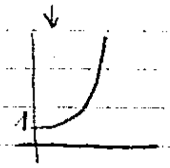
$A = \dfrac{2}{s_0^2+3s_0+2}$ $\quad s_0 \neq \{-1, -2\}$

$s_0 \neq \{\text{natural frequency}\}$

$$V_c^p(t) = \frac{2}{s_0^2+3s_0+2}\, e^{s_0 t}$$

$V_s(t)$	$V_c^p(t)$
e^{-3t}	$\to e^{-3t}$
e^{-5t}	$\to \frac{2}{12}\, e^{-5t}$
1	$\to 1$
e^{-2tJ}	$\to \frac{2}{-2-J6}\, e^{-J2t}$
$e^{t}\ (s_0=1)$	$\to \frac{1}{3}\, e^{t}$

$$\to A = \frac{2}{s_0^2+3s_0+2}\bigg|_{s_0=-3} = 1 \to 1.e^{-3t}$$



Example: Same circuit with state eqn

$$\begin{bmatrix} V_c^{\circ}(t) \\ I_L^{\circ}(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} V_s(t)$$

Interest: Particular solution for exponential family of inputs ($V_s(t) = e^{s_0 t}$)

Guess:
$$\begin{bmatrix} V_c^p(t) \\ I_L^p(t) \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{s_0 t}$$

Substitute the guess into state eqn.

$$s_0 \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{s_0 t} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{s_0 t} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{s_0 t}$$

$$\left(s_0 \cdot I - \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \right) \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} s_0+3 & -1 \\ 2 & s_0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\frac{1}{s_0^2+3s_0+2}\begin{bmatrix}s_0 & 1\\ -2 & s_0+3\end{bmatrix}\begin{bmatrix}0\\ 2\end{bmatrix}=\frac{1}{s_0^2+3s_0+2}\begin{bmatrix}2\\ 2(s_0+3)\end{bmatrix} \qquad s_0\neq\{-1,-2\}$$

$$\begin{bmatrix}V_c^p(t)\\ I_L^p(t)\end{bmatrix}=\frac{1}{s_0^2+3s_0+2}\begin{bmatrix}2\\ 2(s_0+3)\end{bmatrix}e^{s_0t} \;;$$

$$V_c^p(t)=\frac{2}{s_0^2+3s_0+2}\,e^{s_0t}$$

<u>Example:</u> Same circuit with $V_s(t)=\cos 5t$

$$(D^2+3D+2)\,V_c^p(t)=2\cos 5t$$

$$V_c^p(t)=A\cos 5t+B\sin 5t \qquad \times 2$$

$$\dot{V}_c^p(t)=-5A\sin 5t+5B\cos 5t \qquad \times 3$$

$$\ddot{V}_c^p(t)=-25A\cos 5t-25B\sin 5t \qquad \times 1 \;;$$

$$\cos 5t\,(2A+15B-25A)+\sin 5t\,(2B-15A-25B)$$

$$\cos 5t\,(-23A+15B)+\sin 5t\,(-23B-15A)=2\cos 5t$$

$-23B=15A$

$$-23A+15B=2 \quad \times 15 \;;\quad -15\cdot 23A+225B=30$$

$$15A+23B=0 \quad \times 23 \qquad 15\cdot 23A+23^2B=0$$

$(15^2+23^2)B=30$ buradan A ve B bulunur.

<u>or:</u> $(D^2+3D+2)V_c^p(t)=2\underbrace{Re\{e^{j5t}\}}_{\cos 5t}$

$2\times$ $V_c^p(t)=Re\{A_c e^{j5t}\}$ ← Guess ; $\quad A_c\in\mathbb{C}$

$3\times$ $\dot{V}_c^p(t)=Re\left\{\frac{d}{dt}\{A_c e^{j5t}\}\right\}=Re\{A_c j5\, e^{j5t}\}$

$1\times$ $\ddot{V}_c^p(t)=Re\left\{\frac{d^2}{dt^2}\{A_c e^{j5t}\}\right\}=Re\{A_c(-25)\,e^{j5t}\}$

$$(D^2+3D+2)\,V_c^p(t)=Re\{2A_ce^{j5t}\}+Re\{3A_c(j5)\,e^{j5t}\}+Re\{A_c(-25)\,e^{j5t}\}$$

$$=Re\{2e^{j5t}\}$$

$$2A_c\cancel{e^{j5t}}+3A_c(j5)\,\cancel{e^{j5t}}+A_c(-25)\,\cancel{e^{j5t}}=2\cancel{e^{j5t}}$$

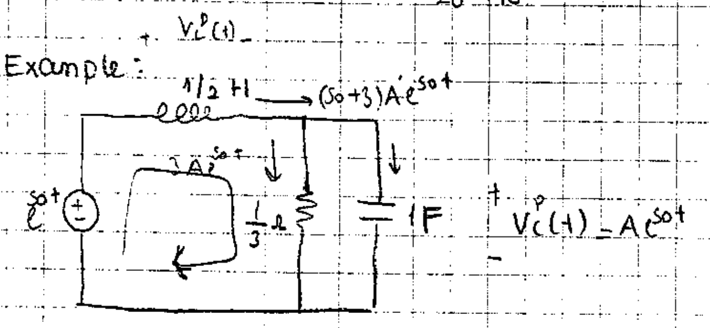
$$2A_c+15A_cj5-25A_c=2$$

$A_c = \dfrac{2}{2+j15-25}$ $\quad V_c^p(t) = Re\left\{\dfrac{2}{23+j15}\, e^{j15}\right\}$

$= Re\left\{\dfrac{21-23-j15}{23^2+15^2}\left(\cos 5t + j\sin 5t\right)\right\}$

$= \dfrac{2}{23^2+15^2}\left(-23\cos 5t + 15\sin 5t\right)$

Example:



$V_c^p(t) = ?$

$V_c^p(t) = A e^{s_0 t}$ $\quad I_c(t) = C \cdot A s_0 e^{s_0 t}\Big|_{C=1} = s_0 A e^{s_0 t}$

$V_L^p(t) = L \cdot D\left[(s_0+3) A e^{s_0 t}\right]$

$V_L^p(t) = L \cdot A s_0 (s_0+3) e^{s_0 t} = \dfrac{A s_0 (s_0+3)}{2} e^{s_0 t}$

KVL; $-e^{s_0 t} + V_L^p(t) + V_c^p(t) = 0 \longrightarrow A = \dfrac{2}{s_0^2 + 3s_0 + 2}$

Phasors: (related to phases)

$f(t) = A e^{s_0 t}$

$\uparrow$ phasor quantity

$f(t) = (1+j)\, e^{j5t}$

$\longleftarrow$ phasor $= 1+j = \sqrt{2}\,\angle 45°$

$2\cos\left(\omega t + \dfrac{\pi}{6}\right) = 2\,Re\left\{e^{j(\omega t + \pi/6)}\right\}$

$= Re\left\{2e^{j\pi/6}\, e^{j\omega t}\right\}$ $\quad 2\,\angle 30$ phasor

$2\cos(\omega t + 30°) \xrightarrow{\text{phasor}} 2\,\angle 30°$

Example: $\underbrace{\cos(4t+30^\circ)}_{A} + \underbrace{\cos(4t+60^\circ)}_{B} = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$= Re\{e^{j(4t+30^\circ)}\} + Re\{e^{j4t+60^\circ}\}$$
$$= Re\{e^{j(4t+30^\circ)} + e^{j(4t+60^\circ)}\}$$
$$= Re\{e^{j4t}(e^{j\pi/6} + e^{j\pi/3})\}$$
$$= Re\left\{\left(\frac{\sqrt{3}}{2} + j\cdot\frac{1}{2}\right) + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right\}$$
$$= Re\left\{e^{j4t}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)(1+j)\right\}$$
$$= Re\left\{e^{j4t}\left(\frac{(\sqrt{3}+1)}{2}\right)\sqrt{2}\;e^{j\pi/4}\right\}$$
$$= \frac{\sqrt{3}+1}{\sqrt{2}}\cos(4t+45^\circ)$$

$$\cos(4t+30^\circ) \rightleftharpoons 1\,\angle 30^\circ \quad (w=4)$$
$$\cos(4t+60^\circ) \rightleftharpoons 1\,\angle 60^\circ \quad (w=4)$$
$$\frac{+}{1\angle 60^\circ + 1\angle 30^\circ} = \left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}+1}{2}(1+j)$$
$$= \frac{\sqrt{3}+1}{2}\cos(4t+45^\circ) \qquad = \frac{\sqrt{3}+1}{\sqrt{2}}\angle 45^\circ$$

Example: $\sum_{k=0}^{N}\cos\left(4t+\frac{\pi}{6}k\right) = \sum_{k=0}^{N} Re\left\{e^{j(4t+\frac{\pi}{6}k)}\right\}$

$$= Re\left\{\sum_{k=0}^{N} e^{j(4t+\frac{\pi}{6}k)}\right\}$$
$$= Re\left\{e^{j4t}\sum_{k=0}^{N} e^{j\frac{\pi}{6}k}\right\}$$

$$\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$$

$$= Re\left\{e^{j4t}\,\frac{1-e^{j\pi/6(N+1)}}{1-e^{j\pi/6}}\right\}$$
$$= Re\,\frac{\{e^{j4t}(1+e^{-j\pi/6})\}}{\|1-e^{j\pi/6}\|^2}\cdot\left(1-e^{j\frac{\pi}{6}(N+1)}\right)$$

$\downarrow$

bunu simdilik bosver

GIPTA

HW#2 ?PS: 3,6,7, 12c (Node pek terih c tme)

(27)

Phasor: $\cos(\omega t + \phi) \longleftrightarrow Ae^{j(\omega t + \phi)}$

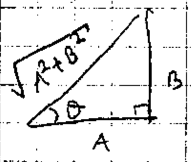
$(Ae^{j\phi})e^{j\omega t}$

$\hookrightarrow$ phasor $\longrightarrow A \angle \phi$

(ω not written but kept a side)

$\rightarrow A\cos(\omega t + \phi_A) + B\sin(\omega t + \phi_B) = ?$

$$\sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos(\omega t + \phi) + \frac{B}{\sqrt{A^2+B^2}}\sin(\omega t + \phi)\right)$$



$$\sqrt{A^2+B^2}\left[\cos(\theta)\cos(\omega t + \phi) + \sin(\theta)\sin(\omega t + \phi)\right]$$

$$\sqrt{A^2+B^2}\cos\left[\omega t + \phi - \tan^{-1}(B/A)\right]$$

$\rightarrow A\cos(\omega t + \phi) \longrightarrow A \angle \phi$

$B\sin(\omega t + \phi)$

$B\cos(\omega t + \phi - 90°) \longrightarrow B\angle \phi - 90$

+

$A\cos(\omega t + \phi) + B\sin(\omega t + \phi) \qquad A\angle\phi + B\angle\phi - 90$

$$= Ae^{j\phi} + Be^{j(\phi - 90°)} = Ae^{j\phi} + Be^{j\phi} \cdot e^{-j\pi/2} \quad -j$$

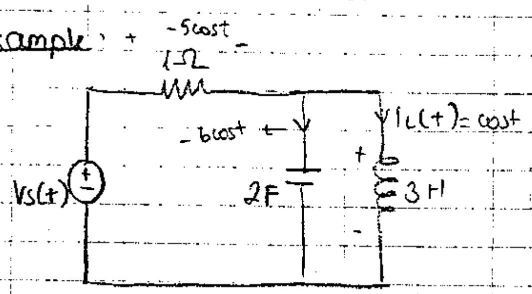
$$= Ae^{j\phi} - jBe^{j\phi} = e^{j\phi}(A - jB)$$

$$= e^{j\phi}\sqrt{A^2+B^2}\angle -\tan^{-1} -B/A$$

$$\longleftarrow = \sqrt{A^2+B^2} \angle \phi - \tan^{-1} B/A$$

$$\sqrt{A^2+B^2}\cos\left(\omega t + \phi - \tan^{-1}\frac{B}{A}\right)$$

Example:



$i_L(t) = \cos t$

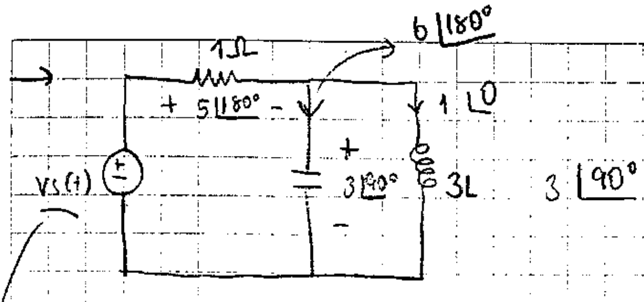
Find $V_s(t) = ?$

$$V_{3H}(t) = L\frac{d}{dt}i_L(t) = -3\sin(t)$$

$$i_C(t) = C\frac{dV_C(t)}{dt} = -6\cos t$$

$$V_s(t) = -3\sin t + (-5\cos t) \; ;$$

$$V_s(t) = \sqrt{34}\cos\left(t - \tan^{-1}\left(\frac{-3}{-5}\right)\right)$$



$-3\sin t \xrightarrow[\text{phasor}]{} (1\ \angle 180°)\ 3\ \angle -90°$

$3\cos(t+90°) \longrightarrow 3\angle 90°$

$V_s(t) \xrightarrow[\text{phasor}]{} 5\ \angle 180° + 3\angle 90°$

$-5+3J \quad = \sqrt{34}\ \angle \tan^{-1}(3/5)+180$

$= -\sqrt{34}\ \angle -\tan^{-1}(3/5)$

$V_s(t) = -\sqrt{34}\ \cos(t - \tan^{-1}(3/5))$

Example: $A\cos(\omega t+\phi) \xrightarrow[\text{phasor}]{} A\angle\phi$

$\downarrow d/dt$

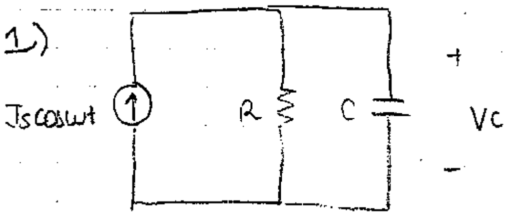
$-A\omega\sin(\omega t+\phi)$

$A\omega\cos(\omega t+\phi+90°)$ } phasor representation $A\omega\ \angle\phi+90°$

How phase is mapped under derivative operation

Example:

1)

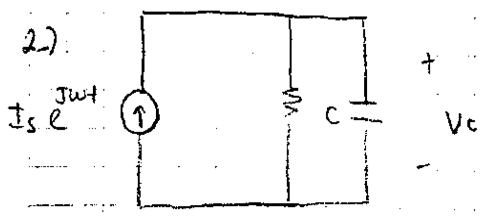


$\left(D+\frac{1}{RC}\right) V_c(t) = \frac{I_s\cos\omega t}{C}$

$V_c(0^-) = V_0$

$V_I^{complete}(t) = c_1 e^{-t/RC} + \underbrace{A\cos\omega t + B\sin\omega t}_{\text{particular solution}}$

2)

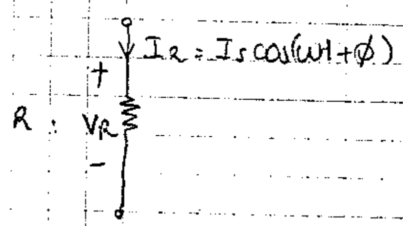
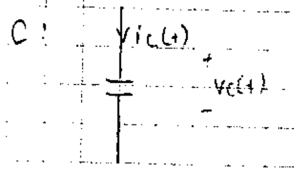
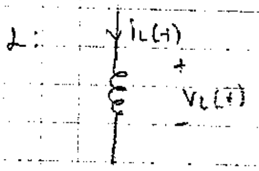


$\left(D+\frac{1}{RC}\right) V_c(t) = I_s \frac{e^{J\omega t}}{C} \quad V_c(0^-) = V_0$

$V_{II}^{complete} = \underbrace{d_1}\, e^{-t/RC} + \underbrace{D}\, e^{J\omega t}$

are complex numbers

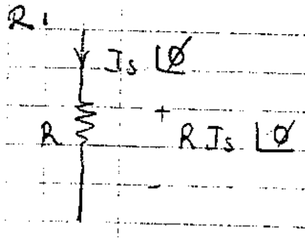
Phasor Circuit Analysis

	- Time - Domain -	- Phasor -
R: 	$I_R = I_s \cos(\omega t + \phi)$ $V_R = I_s R \cos(\omega t + \phi)$	$I_R = I_s \angle \phi$ $V_R = R I_s \angle \phi$
C: 	$I_C(t) = I_s \cos(\omega t + \phi)$ $V_C(t) = \frac{I_s}{C\omega} \sin(\omega t + \phi)$	$I_C = I_s \angle \phi$ $V_C = \frac{I_s}{C\omega} \angle \phi - 90^\circ$
L: 	$I_L(t) = I_s \cos(\omega t + \phi)$ $V_L(t) = L\omega I_s(-\sin(\omega t + \phi))$ $= L\omega I_s \cos(\omega t + \phi + 90^\circ)$	$I_L(t) = I_s \angle \phi$ $V_L(t) = \omega L I_s \angle \phi + 90^\circ$

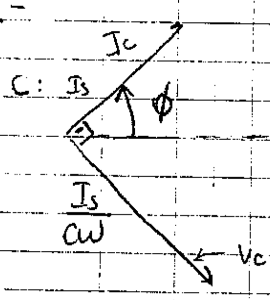
Current and voltage of R are in phase

- Phasor Domain Component -

R:



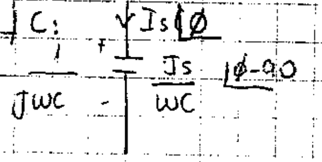
C:



Current phasor leads voltage phasor by 90° (phasor diagram)

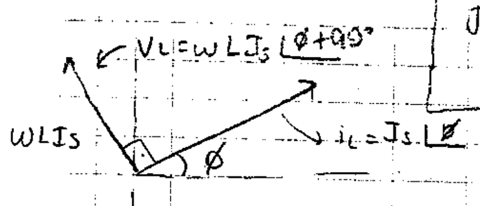
Voltage phasor lags by 90°

C:



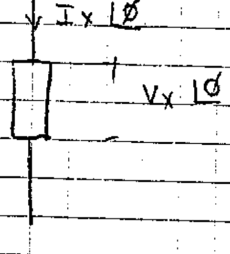
(ω: frequency)

L: (phasor diagram)



Voltage of inductor leads the current by 90°

Impedance, Admittance



$Z = \frac{\text{voltage phasor of component}}{\text{current phasor of component}}$

impedance

$Z = Re\{Z\} + j\,Im\{Z\}$

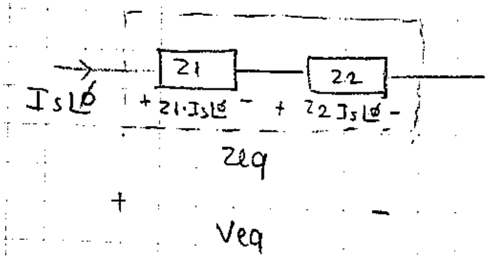
resistance   impedance

Y : Admittance $Y = \frac{\text{Current Phasor}}{\text{Voltage phasor}} = \frac{1}{Z}$

$= \underbrace{Re\{Y\}}_{\text{conductance}} + j\,\underbrace{Im\{Y\}}_{\text{susceptance}}$

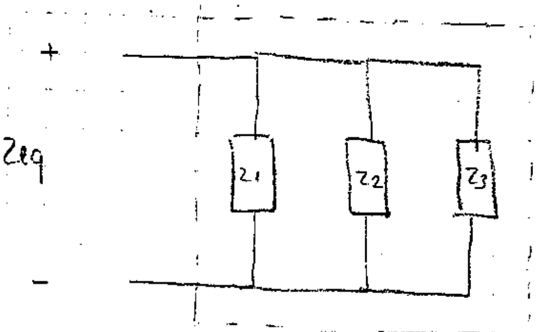
Impedance Combinations

1 Series



$$Z_{eq} = \frac{V_{eq}}{I_s\angle\phi} = \frac{Z_1 \cancel{I_s\angle\phi} + Z_2 \cancel{I_s\angle\phi}}{\cancel{I_s\angle\phi}} = Z_1 + Z_2$$

2 Parallel



$$Y_{eq} = \sum_{k=1}^{3} Z_k$$

$$Z_{eq} = \frac{1}{\sum_{k=1}^{3} 1/Z_k}$$

Similarly Δ-Y conversion can also be used by impedances by replaces

$R_Y \longrightarrow Z_Y$

$R_\Delta \longrightarrow Z_\Delta$

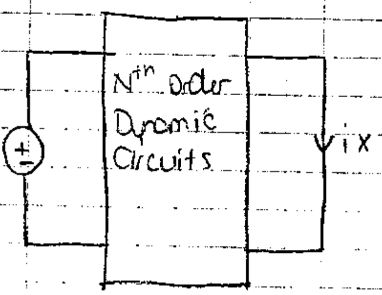
A.C. Steady State Calculations

N^{th} order Dynamic Circuit $\longrightarrow$ particular ?
$\longrightarrow$ homogeneous solution

zero input   zerostate

↓

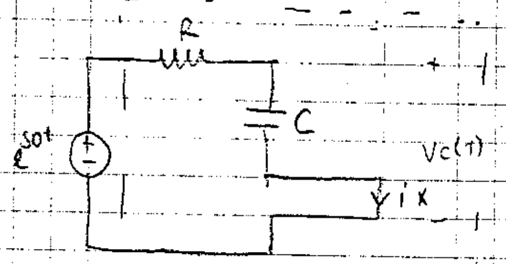
natural frequency
(mode excitation)
mode : $C_k e^{\lambda_k t}$
k^{th} mode of the circuit



$i_x(t) = ?$

$i_x(t) = i_x^h(t) + i_x^p(t)$

$i_x^p(t) = ?$



$i_x^p(t) = ?$

Guess: $\underbrace{v_c(t) = Ae^{s_0 t}}$ phasor

$i_c^p(t) = C \frac{d}{dt} v_c = C \underbrace{(A s_0 e^{s_0 t})}$

↳ CAs_0: current phasor

$V_R(t) = R i_c(t)$

$= \underbrace{RACs_0} e^{s_0 t}$

↳ $RACs_0$ voltage phasor for $V_R(t)$

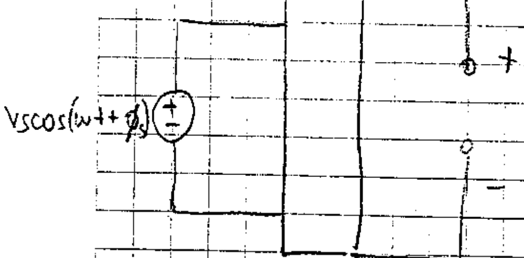
$-e^{s_0 t} + V_R(t) + V_c^p(t) = 0$

$-\cancel{e^{s_0 t}} + RACs_0 e^{s_0 t} + A\cancel{e^{s_0 t}} = 0$

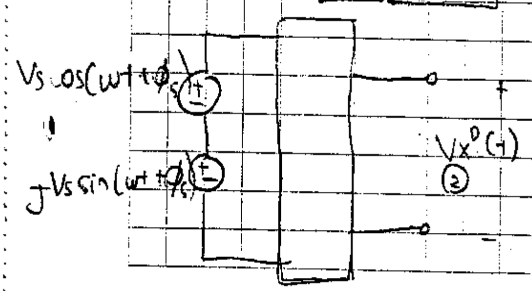
$\boxed{RACs_0 + A - 1 = 0}$

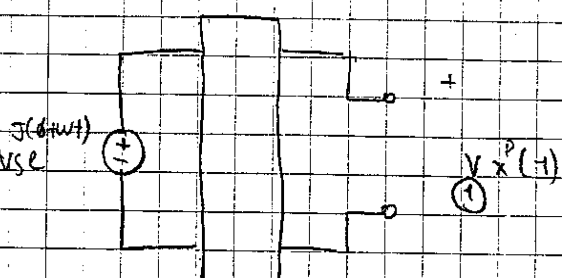
$\boxed{A = \frac{1}{RCs_0 + 1}}$ $\longrightarrow$ $\boxed{V_c^p(t) = Ae^{s_0 t} = \frac{1}{RCs_0 + 1} e^{s_0 t}}$

Sinusoidal Input:



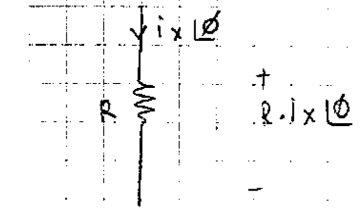
Again we are interested in particular solution

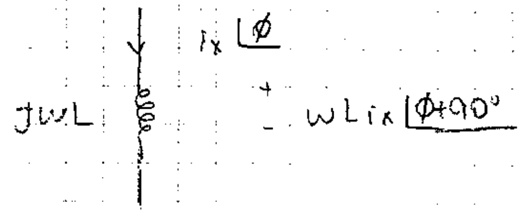




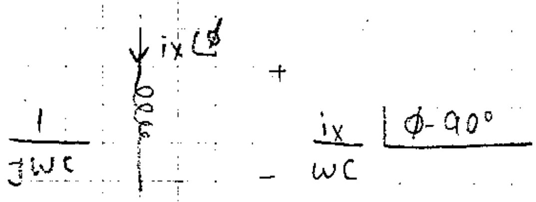
$Re\{V_2{}_x^p(t)\} = V_x^p(t)$

Phasor Domain Rep. components

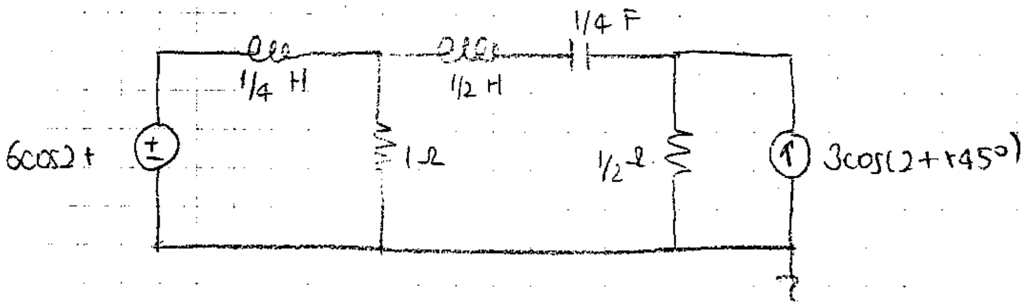




→ quelde (+) Amplitude yapmaya calis.



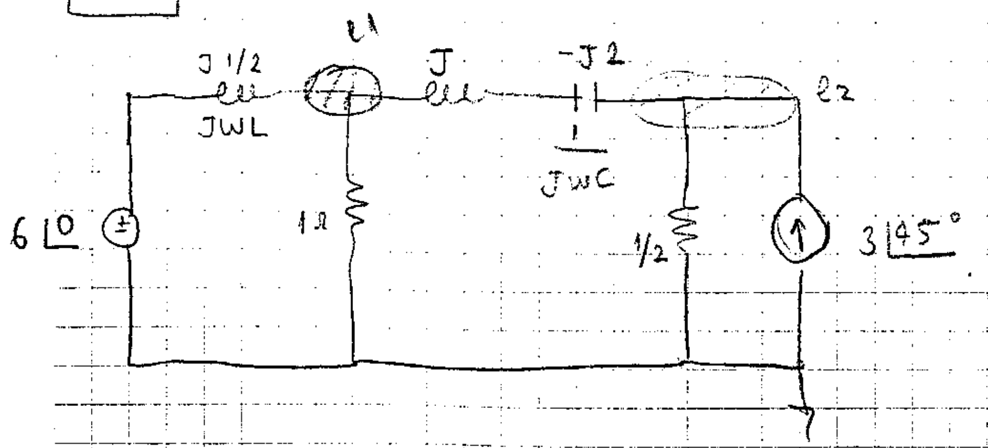
→ Node Analysis example (we are assuming the circuits are stable)



Find steady state value for node voltages

Phasor Domain:

$\boxed{w=2;}$



e_1, e_2 : phasor representation of node voltages. (33)

KCL @ e_1 : $\frac{e_1-6}{J/2} + \frac{e_1}{1} + \frac{e_1-e_2}{-J} = 0$;

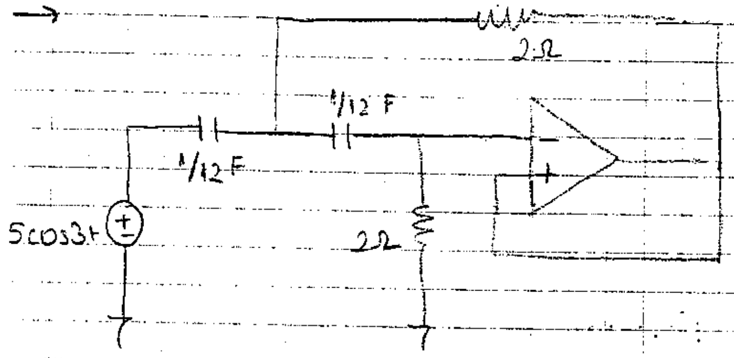
KCL @ e_2 : $\frac{e_2}{1/2} - 3\angle 45^\circ + \frac{e_2-e_1}{J} = 0$

$$\begin{bmatrix} -J+1 & -J \\ -J & 2+J \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -12T \\ 3\angle 45^\circ \end{bmatrix}$$

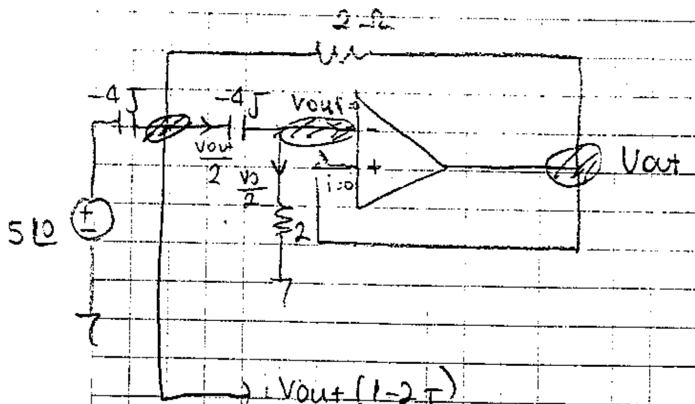
(Dependent source olmazsa reciprocal olurdu)

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{1}{4-J} \begin{bmatrix} 2+J & J \\ J & -J+1 \end{bmatrix} \begin{bmatrix} -12T \\ 3\angle 45^\circ \end{bmatrix} = \begin{bmatrix} ? \\ 3.63\angle 14 \end{bmatrix}$$

$\sqrt{2}\angle -45^\circ$



Assume opamp is in linear region, find $V_{out}^{ss}(t)$ ← steady state



$V_{out}(1-2T)$

$\angle 127 = \cos(3t+127)$

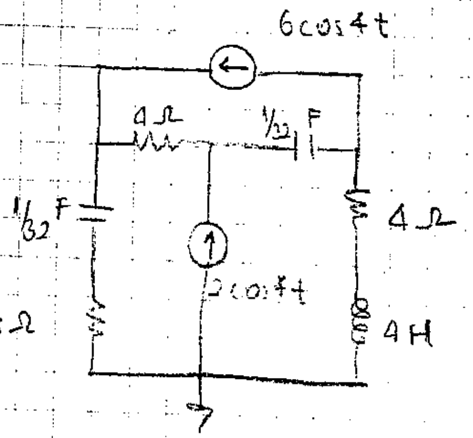
KCL @ $V_{out}(1-2T)$, $\frac{V_{out}(1-2T)-5}{-4T} + \frac{V_{out}}{2} + \frac{V_{out}(1-2T)-V_{out}}{2} = 0$

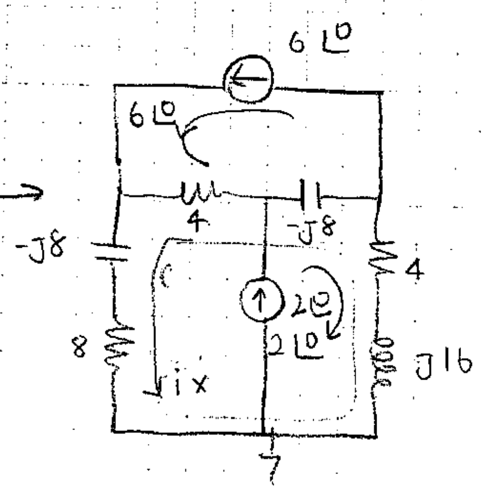
$V_{out}\left(\left(\frac{1-2T}{-2T}\right) + 1 - 2T\right) = \frac{5}{2}T \qquad V_{out} = \frac{5/2\,T}{2-\frac{3}{2}T} = \frac{5J}{5\angle -37^\circ}$

$2 - \frac{3T}{2}$

GIPTA

→ Mesh analysis





KVL across ℓ loop;

$\boxed{w=4}$

; $\frac{32-j32}{16} = 2(1-j) = 2\sqrt{2}\,\angle{-45} = i_x$

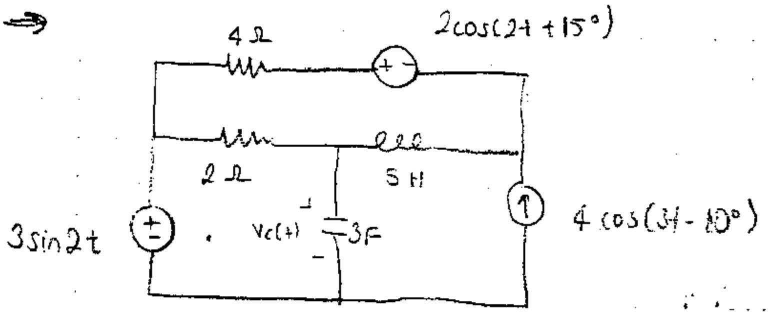
$i_L = 2\angle 0 - i_x = 2 - 2 + 2j = 2j$

$\boxed{i_L = 2\ \angle 90^\circ}$

$\boxed{i_L(t) = 2\cos(\omega t + 90^\circ)}$

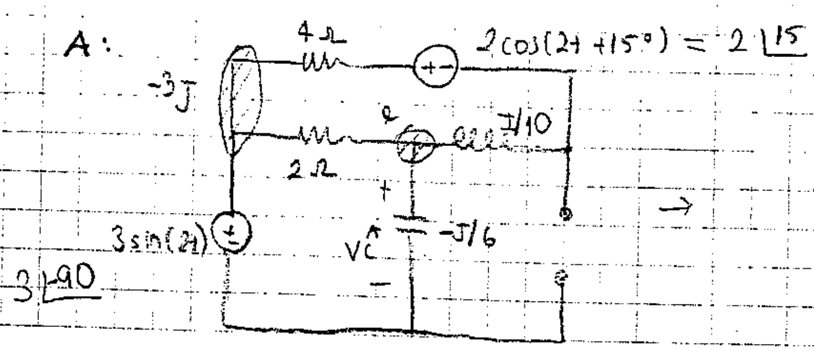
Sources with different frequencies (use superposition)

→

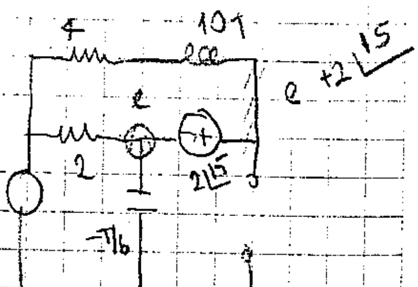


$V_c^{ss}(t) =$

A:



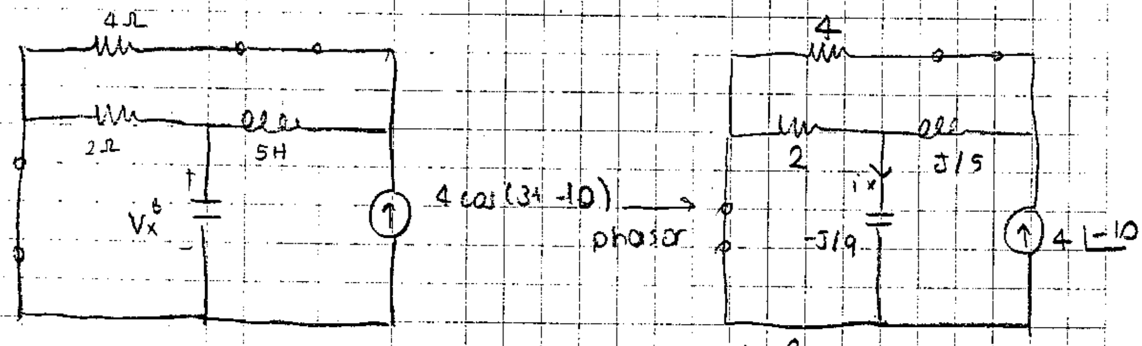
→



$$\frac{e-(-3_T)}{2}+\frac{e}{-T/6}+\frac{(e+2\angle 15^\circ)-(-3_T)}{4+J\sqrt{10}}=0 \qquad e=V_{3F}^{(A)}\angle\theta^{(A)}$$ (35)

$V_{3F}^{(A)ss}(t)=V_{3F}^{(A)}\cos(2t+\theta^{A})$

B:



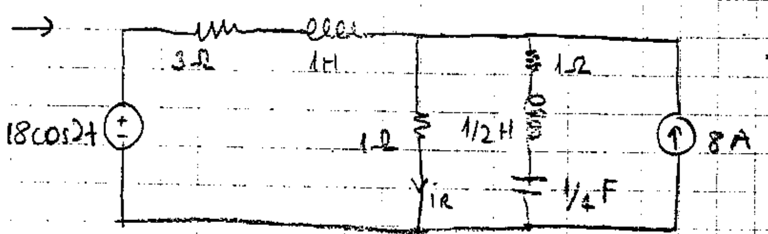
- $I_x=4\angle -10\ \dfrac{4}{4+(2//-J/9+J15)}\cdot\dfrac{2}{2//(-J/9)}$

$V_X^{(B)}=I_x\left(-\frac{J}{9}\right)=V_{3F}^{(B)}\angle\theta^{B}$

$V_{3P}^{(B)ss}=V_{3P}^{(B)}\cos(3t+\theta^{B})\ V$

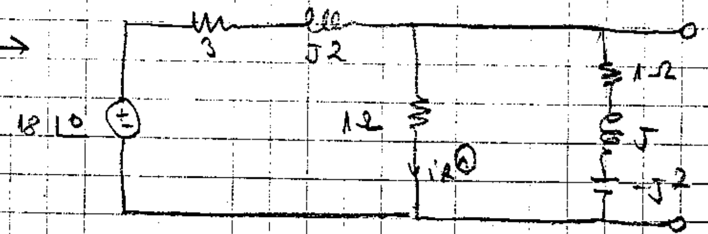
$V_{3F}^{ss(t)}=V_{3F}^{(A)ss}(t)+V_{3F}^{(B)ss}(t)$

→



$i_R^{ss}(t)=?$

$\boxed{A}$ →



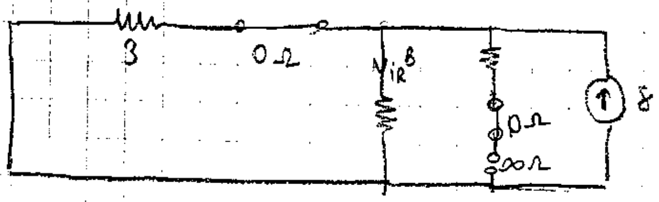
$$i_R^{(A)}=\frac{18}{3+J2}\ \frac{\overset{G_B}{1/1}}{\underset{G_B}{1/1}+\underset{G_A}{1/(3+J2)}+\underset{G_C}{1/(1-J)}}=2(1-J)$$

Check!

$= 2(1-j) = 2\sqrt{2}\angle -45$

$i_R^{Ⓐ ss}(t) = 2\sqrt{2}\cos(2t - 45°)$

Ⓑ



$w = 0$

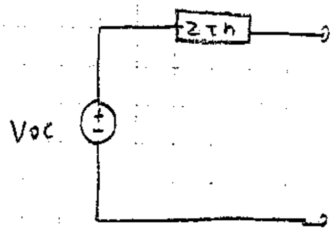
$L \to jwL \quad C \to \frac{-j}{wC}$

$i_R^{Ⓑ} = 8 \cdot \frac{1}{1 + 1/3} = 6 \qquad i_R^{B\, ss}(t) = 6\angle 0$

$i_R^{ss}(t) = i_R^{Ⓐ ss}(t) + i_R^{Ⓑ ss}(t)$

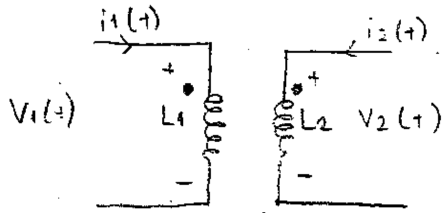
$$\boxed{i_R(t) = 2\sqrt{2}\cos(2t - 45°) + 6A}$$

Thevenin - Norton Equivalent



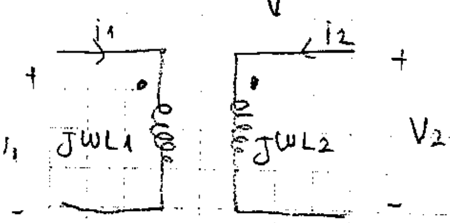
- Voc: Phasor - open circuit voltage
- Zth: Thevenin impedance.
- : Their circulation is exactly the same calculation for DC Circuits.

Coupled Inductors



$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} d/dt\, i_1(t) \\ d/dt\, i_2(t) \end{bmatrix}$$

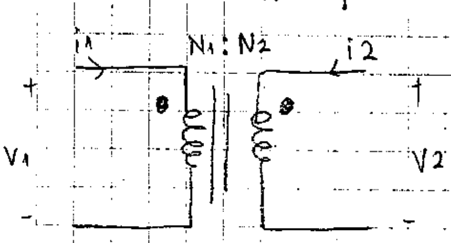
phasor



phasor

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} jwL_1 & jwM \\ jwM & jwL_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Transformers

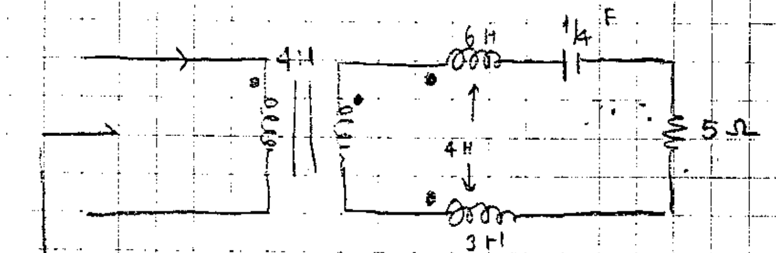


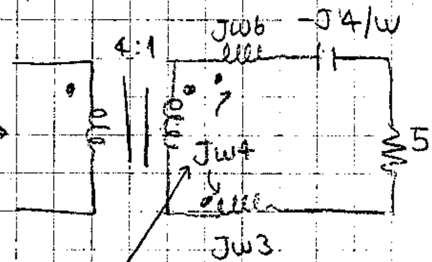
$V_2/V_1 = N_2/N_1$

$i_1/i_2 = -N_2/N_1$

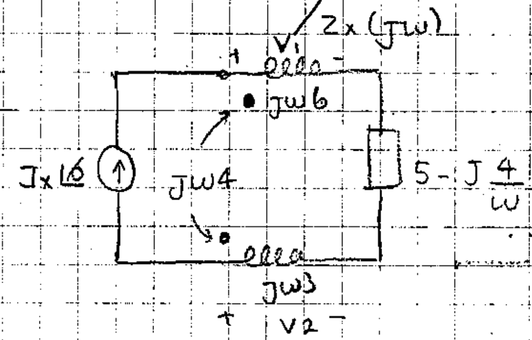
Time domain representation is equal phasor domain rep.

→ PS II pr. 5





Find the input impedance



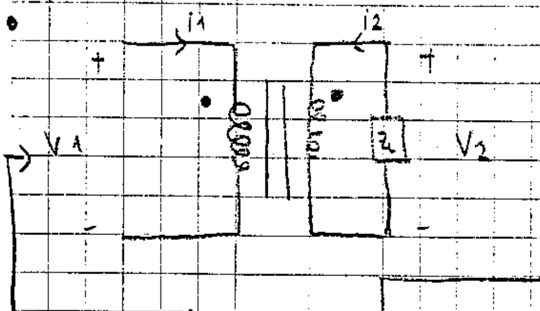
$$Z_x(j\omega) = \frac{V_x}{I_x\angle 0} = \frac{V_1 + I_x(5 - j\frac{4}{\omega})\angle 0 - V_2}{I_x\angle 0}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega 6 & j\omega 4 \\ j\omega 4 & j\omega 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad I_1 \to I_x\angle 0, \quad I_2 \to -I_x\angle 0$$

then; $V_1 - V_2 = \begin{bmatrix} j\omega 2 & j\omega 1 \end{bmatrix} \begin{bmatrix} I_x\angle 0 \\ -I_x\angle 0 \end{bmatrix} = (j\omega)\, I_x\angle 0$

$$= \frac{I_x\angle 0\,(j\omega + 5 - 4j/\omega)}{I_x\angle 0}$$

$$= 5 + j\left(\omega - \frac{4}{\omega}\right)$$



$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \frac{i_1}{i_2} = \frac{-N_2}{N_1}$ } transformer

Load; $V_2 = Z_L(-i_2)$

$\frac{V_1 N_2}{N_1} \qquad -\frac{i_1 N_1}{N_2}$

$Z_{eq} = \left(\frac{N_1}{N_2}\right)^2 Z_L$

$$\boxed{\frac{V_1}{i_1} = Z_L\left(\frac{N_1}{N_2}\right)}$$

derive it

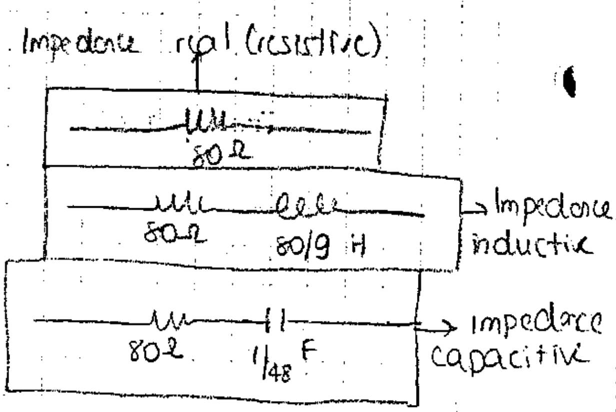
GIPTA

$$Z_{j\omega} = \left(\frac{4}{1}\right)^2 Z_x(j\omega) = 16\left(5 + j\left(\omega - \frac{4}{\omega}\right)\right)$$

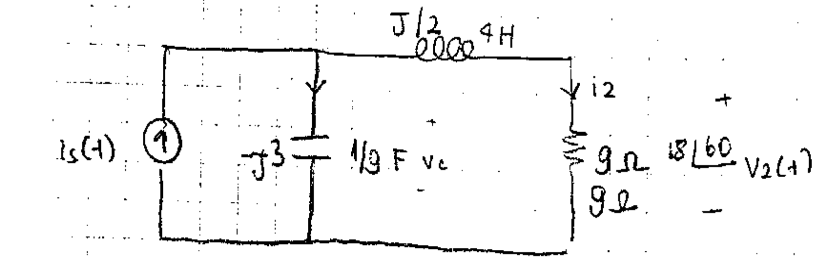
Assume $\omega = 2 \longrightarrow Z(j2) = 80$

$\omega = 3 \longrightarrow Z(j3) = 80 + \frac{80}{3} j$

$\omega = 1 \longrightarrow Z(j) = 80 - 48 j$

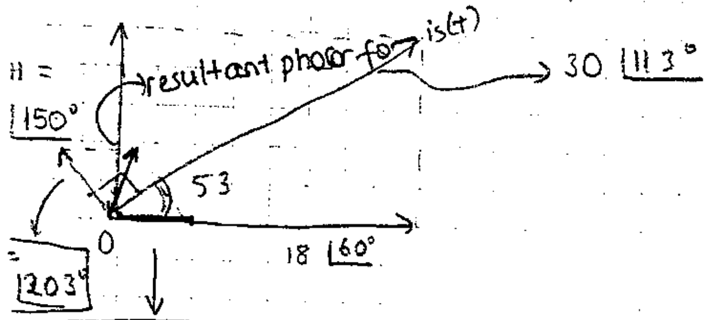


Phasor Diagram Zps - II problem 4



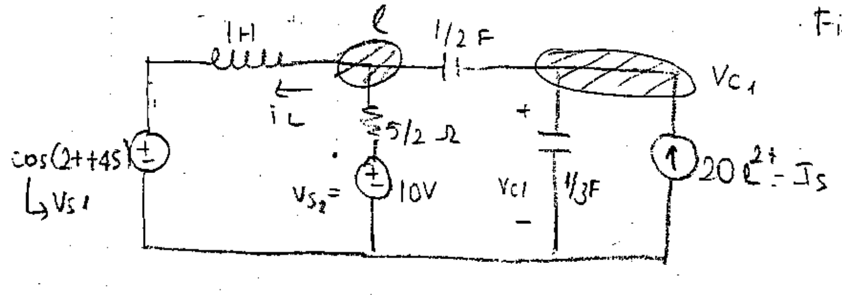
Find is=?

$\overset{ss}{V_2}(t) = 18 \cos(3t + 60°)$



$\boxed{i_2 = 2\ \angle 60°}$

Zps I : 12-d-)



Find homogenous, particular soln.

KCL V_{c1}: $\frac{1}{3} D V_c + \frac{1}{2} D (V_{c1} - e) - I_s = 0$

KCL e: $\frac{1}{2} D (e - V_{c1}) + (e - V_s) \cdot \frac{2}{5} + i_L(0^-) + \int_0^t (e(z) - V_{s1}(z))\, dz = 0 \quad \times 10$

$$\begin{bmatrix} 5D & -3D \\ -5D & 5D + 4 + 10D' \end{bmatrix} \begin{bmatrix} V_c \\ e \end{bmatrix} = \begin{bmatrix} 6 I_s \\ -10 i_L(0^-) + 4 V_{s2}(t) + 10\cancel{D'} V_{s1}(t) \end{bmatrix}$$

Take derivative of 2^{nd} Row

$$\begin{bmatrix} 5D & -3D \\ -5D^2 & 5D^2+4D+10 \end{bmatrix}\begin{bmatrix} V_C \\ \ell \end{bmatrix} = \begin{bmatrix} 6I_s \\ 4DV_{S2}(t)+10V_{S1}(t) \end{bmatrix}$$

zero input; RHS $\rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} V_C(t) \\ \ell(t) \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{\lambda t} \longrightarrow \begin{bmatrix} 5\lambda & -3\lambda \\ -5\lambda^2 & 10+4\lambda+5\lambda^2 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix}\cancel{e^{\lambda t}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$5\lambda(5\lambda^2+4\lambda+10) - 15\lambda^3 = 0$ $\qquad \lambda = 0$

$5\lambda^2+4\lambda+10-3\lambda^2 = 2\lambda^2+4\lambda+10 \xrightarrow{1/2} \lambda^2+2\lambda+5=0$

$\dfrac{-b \mp \sqrt{4-4.1.5}}{2} = \dfrac{-2\mp 4i}{2} = -1\mp 2i$

Turn back state;

- (state variable sayısı = natural frequency sayısı) 0 zaman $\lambda=0$'da natural frequency vardır. (Emin olabilirsin buradan)

- $V_{C1}^h(t) = c + a_1 e^{-t}\cos 2t + a_2 e^{-t}\sin 2t$ (only stable)

- $\begin{bmatrix} 5D & -3D \\ -5D^2 & 5D^2+4D+10 \end{bmatrix}\begin{bmatrix} V_C^p \\ \ell^p \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} e^{s_0 t}$ $\quad$ $S_0 = 0$ $\quad S_0 = \mp 2$ $\quad S_0 = 2$

$$\begin{bmatrix} 5D & -3D \\ -5D^2 & +5D^2+4D+10 \end{bmatrix}\begin{bmatrix} V_{C1}^p(t) \\ \ell(t) \end{bmatrix} = \begin{bmatrix} 120e^{2t} \\ 0 \end{bmatrix}$$ by superposition (only $I_s \neq 0$)

$\begin{bmatrix} V_C^{p(1)} \\ \ell^{p(1)} \end{bmatrix} = \begin{bmatrix} A^{(1)} \\ B^{(1)} \end{bmatrix} e^{2t}$; $\quad 10Ae^{2t} - 6Be^{2t} = 120e^{2t}$ $\quad$ $5A-3B = 60$

$-20Ae^{2t}+38Be^{2t} = 0$ $\quad$ $\dfrac{13B}{2} = 60$

$19B = 10A \quad 5A = \dfrac{19}{2}B$ $\quad$ $B = \dfrac{120}{13}$ $\quad A = \dfrac{456}{26}$

$A = \dfrac{228}{13}$

- $V_C^{p(2)} = ?$ due to $10\cos(2t+45) = 10\,\mathrm{Re}\{e^{j45°} \cdot e^{j2t}\}$

$= 10\,\underline{|45}$ phasor representation

$$\begin{bmatrix} 5D & -3D \\ -5D^2 & 5D^2+4D+10 \end{bmatrix}\begin{bmatrix} V_C \\ \ell \end{bmatrix} = \begin{bmatrix} 0 \\ \underbrace{100\,\underline{|45}}_{\Sigma}\, e^{j2t} \end{bmatrix}$$

$$\begin{bmatrix} V_C^{p(2)} \\ \ell^{p(2)} \end{bmatrix} = \begin{bmatrix} A^{(2)} \\ B^{(2)} \end{bmatrix} e^{j2t}$$

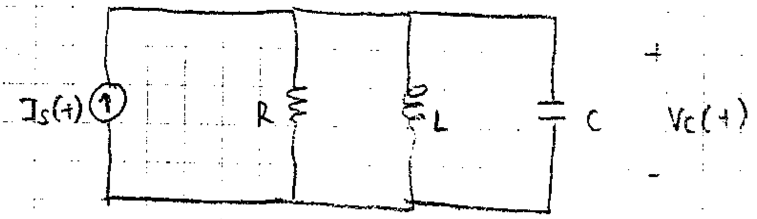
$$\begin{bmatrix} 5+j2 & -3+j2 \\ 5\cdot 4 & -5\cdot 4+j8+10 \end{bmatrix}\begin{bmatrix} A^{(2)} \\ B^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 100\angle 45 \end{bmatrix}$$

$V_c^p(t) = Re\{A^{(2)} e^{j2t}\}$

$= Re\{\sqrt{10}\ \angle \tan^{-1}(1/3)\ e^{j2t}\}$

$= \sqrt{10}\ \cos(2t + \tan^{-1}(1/3))$

Existance of Steady State Solution



$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC}\right) V_c(t) = \frac{1}{C} \frac{d}{dt} I_s(t)$

$(D^2 + 2\alpha D + \omega_0^2)$

→ resonance frequency

→ damping factor

Let's say we want to DE for $I_L(t)$

$V_c(t) = L \frac{d}{dt} I_L(t) = L D I_L(t)$

$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC}\right) \underbrace{L D I_L(t)}_{V_c(t)} = \frac{1}{C} D I_s(t)$ divide by L.

$\left(D^2 + \frac{1}{RC} D + \frac{1}{LC}\right) I_L(t) = \frac{1}{LC} I_s(t)$

$D.D^{-1} = I \longrightarrow \frac{d}{dt}\left(\int_{0^-}^{t} f(\tau)d\tau\right) = f(t)$ Leibnit's Rule.

↓

identity op.

Let set $C=1$, $L=1/2$, $R=1/3$

$(D^2+3D+2)\ I_L(t) = 2 I_s(t)$

Find the particular soln for;

a.) $I_s(t) = I_{s0}\ e^{-5t} \xrightarrow{\text{make a guess}} I_L^p(t) = Ae^{-5t}$

↓

$I_L^p(t) = A\ e^{s_0 t}$ → substitude guess into D.E → $A(s_0^2+3s_0+2) I_L^p(t) = 2I_{s0}$

↓

$s_0 = -5$

$A = \frac{2 I_s}{s_0^2 + 3s_0 + 2}\Big|_{s_0=-5}$ $\quad I_L^p(t) = 1/6\, I_s\, e^{-5t}$

b- $I_s(t) = I_{s0}\, e^{-4t} \xrightarrow[\text{make guess}]{} I_L^p(t) = A\, e^{s_0 t}$, $s_0 = -4$

$A = \frac{2 I_{s0}}{s^2 + 3s_0 + 2}\Big|_{s_0=-4} = \frac{1}{4} I_{s0}$

$I_L^p(t) = \frac{1}{4} I_{s0}\, e^{-4t}$

c- $I_s(t) = I_{s0}\, e^{-jt} \longrightarrow A = \frac{2 I_{s0}}{s_0^2 + 3s_0 + 2}\Big|_{s_0=-j}$

$I_L^p(t) = \underbrace{\frac{2}{1-3j}}_{A} I_{s0}\, e^{-jt} \longleftarrow \frac{2}{1-3j} I_{s0}$

d* $I_s(t) = I_{s0}\, e^{-t} \longrightarrow I_L^p(t) = A\, e^{s_0 t}\Big|_{s_0=-1}$; $A = \frac{2 I_{s0}}{s_0^2 + 3s_0 + 2}\Big|_{s_0=-1} = 2\frac{I_s}{0}$!

Wrong guess

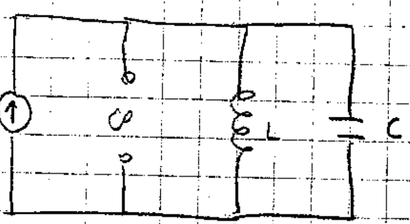
$I_L^p(t) = A t e^{-t}$ ← correct guess

$(D^2 + 3D + 2) I_L(t) = 2 I_{s0}\, e^{-t}$

$I_L^p(t) = A t e^{-t}$ → since nat frequencies are $\{-1, -2\}$ and input excites natural frequency.

d** $R \to \infty$ $\quad G = \frac{1}{R} = 0$

(ohmic losses are zero) $\quad I_s(t)$



$(D^2 + 4) I_L(t) = 4 i_s(t) \longrightarrow$ Nat. freques $= \{+2j, -2j\}$

ii-) • $I_s(t) = I_0 \cos t \longrightarrow I_L^p(t) = R$

$= Re\{I_{s0}\, e^{jt}\} \longrightarrow I_L^p(t) = Re\{A e^{jt}\} \longrightarrow A = \frac{4 I_{s0}}{s_0^2 + 4}\Big|_{s_0 = -j}$

$= \frac{4}{3} I_{s0}$

i) • $I_s(t) = I_{so}\, e^{-t} \longrightarrow I_L^p(t) = A e^{-t} \longrightarrow A = \left. \dfrac{4 I_{so}}{s_o^2+4} \right|_{-1} = \dfrac{4}{5} I_{so}$

iii) • $I_s(t) = I_{so} \cos(2t) = Re\{ I_{so}\, e^{2jt} \} \xrightarrow[\text{guess}]{} I_L^p(t) = Re\{ A e^{j2t} \}$

$\downarrow$

$(D^2+4) I_L(t) = 4 I_{so} \cos 2t$ $\longleftarrow$ wrong guess $\quad A = \left. \dfrac{4 I_{so}}{s_o^2+4} \right|_{s_o = 2j} = \dfrac{4 I_{so}}{0}$

$$\left.\begin{aligned} I_L(t) &= A t \cos 2t + B t \sin 2t \\ &= t\, D \cos(2t + E) \end{aligned}\right\} \text{Correct guess!}$$

• Input excites the natural frequency of $\mp 2j$

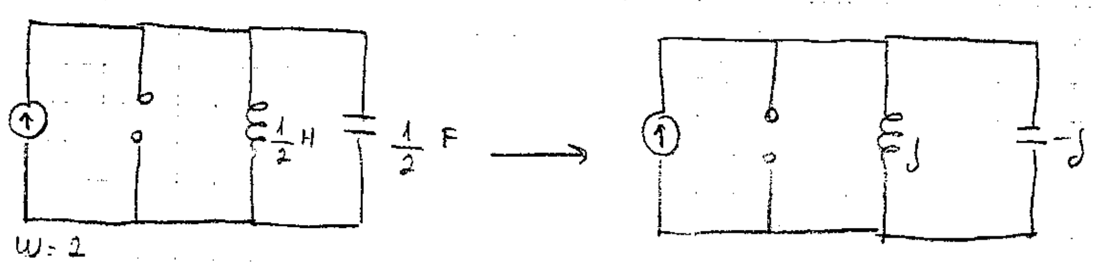
$\lambda = \{ 2j, -2j \}$ and $I_s(t) = I_{so} \cos 2t$

$\downarrow$

natural frequencies.

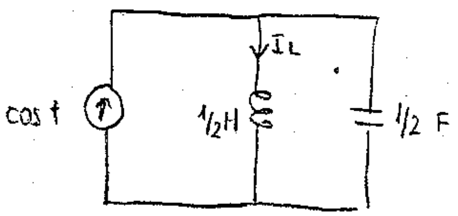
$= I_{so} \dfrac{e^{j2t} + e^{-j2t}}{2}$

In phasor domain;



$j \,//\, -j = \dfrac{1}{0}$!!

• The steady state solution is the part complete solution due to input.



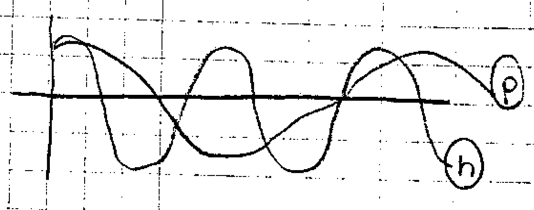
from part ii)

$I_L^p(t) = \dfrac{4}{3} \cos t$

$$\left.\begin{aligned} I_L(0^+) &= I_0 \\ \dot{I}_L(0^+) &= 0 \end{aligned}\right\} I_L^h(t) = A\cos 2t + B \sin 2t$$

$I_L^{comp}(t) = A \cos 2t + B \sin 2t + \dfrac{4}{3} \cos t$

$I_L^{comp}(t) = \left(I_0 - \dfrac{4}{3}\right) \cos 2t + \underbrace{\dfrac{4}{3} \cos t}_{\text{steady state soln}}$ A (B = 0 bulduk türevden)



Md - 1

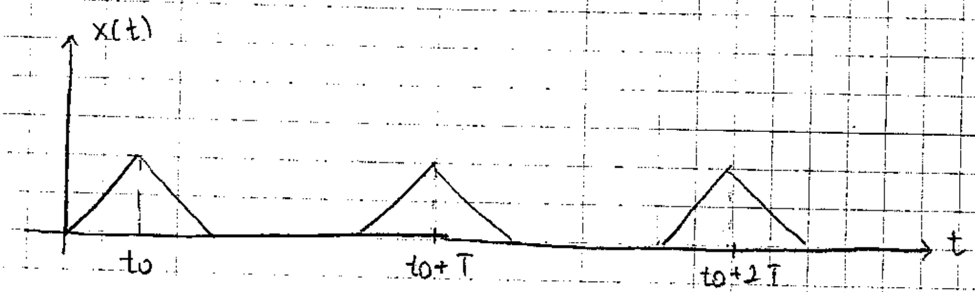
RMS (Effective Values)

$x(t)$: Periodic Signal

$x(t) = x(t+T)$ for $T \neq 0$

$= x(t-kT)$ k is an integer

T: fundamental period of the waveform $x(t)$



$$RMS = \sqrt{\frac{1}{T}\underbrace{\int_{t_0}^{t_0+T}[x(t')]^2 dt'}_{\text{integration over a full period}}}$$

$$= \sqrt{\frac{1}{T}\int_0^T [x(t')]^2 dt'} \quad \text{in Volts}$$

Root Mean Square

$\rightarrow x(t) = A\cos\omega t \longrightarrow RMS = \sqrt{\frac{1}{T}\int_0^T A^2\cos^2\omega t\, dt} = \sqrt{\frac{A^2}{T}\int_0^T \frac{1+\cos 2\omega t}{2}}$

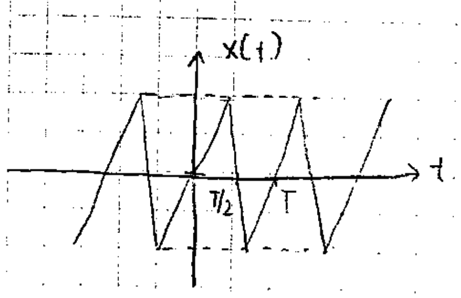
$$= \sqrt{\frac{A^2}{T}\left\{\int_0^T \frac{1}{2}dt + \underbrace{\int_0^T \frac{\cos 2\omega t}{2}dt}_{0}\right\}}$$

$\rightarrow$ frequency 2ω

integration of cosine over "2" full period

$$\boxed{= \frac{A}{\sqrt{2}}}$$

→ $x(t)$: Triangular waveform



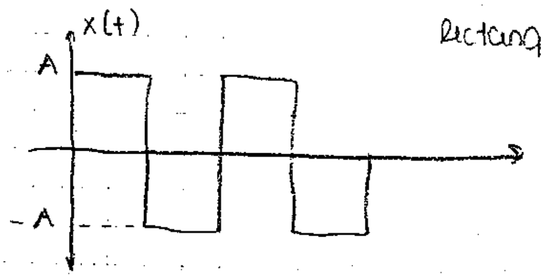
$$RMS = \sqrt{\frac{1}{T}\int_0^T (x(t))^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} (x(t))^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} \left(\frac{2A}{T}t\right)^2 dt}$$

$$= \sqrt{\frac{4A^2}{T^3}\,\frac{t^3}{3}\Big|_{-T/2}^{T/2}} = \frac{A}{\sqrt{3}}$$

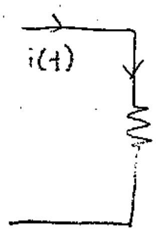
→



Rectangular waveform

$RMS = A$

• Why we are introducing RMS ?



$i(t)$: periodic current waveform

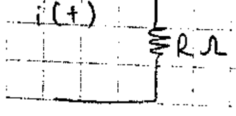
Energy absorbed by $R \Rightarrow \underset{[0,T]}{E} = \int_0^T P_R(\tau)d\tau$ in $[0,T]$

$$= \int_0^T R.[i(t')]^2 dt' = R\int_0^T [i(t')]^2 dt' = T\left(\frac{1}{T}\int_0^T [i(t')]^2 dt'\right) R$$

$$\boxed{= \underbrace{T}_{\text{period}}.\underbrace{(i_{RMS})^2}_{\text{RMS value of}} R}$$

Average power: (over $R\Omega$ resistor)

① Assume $i(t)$ is periodic with T



$$P_{av} = \frac{1}{T}\int_0^T p(t')dt' = \frac{1}{T}\int_0^T [i(t)]^2 R dt = \underbrace{(i_{RMS})^2}_{\text{scalar}}.R$$

$p(t)$ instantaneous power $[\, p(t) = V(t)\, i(t) \,]$

$P_{av} = (i_{RMS})^2 R$

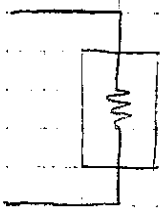
2- If $x(t)$ is almost (not exactly) periodic $\left(x(t) = \cos(2\pi t) + \cos(t) \right)$

$T = 1, 2, 3, \ldots$; $T = 2\pi, 4\pi, 6\pi$

(okek'lerde periodic olacak sadece)

— quasi periodic —

$$i_{RMS}^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T [i(t)]^2 dt \quad ; \quad P_{av} = i_{RMS}^2 \cdot R$$



$P_{av} = 5$ Watts

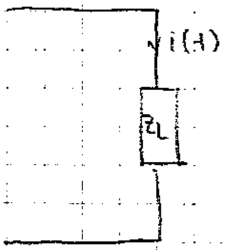
Energy absorbed in 10 periods $= \int_0^{10T} p(t')\, dt'$

$$= T \left(\frac{1}{T} \int_0^{10T} p(t')\, dt' \right)$$

$$= T \left(\frac{10}{T} \int_0^T p(t')\, dt' \right)$$

$$= 10\, P_{av} = 50 \text{ Joule}$$

Average and Instantaneous Power



$$\left. \begin{aligned} v(t) &= V_m \cos(\omega t + \theta_v) \\ i(t) &= I_m \cos(\omega t + \theta_i) \end{aligned} \right\} \rightarrow \begin{aligned} V &= V_m \angle \theta_v \\ i &= I_m \angle \theta_i \end{aligned}$$

$$Z_L = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$$

$$p(t) = V(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

(cos A cos B açılımını uyguladık)

$$= V_m I_m \frac{\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)}{2}$$

$\theta_v = \theta_i + \theta_{ZL}$

$$Z_L = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = |Z_L| \angle \theta_{ZL}$$

$$= \frac{V_m I_m}{2} \cos(\theta_{ZL}) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_{ZL} + 2\theta_i)$$

$\cos(2\omega t + 2\theta_i)\cos(\theta_{ZL}) - \sin(2\omega t + 2\theta_i)\sin(\theta_{ZL})$

• AC dediğimiz şey sinus.

$= \frac{V_m I_m}{2} \cos\theta_{zL}\left(1+\cos(2wt+2\theta_i)\right) - \frac{V_m I_m}{2} \sin\theta_{zL} \sin(2wt+2\theta_i)$ real numbers

$P_{AV} = \frac{V_m I_m}{2} \cos(\theta_{zL}) = \frac{V_{RMS}\sqrt{2}\,(I_{RMS}\sqrt{2})}{2}\cos(\theta_{zL}) = \underbrace{V_{RMS} I_{RMS} \cos(\theta_{zL})}_{\text{Average power consumed by load.}}$ Watts

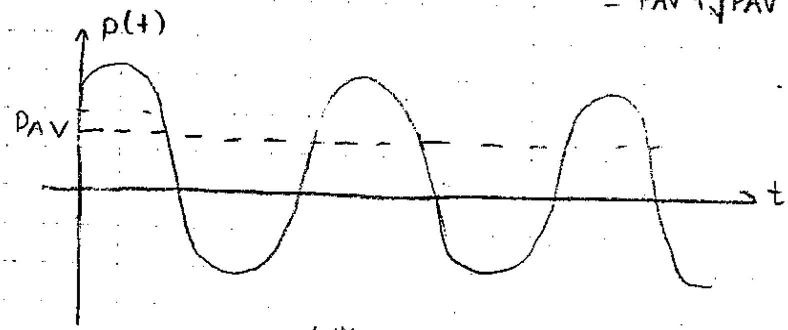
$Z_L = |Z_L| \angle\theta_{zL}$

• Remember: For AC $\longrightarrow V_{RMS} = \frac{V_m}{\sqrt{2}}$

• $Q_m = \frac{V_m I_m}{2} \sin(\theta_{zL}) = V_{RMS} I_{RMS} \sin(\theta_{zL})$ VAR (Volt-amphere Reactive)

$p(t) = P_{av}\left(1+\cos(2wt+2\theta_i)\right) - Q\sin(2wt+2\theta_i)$

$= P_{AV} + \sqrt{P_{AV}^2+Q^2}\cos\left(2wt+2\theta_i+\tan^{-1}\frac{Q}{P}\right)$



$E_{abs} = \int_{t_0}^{t_0+x} p(z)\,dz \qquad X = kT + r(x) \qquad \approx X \cdot P_{AV}$

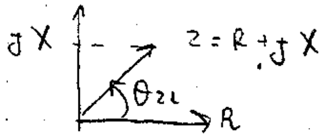
$T = 2\,sec$

$X = 5.1\,sec$; • $5.1\,sec = 2T + 1.1\,sec$.

floor op. ceill op.

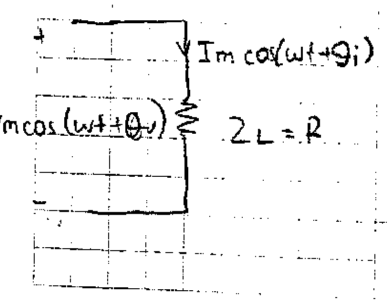
$\left\lfloor \frac{X}{T} \right\rfloor P_{AV} < E_{Absorbed} < \left\lceil \frac{X}{T} \right\rceil P_{AV}$ ...... Anladım! ......

$P_{AV} = V_{RMS} I_{RMS} \cos(\theta_{zL})$



Special Cases

① Resistor

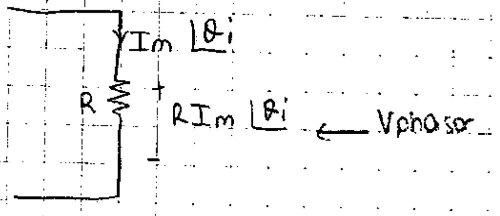


$P(t) = \sqrt{P_{AV}^2 + Q^2}\cos\left(2wt+2\theta_i+\tan^{-1}\frac{Q}{P}\right)$

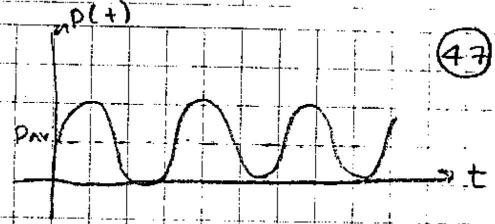
$P_{AV} = V_{RMS} I_{RMS} \cos(\cancel{\theta_{zL}})^{1} = V_{RMS} I_{RMS} = I_{RMS}^2 \overbrace{|Z_L|}^{R} = I_{RMS}^2 R$

$Q = V_{RMS} I_{RMS} \sin(\cancel{\theta_{zL}})^{0} = 0$

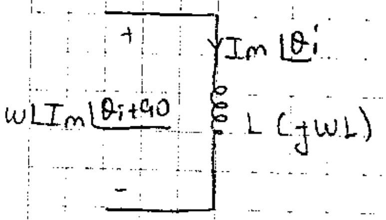
$\theta_{zL} = 0$



$V_{RMS} = R\,I_{RMS}$



Special Case 2:

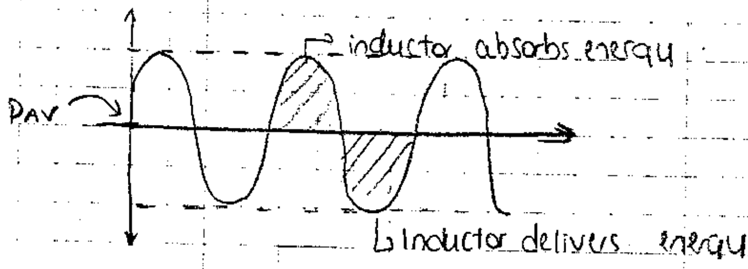


$$Z_L = j\omega L = \omega L \angle 90^\circ \quad (\theta_{ZL})$$

$$P_{AV} = V_{RMS}\, I_{RMS} \cos(\theta_{ZL}) = 0 \text{ Watts}$$

$$Q = V_{RMS}\, I_{RMS} \sin(\theta_{ZL}) = V_{RMS}\, I_{RMS} \quad \text{VAR}$$

$$p(t) = Q \cos(2\omega t + 2\theta_i + 90^\circ)$$



Special Case 3:

$$Z_C = \frac{1}{j\omega C} \qquad \theta_{ZL} = -90^\circ$$

$$P_{AV} = 0 \text{ W}$$

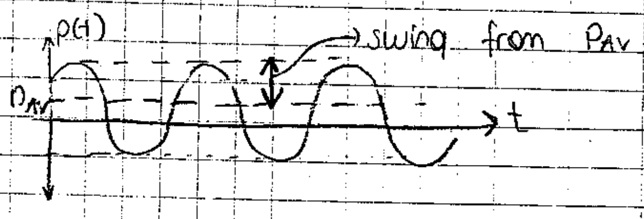
$$Q = -V_{RMS}\, I_{RMS} \quad \text{VAR}$$

Note: $L \rightarrow P = 0, \quad Q > 0$

$C \rightarrow P_{AV} = 0 \quad Q < 0$

Meaning of Q

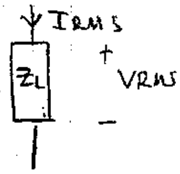
$$p(t) = P_{AV} + \sqrt{P_{AV}^2 + Q^2}\, \cos\left(2\omega t + 2\theta_i + \tan^{-1}\frac{Q}{P_{AV}}\right)$$



Swing from P_{AV};

$$\sqrt{P_{AV}^2+Q^2} = \sqrt{(V_{RMS} I_{RMS}\cos(\theta_{ZL}))^2 + V_{RMS}^2 I_{RMS}^2 \sin^2\theta_{ZL}} = V_{RMS} I_{RMS}$$

(apparent power)



② Average Stored Energy in capacitor

$$E_{cap}(t) = \frac{1}{2} C V_c^2(t) \quad \text{Joule}$$

A.C. (ω)

$$E_{cap}^{average} = \frac{1}{T}\int_0^T E_{cap}(\tau)\,d\tau = \frac{1}{2} C \frac{1}{T}\int_0^T V_c^2(\tau)\,d\tau$$

$$= \frac{1}{2} C V_{RMS}^2$$

$$Q^{cap} = V_{RMS} I_{RMS} \sin(\underbrace{\theta_z}_{-90^\circ})$$

$$P^{cap} = 0$$

$$Q^{cap} = -V_{RMS} I_{RMS}$$

$$= -V_{RMS} \frac{V_{RMS}}{|\underbrace{Z_{cap}}_{\frac{1}{j\omega C}}|} = -\omega C V_{RMS}^2$$

$$\boxed{I_c^{ph} = \frac{V_c^{ph}}{Z_c}}$$

- $Q^{cap} = -2\omega E_{cap}^{average}$

For an Inductor:

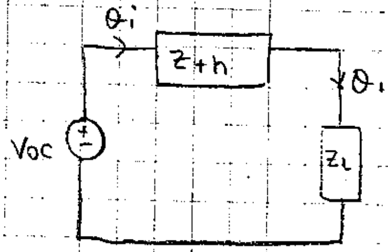
$$E_L^{av} = \frac{1}{T}\int_0^T \frac{1}{2} L i_L^2(\tau)\,d\tau = \frac{1}{2} L (I_{RMS})^2$$

$$Q^{ind} = V_{RMS} I_{RMS} \sin(\underbrace{\theta_{ZL}}_{90^\circ}) = V_{RMS} I_{RMS} = (I_{RMS} |\underbrace{Z_L}_{j\omega L}|) I_{RMS}$$

$$= \omega L I_{RMS}^2$$

- $Q_{ind} = 2\omega E_L^{AVE}$

Conservation of P & Q :

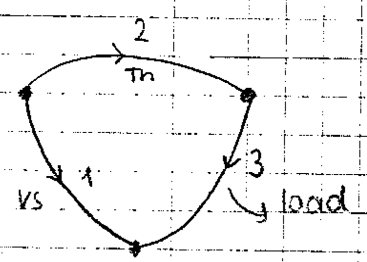


Telleaen's Theorem

$$\sum_{k=1}^{3} P_k(t) = 0$$

$\downarrow$

$V_k(t)\, i_k(t)$



$$V_k^{(t)} = V_m^k \cos(\omega t + \theta_V^k)$$

$$i_k^{(t)} = I_m^k \cos(\omega t + \theta_I^k)$$

$$P_2(t) + P_3(t) = -P_1(t)$$

$P_1(t)$ → power delivered by Vs

$P_3(t)$ → power absorbed

$P_2(t)$ → power absorbed

$P_{AV}^{②} \{1 + \cos(2\omega t + 2\theta_i)\}$ $- Q^{②}_2 \sin(2\omega t + 2\theta_i)$	$P_{AV}^{③} \{1 + \cos(2\omega t + 2\theta_i)\}$ $- Q^{③} \sin(2\omega t + 2\theta_i)$

$$\sum_k P_{AVG}^k = \sum_{k'} P_{AV}^{k'}$$

$\sum_k P_{AVG}^k$ → absorbed, $\sum_{k'} P_{AV}^{k'}$ → delivered

$$\theta_i = \theta_i^{①} = \theta_i^{②} = \theta_i^{③}$$

$$\sum_k Q^k (\text{absorbed}) = \sum_{k'} Q^k (\text{delivered})$$

P & Q is conserved

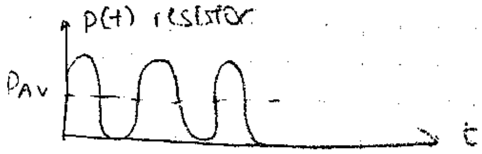
Complex Power

P_{AV}, Q : reactive power (VAR : voltage Amphere reactive)

P_{AV} → average power

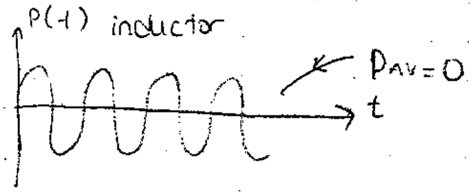
$$p(t) = P_{AV} + P_{AV}\cos(2\omega t + 2\theta_i) - Q\sin(2\omega t + 2\theta_i)$$

$p(t)$ ↑ instantaneous power



$P_{AV} = V_{RMS} I_{RMS} \cos(\theta_Z)$

$Q = V_{RMS} I_{RMS} \sin(\theta_Z)$



$Z = j\omega L$
$\theta_Z = 90°$
$P_{AV} = 0$
$Q = V_{RMS} I_{RMS}$.

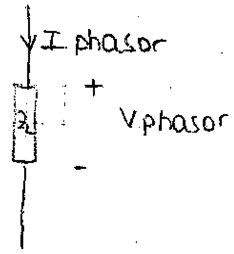
<u>Complex Power</u> → Angle of load impedance

① $S = P + jQ = V_{RMS} I_{RMS} (\cos\theta_{ZL} + j\sin\theta_{ZL})$

$= V_{RMS} I_{RMS} e^{j\theta_{ZL}}$

② $S = \frac{1}{2} V I^* = \frac{1}{2} |V| e^{j\theta_V} (|I| e^{j\theta_i})^* = \frac{|V|}{\sqrt{2}} \frac{|I|}{\sqrt{2}} e^{j\overbrace{(\theta_V - \theta_i)}^{\theta_{ZL}}}$

$= V_{RMS} \cdot I_{RMS} \, e^{j\theta_Z}$

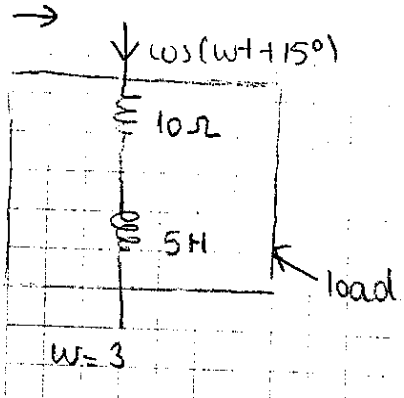


$Z_L = \frac{|V| \angle\theta_V}{|I| \angle\theta_i}$ | $\theta_{ZL} = \theta_V - \theta_i$

③ $S = \frac{1}{2} V I^* = \frac{1}{2} (Z_L I) I^* = \frac{|I|^2}{2} Z_L = I_{RMS}^2 Z_L$ → load impedance (Z_L)

④ $S = \frac{1}{2} V \left(\frac{V}{Z_L}\right)^* = \frac{|V|^2}{2 Z_L^*} = \frac{V_{RMS}^2}{Z_L^*}$ → conjugate

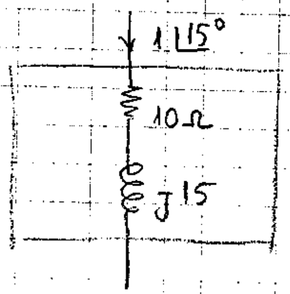
→



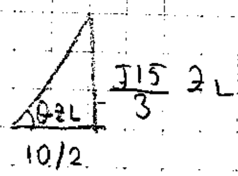
Find P_{AV} and Q of load

i-) Method 1

$P_{AV} = V_{RMS} I_{RMS} \cos(\theta_{ZL})$



$$V = (10 + j15)\angle 15^\circ = 5(2 + j3)\angle 15^\circ$$
$$= 5\sqrt{13}\angle 15^\circ + \tan^{-1}(3/2)$$



Method 1:

$$P_{AV} = V_{RMS}\, I_{RMS} \cos(\theta_{ZL})$$
$$= \frac{5\sqrt{13}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{2}{\sqrt{13}} = 5 \text{ Watts}.$$

$$Q = V_{RMS}\, I_{RMS} \sin(\theta_{ZL})$$
$$= \frac{5\sqrt{13}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{13}} = 7{,}5 \text{ VAR's}$$

Method 2:

$$S = \frac{1}{2}\left(5\sqrt{13}\angle 15 + \tan^{-1}(3/2)\right)\left(1\angle 15^\circ\right)$$
$$= \frac{1}{2}\, 5\sqrt{13}\angle \tan^{-1}(3/2) = \frac{5\sqrt{13}}{2}(\cos\theta_x + j\sin\theta_x) = \frac{5\sqrt{13}}{2}\left(\frac{2}{\sqrt{13}} + j\frac{3}{\sqrt{13}}\right)$$
$$= 5 + j(7{,}5) \text{ VA (Volt-Amperes)}$$

Method 3

$$S = I_{RMS}^2 Z_L = \frac{1}{2}(10 + j15) = 5 + j(7{,}5) \qquad I_{RMS} = \frac{1}{\sqrt{2}}$$

Method 4

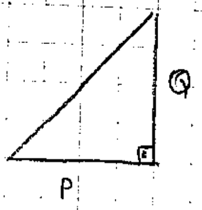
$$S = \frac{V_{RMS}^2}{Z_L^*} \quad \text{where } V_{RMS} = \frac{5\sqrt{13}}{\sqrt{2}}$$

$$S = \frac{25 \cdot \frac{13}{2}}{10 - j15} = \frac{5 \cdot \frac{13}{2}}{2 - j3} = \frac{5 \cdot \frac{13}{2}(2 + j3)}{13} = 5 + j(7{,}5)$$

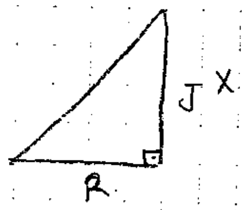
Power Triangle and Impedance Triangle

$$S = I_{RMS}^2 Z_L = P + jQ$$

power Triangle (Δ)

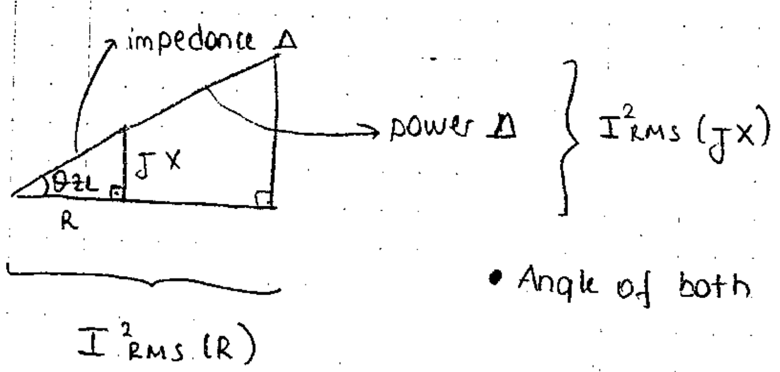


Impedance Triangle (Δ)



$P = I_{RMS}^2 R$

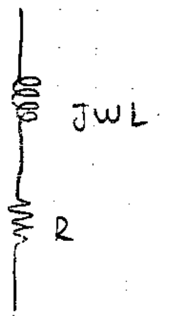
$Q = I_{RMS}^2 X$

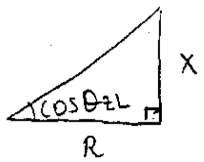


• Angle of both Δ's are the same

Power Factor (PF)

Inductive load



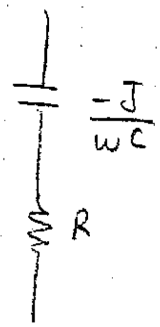


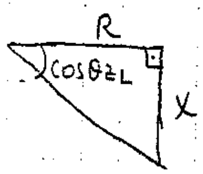
$S = P + JQ$

$P = I_{RMS}^2 R$ } $P > 0$

$Q = I_{RMS}^2 X$ } $Q > 0$

Capacitive load





$S = P + JQ$

$P = I_{RMS}^2 R$ } $P > 0$

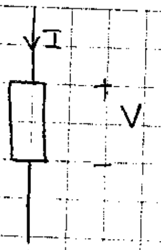
$Q = I_{RMS}^2 X$ } $Q < 0$

• power factor : $pf = \cos(\theta_{ZL})$ ← ΔS (angle of S)

• lagging

(Current lags the voltage = inductive)

• $pf = \cos(\theta_{ZL})$ • leading



$V = 100\ V$ (RMS) (We do not need for this method)

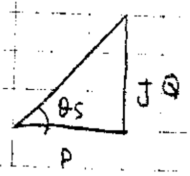
$P = 5kW$

$pf = 0{,}8$ lagging

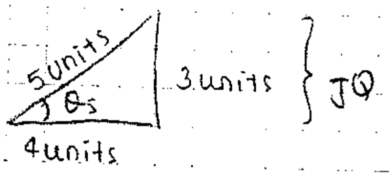
a.) Find VAR

b.) Find apparent power

a.) $Q = ?$



$\cos\theta_s = 0{,}8 \longrightarrow \frac{4}{5}$



$Q = 3750$ VAR since lagging

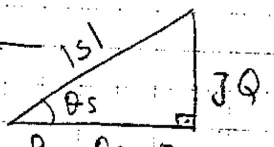
$Q = P\tan\theta_s = 5000\,\frac{3}{4} = 3750$ (lagging)

b.) Apparent Power: $V_{RMS}\, I_{RMS}$

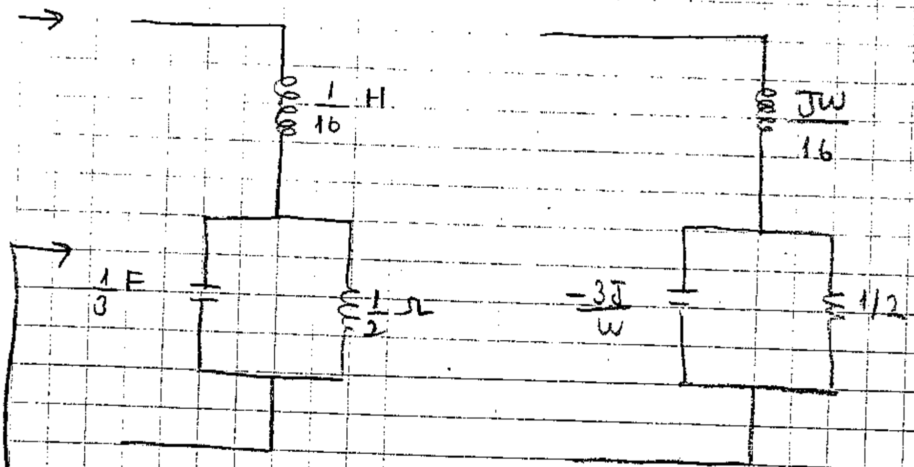
$S = V_{RMS}\, I_{RMS}\,(\cos\theta_{ZL} + j\sin\theta_{ZL})$

$|S| = V_{RMS}\, I_{RMS}$ $\qquad |S| = \frac{5000}{\cos(\theta_s)} = \frac{5000}{4/5} = 6250$ VA

apparent power



$\theta_s = \theta_{ZL}$



$Z(jw) = ?$ $\longrightarrow$ phasor domain

($\omega \neq 0$) Note that

$$Z(j\omega) = j\frac{\omega}{16} + \frac{\frac{-3j}{\omega}\cdot\frac{1}{2}}{-\frac{3j}{\omega}+1/2} = \frac{288 + j\omega(\omega^2-12)}{16(36+\omega^2)}$$ p.40 of notes

(54)

$\rightarrow$ cosine of angle with V

; $\omega = \sqrt{12} \longrightarrow Z(j\omega) = \dfrac{288}{16(36+12)}$ = purely Real pf = 1 unity power factor

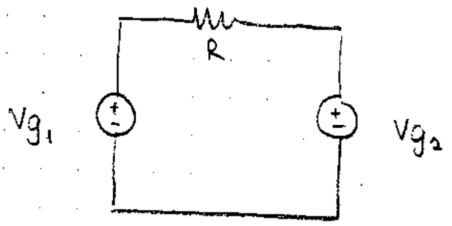
; $\omega < \sqrt{12} \longrightarrow \operatorname{Im}\{Z(j\omega)\} < 0 \longrightarrow$ capacitive load.

$$pf: \frac{288}{\sqrt{288^2+(\omega(\omega^2-12))^2}} \leftarrow \cos(\theta_{Zl}) \quad \text{(leading)}$$

; $\omega > \sqrt{12} \longrightarrow \operatorname{Im}\{Z(j\omega)\} > 0 \longrightarrow$ Inductive load

$$\text{p.f.} = \frac{288}{\sqrt{288^2+(\omega(\omega^2-12))^2}} \quad \text{(lagging)}$$

Superposition in AC Power



$V_{g_1} = V_1\cos(\omega_1 t + \theta_1)$

$V_{g_2} = V_2\cos(\omega_2 t + \theta_2)$

$i(t) = I_1\cos(\omega_1 t + \phi_1) + I_2\cos(\omega t + \phi_2)$ A (By superposition of V_{g_1} & V_{g_2})

$$P_{AV} = \frac{1}{T}\int_0^{t}(i(t))^2 R\,dt' = \frac{R}{T}\int_0^{T}\left[\begin{array}{l} I_1^2\cos^2(\omega_1 t+\phi_1) + I_2^2\cos^2(\omega_2 t+\phi_2) \\ +2I_1I_2\cos(\omega_1 t+\phi_1)\cos(\omega_2 t+\phi_2)\end{array}\right]dt'$$

$$= R\Big(\underbrace{\frac{1}{T}\int_0^{T} \text{------}\,dt'}_{(I_{1,RMS})^2} + \underbrace{\frac{1}{T}\int_0^{T} \text{---}\,dt'}_{(I_{2,RMS})^2} + \underbrace{\frac{2}{T}\int_0^{T} I_1I_2\cos(\omega_1 t'+\phi_1)\cos(\omega_2 t'+\phi_2)dt'}\Big)$$

Since $\omega_1 \neq \omega_2$;

$$\frac{I_1I_2}{T}\int_0^{T}\left[\begin{array}{l}\cancel{\cos((\omega_1-\omega_2)t'+\phi_1-\phi_2)}^{\,0} + \\ \cancel{\cos((\omega_1+\omega_2)t'+\phi_1+\phi_2)}^{\,0}\end{array}\right]dt'$$

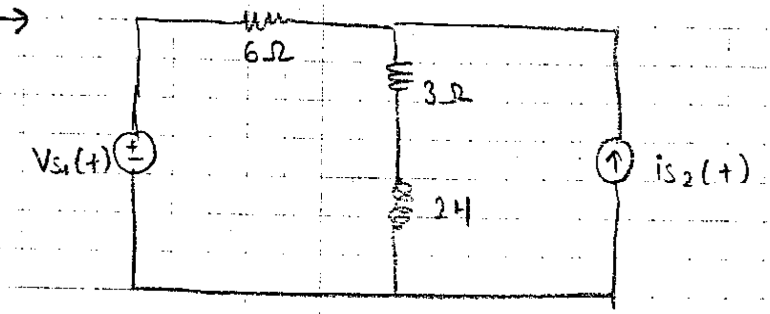
GIPTA

55

$$= R_1 \, (I_1^{RMS})^2 + R_1 \, (I_2^{RMS})^2$$

Superposition in AC power is possible for inputs with different frequencies.

→

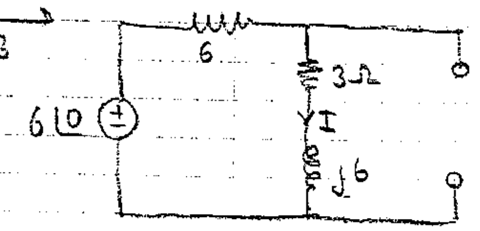


$V_{s_1}(t) = 6\cos(3t)$

$i_{s_2}(t) = 2\cos(4t + 30°)$ A

$P_{3\Omega}^{AV} = ?$

→ $\omega = 3$

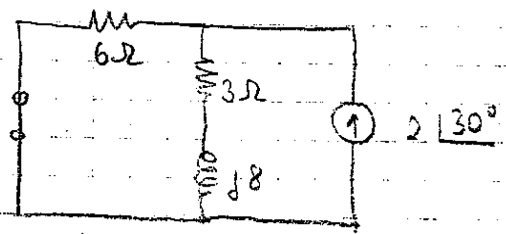


$$I = \frac{6}{9 + j6} = \frac{2}{3 + j2} = \frac{2}{\sqrt{13}} \angle -\tan^{-1}(2/3)$$

$$= \frac{2}{\sqrt{13}} \angle -\tan^{-1}(2/3)$$

$$P_{3\Omega}^{AVG,\, \omega=3} = (I_{RMS}^{3\Omega})^2 R = \left(\frac{2}{\sqrt{2}\,\sqrt{13}}\right)^2 3 = \frac{2}{13} 3 = \frac{6}{13} \text{ Watts.}$$

→ $\omega = 4$;

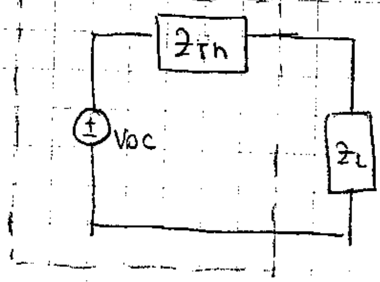


$$I^{(2)} = 2\angle 30 \, \frac{\frac{1}{3+j8}}{\frac{1}{3+j8} + \frac{1}{6}} = 2\angle 30 \, \frac{6}{9 + j8} = 2\angle 30 \, \frac{6}{\sqrt{145} \, \tan^{-1}(8/9)}$$

$$P_{3\Omega}^{AVG,\, \omega=4} = (I_{RMS}^{3\Omega,\, \omega=4})^2 \cdot 3 = \left(\frac{2 \cdot 6}{\sqrt{2}\,\sqrt{145}}\right) \cdot 3 = \frac{216}{145} \text{ watts.}$$

$$P_{3\Omega}^{AVG} \cong \frac{1}{2} + \frac{3}{2} \cong 2 \text{ Watts.}$$

Maximum Power Transfer



Voc, Z_{Th}: fixed (given to us (not a function of load)

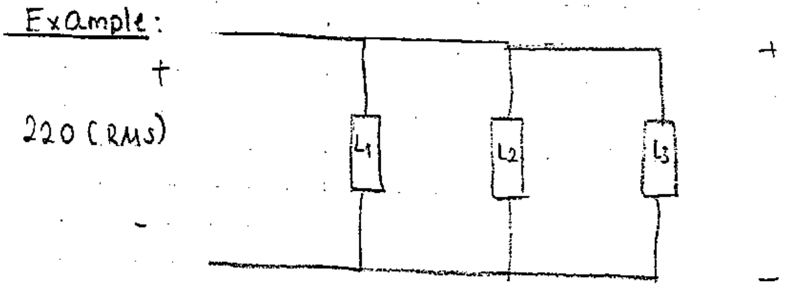
Select Z_{load} s.t. P_{AV} transferred to load is maximized.

$Z_{Th} = R_{Th} + jX_{th}$ $\quad$ $Z_L = R_L + jX_L$

$$I_{phase}^{load} = \frac{V_{oc}}{(R_{Th}+R_L)+j(X_{th}+X_L)} \longrightarrow \frac{(V_{oc}^{RMS})^2}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2} \cdot R_L$$

$$\left.\begin{aligned}\frac{\partial P_{AV}}{\partial R_L}&=0\\ \frac{\partial P_{AV}}{\partial X_L}&=0\end{aligned}\right\}\text{Solve together} \longrightarrow \begin{aligned}R_L&=R_{Th}\\ X_L&=-X_{Th}\end{aligned} \longrightarrow \boxed{Z_L = Z_{th}^*}$$

Example:



Load 1; 16 kW, 18 kVAR

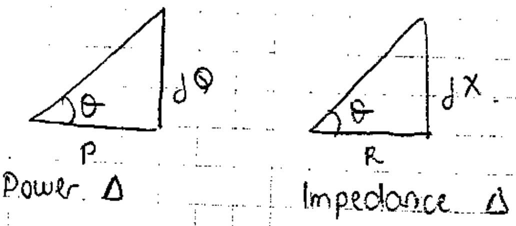
Load 2; 10kVA, at pf 0,6 leading

Load 3; 8kW at unity pf

a) Find pf at the source side.

b.) Find the impedance seen by the source.

a.)

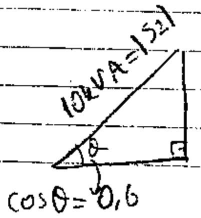


pf cos θ

$S_{total}^{source} = S_{L1} + S_{L2} + S_{L3}$

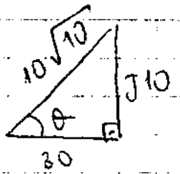
$S_{L1} = 16 + j18$ kVA

$S_{L2} = 6 - j8$ kVA
leading



$S_{L3} = 8$ kVA

$S_{Total} = 30 + j10$ kVA



p.f: $\cos\theta = \dfrac{30}{10\sqrt{10}} = \dfrac{3}{\sqrt{10}}$ lagging.

b-) $|S_{Total}| = V^{RMS}_{source} \cdot I^{RMS}_{source}$

$10\sqrt{10} = 220 \cdot I^{RMS}_{source}$

$I^{RMS}_{source} = \dfrac{\sqrt{10}}{22}$ kA (RMS)

$S^{sources}_{Total} = \left(I^{RMS}_{source}\right)^2 Z_{Total}$

$$Z_{Total} = \frac{(30+j10)10^3}{\left(\frac{\sqrt{10}}{22}\right)^2 10^6} = 22^2 \frac{(3+j)10^4}{10^7} = 22^2\ 10^{-3}(3+j)$$

$$= 0,484\,(3+j)\ \Omega$$

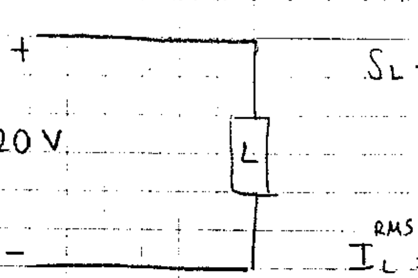
Power Factor Compensation

Example: A mill consumes 100 kW, 220 V (RMS) at pf: 0,85 & lagging

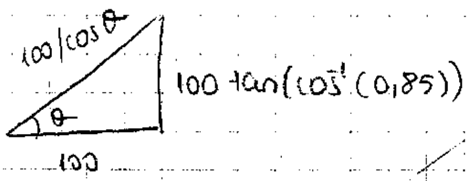
a-) Find RMS current supplied by 220V source to will.

b-) Find the current RMS if pf were 0,95 lagging.

a-)



$S_L = 100 + j$

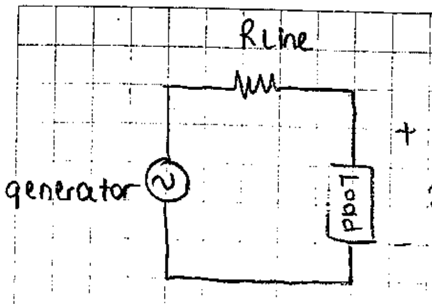


$$I_L^{RMS} = \frac{|S_L|}{220V} = \frac{(100/0,85)\,kVAR}{220V} = 534,8\ A$$

$$I_L^{RMS} = \frac{|S_L|}{220V} = \frac{10/0,95 \times 10^3}{220V} = 478\ A$$

Third part: c-) If there is a line connecting the source to the load & if the $R_{line} = 0,1\ \Omega$; Find power loss over the line for

p.f: {0.85, 0.95}



i.) $pf = 0.85 \rightarrow I_{Load} = 534.8$ A (RMS)

$P_{Line} = I^2_{Load,RMS} \times (0.1) = 28,6$ kW

ii.) $pf = 0.95 \longrightarrow P_{Line} = I^2_{Line,RMS}\ (0.1)$

468 A

$= 22.9$ kW

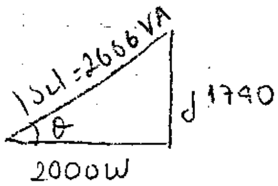
d.) Define efficiency as $\frac{\text{Power Delivered}}{\text{Power generated}} = \text{efficiency}$

i.) pf : 0,85 $\quad eff = \frac{100}{100+28.6} = 77\ \%$

ii.) pf : 0.95 $\quad eff = \frac{100}{100+22.9} = 82\ \%$

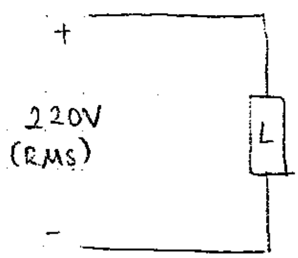
Example: Load requires 2kW at 0,75 p.f. lagging at 220V (RMS) Calculate the reactive power supplied by the compensating capacitor to make pf 0.9 lagging. Find the impedance and the capacitance in Farads (Assume 220V RMS, 50 Hz line)

$S_L = 2000 + j1740$



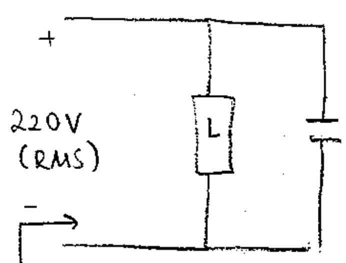
$\cos\theta = 0,75$

Before



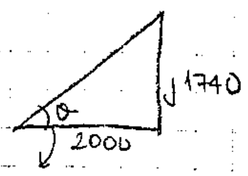
p.f : 0.75 (lag)

After



change pf to 0,9 (lagging)

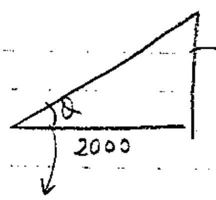
Before



$\cos\theta = 0,75$

$S_{before} = 2000 + j1740$

After



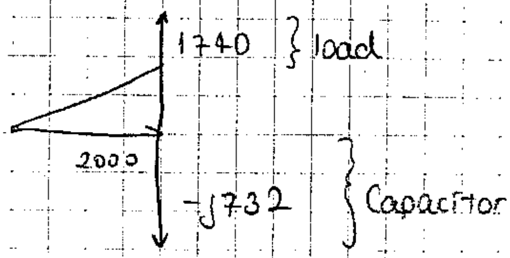
$j2000\tan(\cos^{-1}(0.9)) = j\,968$

$\cos\theta = 0.9$

$S^{desired}_{after} = 2000 + j\,968$

$S^{desired} = S^{load} + S^{cap}$

$S^{cap} = -j\,(1740 - 968) = -j\,772$



Impedance Calculation:

$$S^{cap} = \frac{(V^{RMS}_{cap})^2}{Z^*_{cap}} \qquad -j\,772 = \frac{(220)^2}{Z^*_{cap}} \quad ; \quad Z^*_{cap} = \frac{48\,400}{772}\,j$$

$Z^*_{cap} = -j\,61.07$

$$Z_{cap} = \frac{-j}{w\,C_{cap}}$$

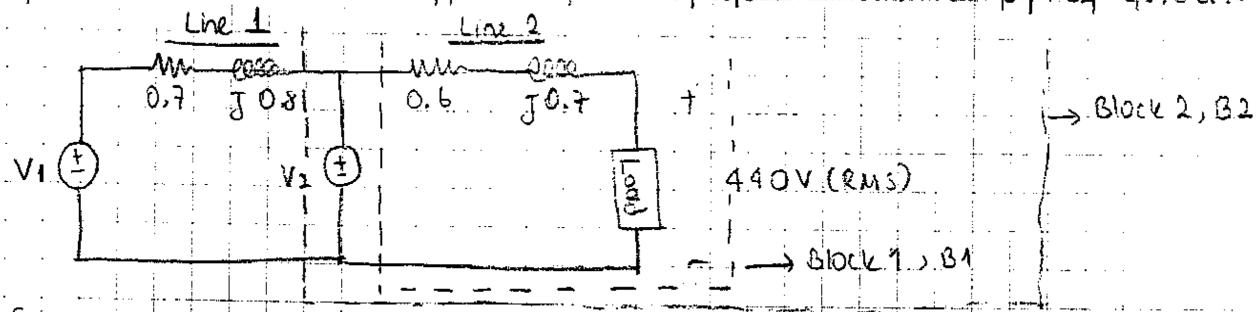
$$61.07 = \frac{1}{(2\pi 50)\,C_{cap}} \quad ; \quad C_{cap} = \frac{1}{100\pi\,(61.07)} = 52\,\mu F$$

$Z_{cap} = -j\,61.07$

Volt - Amphere Method for Power Calculations

Example: Two generators supply to the load 10 kW at 0,8 pf lagging

The generator 2 supplies 5 kW at 0,6 pf lagging, Find the voltages of V_1 & V_2 (RMS), the apparent power of generator 1 & pf of generator 2.



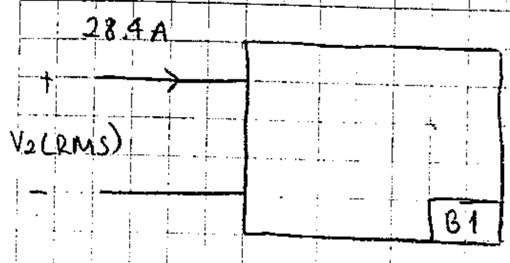
$S_{load} = 10\,000 + j\,7500$ VA

$S_{Line,2} = ?$

$$S_{Line,2} = (I^{RMS}_{load})^2\,(0{,}6 + j\,0{,}7) \qquad I^{RMS}_{load} = \frac{|S_{load}|}{440\,V} = \frac{10\,000/0{,}8}{440} = 28.4\,A$$

$S_{Line,2} = (I^{RMS}_{load})^2\,(0{,}6 + j\,0{,}7) = 484 + j\,565$

$S_{B_1} = S_{Line_2} + S_{load} = 10{,}484 + j\,8065$

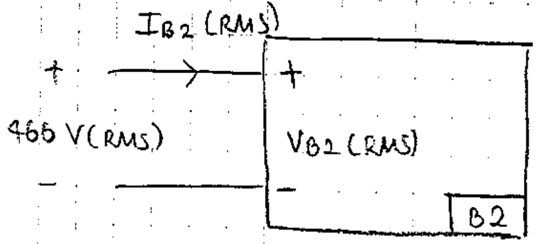


$$\underbrace{|S_{B1}|}_{\sqrt{(0{,}484)^2+(8065)^2}} = V_{B1}^{RMS} \cdot I_{B1}^{RMS}$$

$$V_{B1}^{RMS} = V_2^{RMS} = 466\,V \text{ (RMS)}$$

$$S_{G2}^{\text{supplied}} = 5000 + j\,5000\,\frac{4}{3} = 5000 + j6666$$

$$S_{B2} = S_{B1} - S_{G1}^{\text{supplied}} = 5484 + j\,1400$$



$$|S_{B2}| = 466\; I_{B2}^{RMS}$$

$$I_{B2}^{RMS} = \frac{|5484 + j1400|}{466} = 12.14\text{ A (RMS)}$$

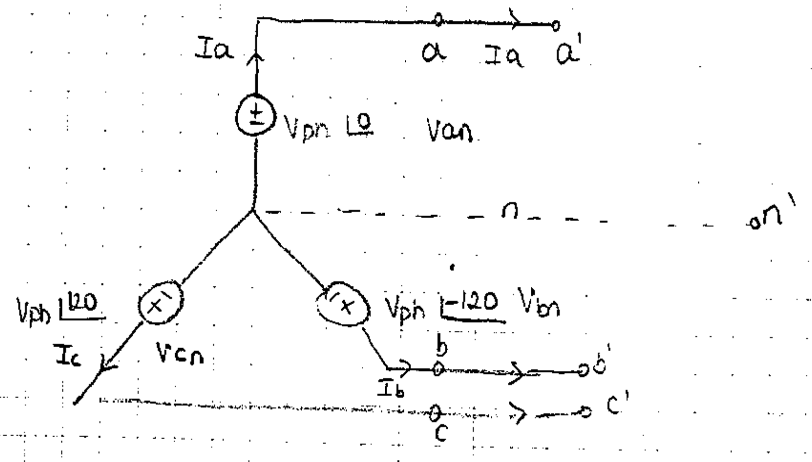
$$S_{line1} = (12.14)^2\left[0{,}7 + j\,0{,}8\right] = 103 + j1$$

$$S_{G1}^{\text{supplied}} = S_{line1} + S_{B2} = 5587 + j\,1516$$

$$V_1 = \frac{|S_{G1}|}{12.14} = 477\text{ V}$$

Apparent Power $V_{G1} = |S_{G1}| \cong 5784$

3 Phase Circuits



Van, Vbn, Vcn : phase voltages
↳ Va − Vn

Ia, Ib, Ic : phase currents

} Phase

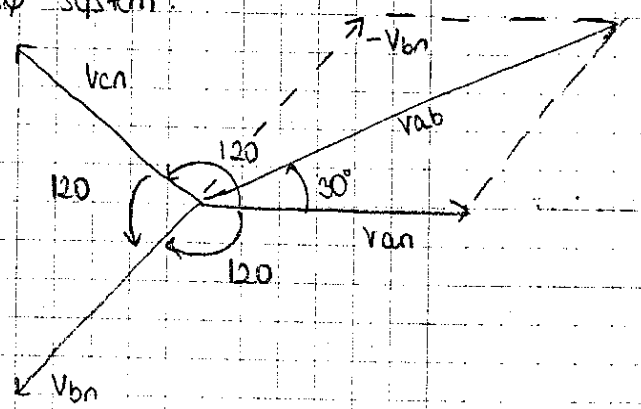
Vab, Vac, Vbc : line voltages

Iaa', Ibb', Icc'

} line

In 3-ϕ systems, a, b, c are called the lines connecting to another 3ϕ system,

phasor diagram



V_{an}, V_{bn}, V_{cn} : phase voltages

$V_{ab} = ?$ $\quad V_a - V_b = V_{an} - V_{bn}$

$V_{ab} = \sqrt{3}\, V_{ph} \angle 30^\circ$

or

$V_{ab} = V_{an} - V_{bn} = V_{ph}\angle 0 - V_{ph}\angle -120$

$= V_{ph}\angle 0^\circ + 1\angle 180\, V_{ph}\angle -120$

$= V_{ph}\angle 0^\circ + V_{ph}\angle 60^\circ = V_{ph}\left[1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right]$

$= \sqrt{3}\, V_{ph}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3}\, V_{ph}\angle 30^\circ$

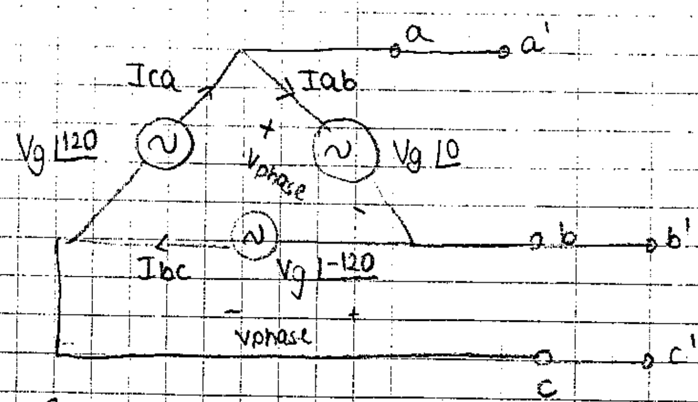
$V_{ac} = \sqrt{3}\, V_{ph} \angle -30^\circ$

$V_{cb} = \sqrt{3}\, V_{ph} \angle 90^\circ$

- For Y-connected load/generator:

  $I_{line} = I_{phase}$

- Δ-connected



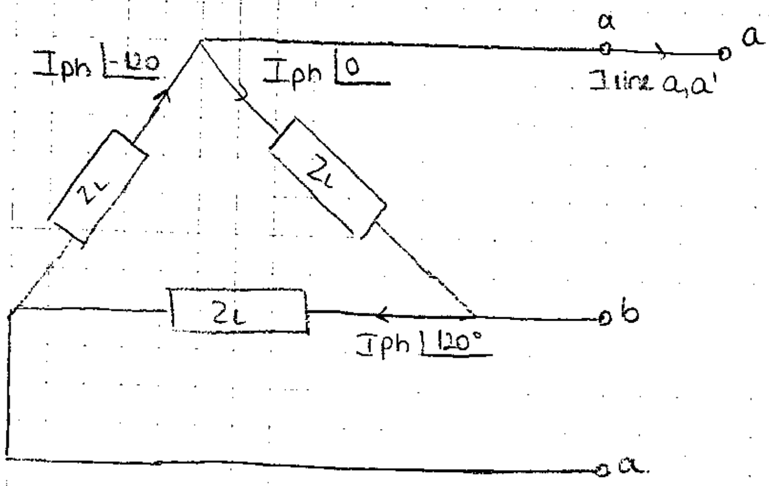
phase $\begin{cases} V_{an}, V_{bn}, V_{cn} : \text{phase voltages} \\ I_{ab}, I_{ca}, I_{bc} : \text{phase currents} \end{cases}$

line $\begin{cases} V_{ab}, V_{ac}, V_{bc} & \text{; line voltages} \\ I_{aa'}, I_{bb'}, I_{cc'} & \text{; line currents} \end{cases}$

- For Δ-load

$V_{phase} = V_{line}$

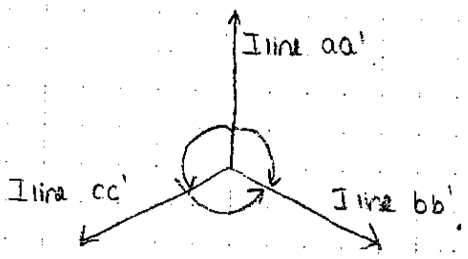
Line Current & Phase Current



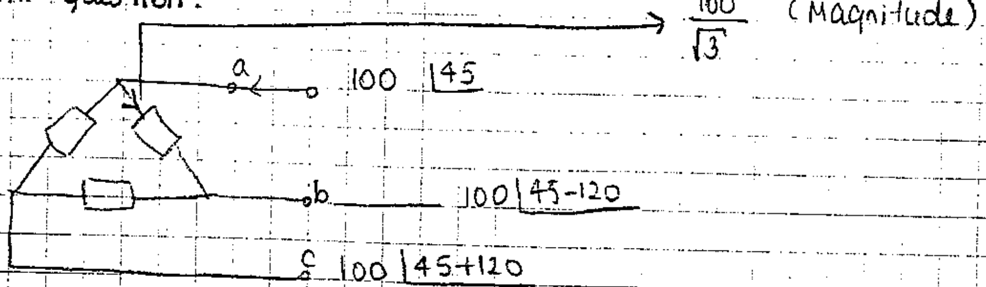
$I_{line\ aa'} = I_{ph}\angle{-120} - I_{ph}\angle 0 = \sqrt{3}\, I_{ph}\angle 210^\circ$

$I_{line\ bb'} = \sqrt{3}\, I_{ph}\angle{-30^\circ}$

$I_{line\ aa'} = \sqrt{3}\, I_{ph}\angle 90^\circ$

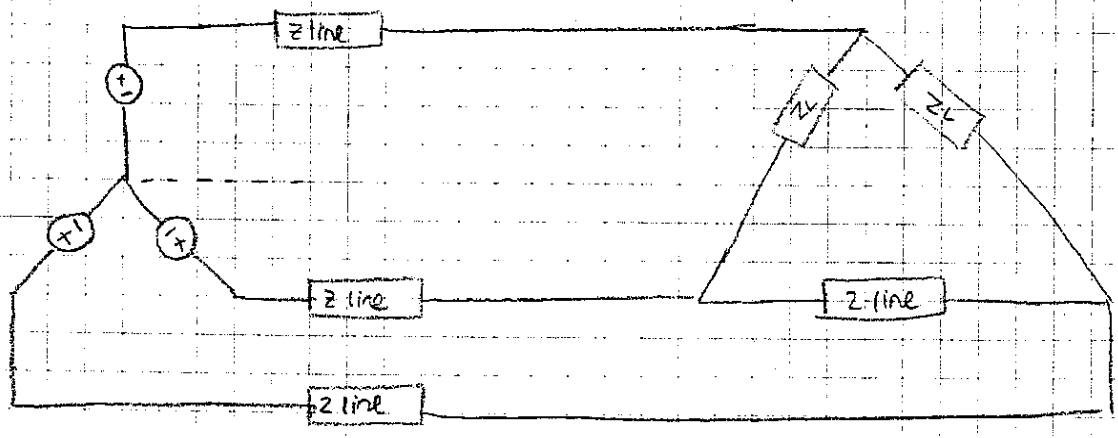


Mini Question:



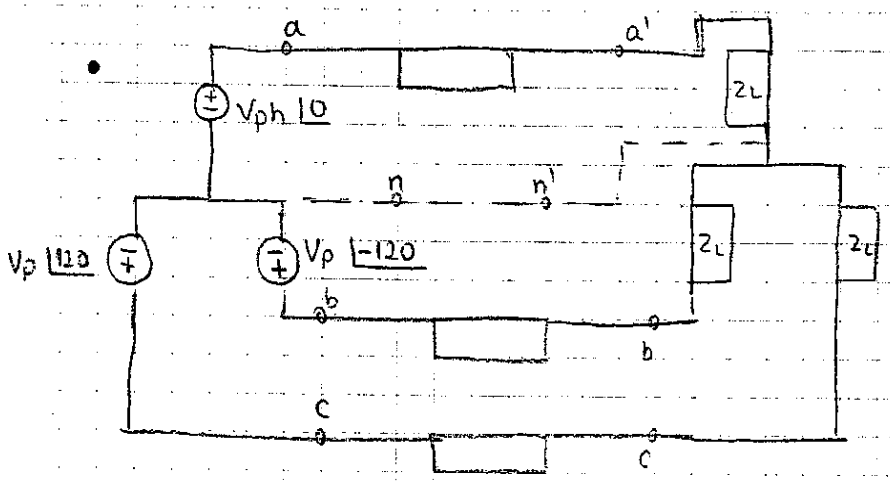
Notes:

1. Balanced 3ϕ systems:

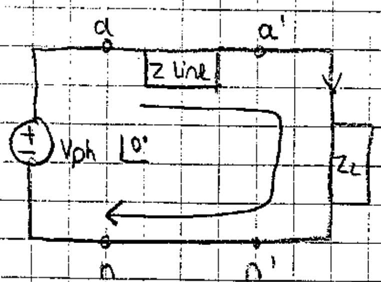


- In EE202 we only study balanced 3ϕ systems

Balanced system is the system having identical components on each phase. (apart from 120° of phase shift in the external input)



If the system is balanced, I can solve only phase a circuit (that is the circuit between a and n)

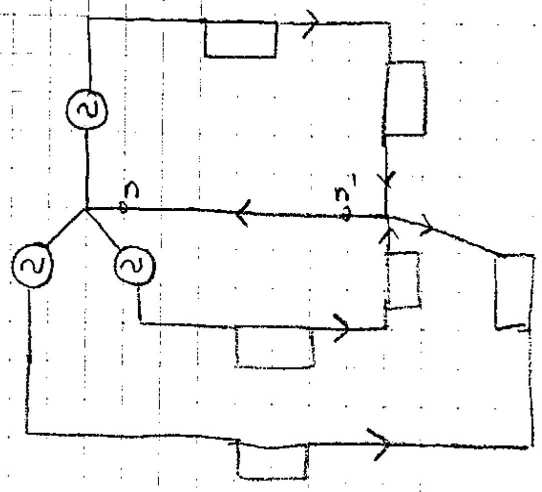


$I_{load,\,an'} = 51 \angle 12^\circ$

$I_{load,\,bn'} = 51 \angle 12^\circ - 120^\circ$

$I_{load,\,cn'} = 51 \angle 12 + 120^\circ$

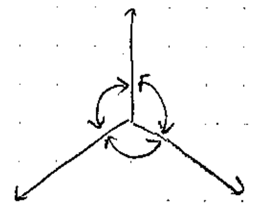
2. Neutral Connection



$I_{n'n} = ?$ (Assume balanced system)

If balanced $I_{n'n} = 0A$!

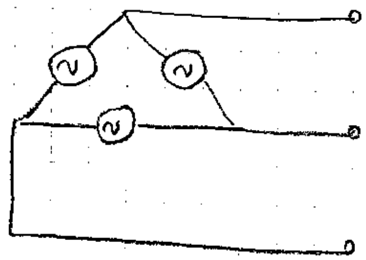
Y



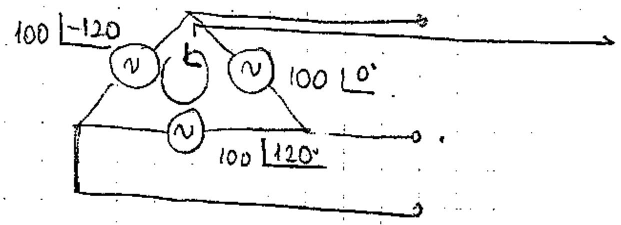
phasor diagram for load currents

So, the line between n & n' carries 0A of current, so it can be removed without any harm to the circuit

3. Δ Connected Generators



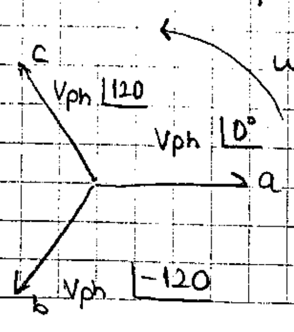
Generators arenot, in practice connected in Δ configuration. Since, if the system is even a little unbalanced.



Current can be very large.

- Generators are connected in Y configuration.

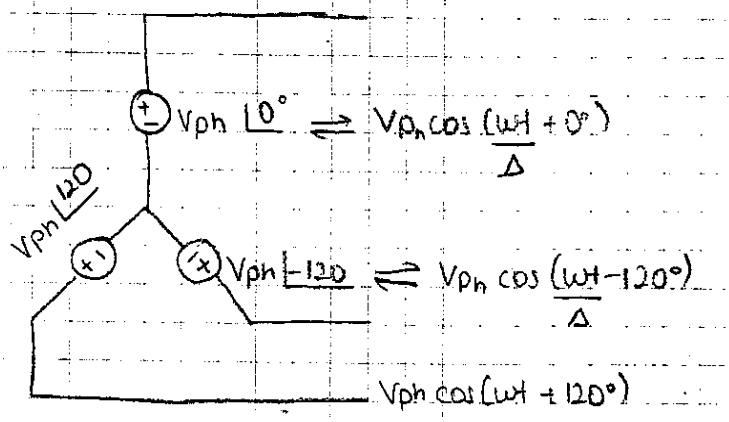
4- a, b, c Sequence (positive sequence)



a, b, c sequence (positive sequence)

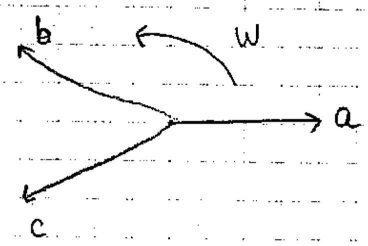
a has 0° then

b has 0° then c has 0°



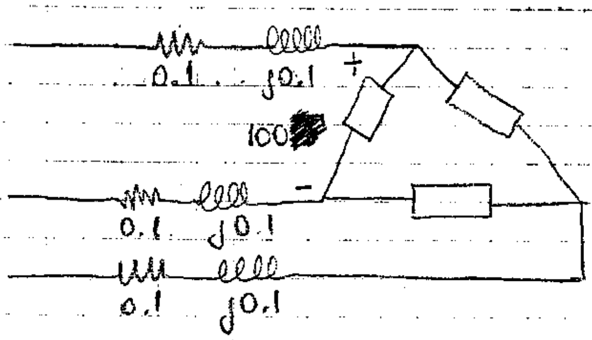
- a c b sequence (negative sequence)

a ⟶ c ⟶ b reaches 0° phase.



Example:

3ϕ generator



A 3ϕ load consumes 1200 Watts at p.f 0.8

a.) Find power supplied by the generator

b.) Find Vph (RMS) of the generator

i.) If generator Δ connected

ii.) If generator Y connected

a.) $S_{3\phi}^{load} = 1200 + j800$

Total complex power consumed by load

$S_\phi = \frac{S_{3\phi}}{3} = 400 + j300$

$|S_\phi| = |V_{ph}^{load}|\,|I_{ph}^{load}|$

$500 = 100\,|I_{ph}^{load}|$

$I_{ph,RMS}^{load} = 5A$

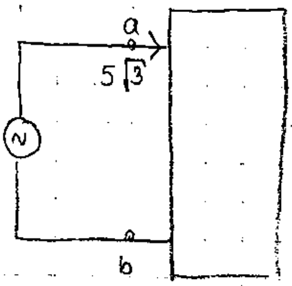
$I_{line}^{RMS} = I_{ph}^{RMS}\sqrt{3} = 5\sqrt{3}$ A (RMS)

$S_\phi^{line} = (I_{line}^{RMS})^2\,(0.1 + j0.1) = 7.5 + j7.5$

$S_{3\phi}^{line} = 22.5 + j22.5$

a.) $S^{supplied} = (1200 + j900) + (22.5 + j22.5) = 1222.5 + j922.5$

b.)



per phase power supplied: $407.5 + j307.5$

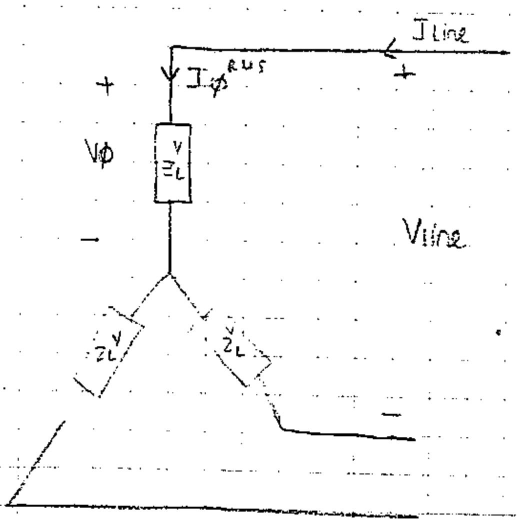
↑ supplied

$|S_{perphase\ supplied}| = 5\sqrt{3}\,V_{ph} \longrightarrow V_{ph} = \dfrac{\sqrt{(407.5)^2 + (307.5)^2}}{5\sqrt{3}}$

i) $V_{ph_\Delta} = \sqrt{3}\,V_{ph_Y} = \dfrac{\sqrt{(407.5)^2 + (307.5)^2}}{5}$

3ϕ power calculations

Y-connection



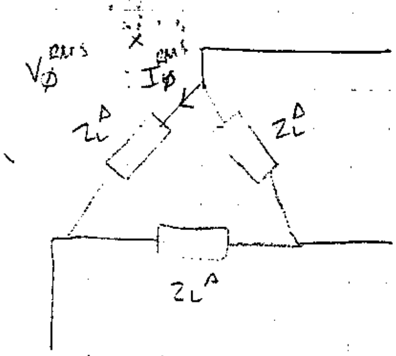
$S^{total} = 3S_\phi = 3\,(I_\phi^{RMS})^2\, Z_L^Y$ ← angle of

$S^{total} = 3\,I_\phi^{RMS} \cdot \underbrace{I_\phi^{RMS}\,|Z_L^Y|}_{V_\phi^{RMS}}\; e^{j\angle Z_L^Y}$

$= 3 I_{line}^{RMS}\,\dfrac{V_{line}^{RMS}}{\sqrt{3}}\, e^{j\angle Z_L^Y} = \sqrt{3}\, I_{line}^{RMS}\, e^{j\angle Z_L^Y}$

Δ-connection

$I_\phi^{RMS} = \dfrac{I_{line}^{RMS}}{\sqrt{3}}$



$S_{3\phi} = 3S_\phi = 3(I_\phi^{RMS})^2\, Z_L^\Delta$

$= 3 I_\phi^{RMS}\, I_\phi^{RMS} \cdot |Z_L|\, e^{j\theta_L}$

$= 3 I_\phi^{RMS}\, V_\phi^{RMS}\, e^{j\theta_L}$

$= 3\dfrac{I_{line}^{RMS}}{\sqrt{3}}\, V_{line}^{RMS}\, e^{j\theta_L}$

$= \sqrt{3}\, I_{line}^{RMS}\, V_{line}^{RMS}\, e^{j\theta_L}$

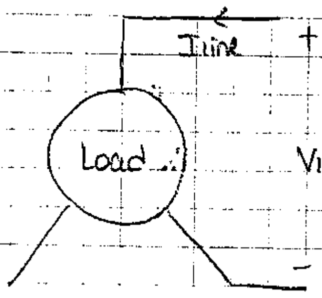
(Same as Y connection)

Cont. Y-Connection

$P = \sqrt{3}\, I_{line}^{RMS}\, V_{line}^{RMS} \cos(\theta_{ZL})$

$Q = \sqrt{3}\, I_{line}^{RMS}\, V_{line}^{RMS} \sin(\theta_{ZL})$

Contn. Δ-Connection



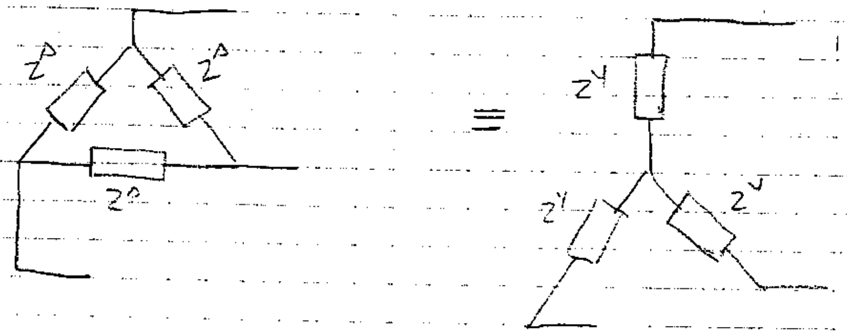
To calculate the total power consumed by load you need.

$P_{3\phi} = \sqrt{3}\, V_{line}^{RMS}\, I_{line}^{RMS} \cos(\theta_L)$

$Q_{3\phi} = \sqrt{3}\, V_{line}^{RMS}\, I_{line}^{RMS} \sin(\theta_L)$

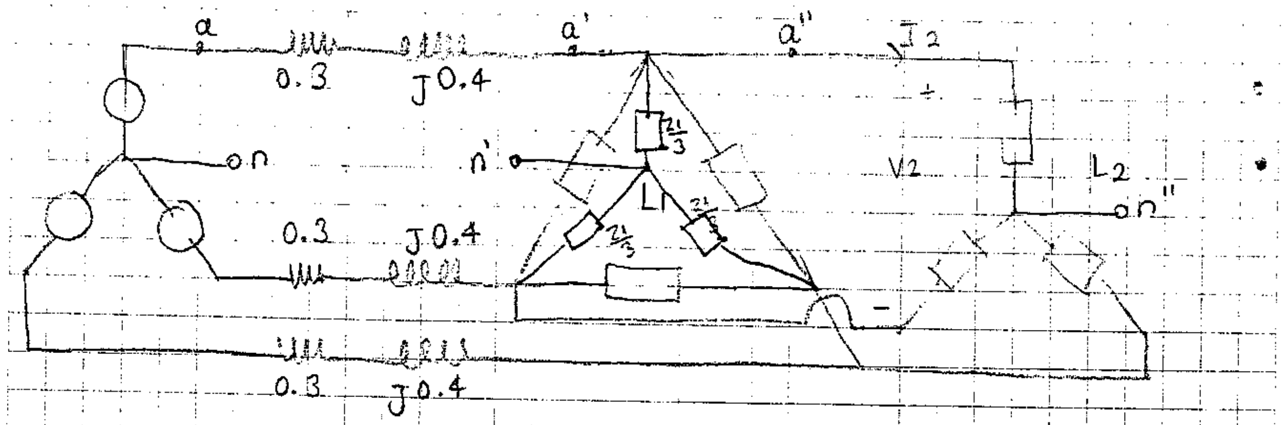
You do not need the type of connection (Y or Δ)

Remember



$* Z^{\Delta} = 3 Z^{Y}$

Example:

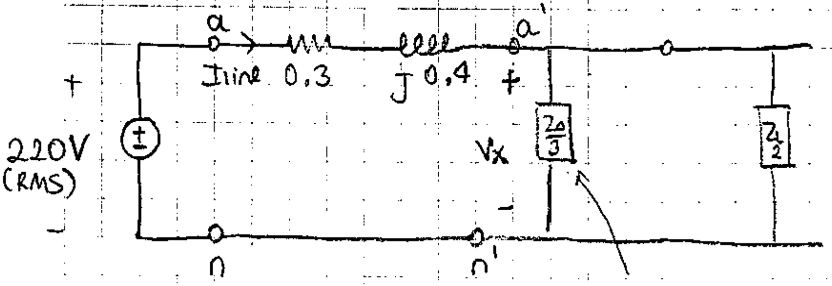


Generator produces 380 V (line to line) RMS voltage at 9 kW and 9 kVAR

L1: 6 kW at 0.6 pf lagging

Find V2, I2 and power absorbed by load 2.

Solution 1 : Using single phase equivalent circuit.



L1 : after Δ→Y transformation

$V_{an} = \frac{380}{\sqrt{3}} = 220 \text{ V (RMS)}$

$S^{gen}_{supplied} = 3000 + j\,3000 \longrightarrow |S_\phi^{gen}| = 220\, I_{line}^{RMS} \longrightarrow I_{line}^{RMS} = 19.3 \text{ A}$

perphase power

$S_\phi^{line} = (I_{line}^{RMS})^2 (0.3 + j\,0.4) = 112 + j\,149$

$S_\phi^{L1+L2} = S_\phi^{gen} - S_\phi^{line} = 2888 + j\,2851$

power consumed by load #1, #2

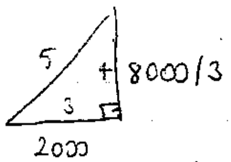
$|S_\phi^{L1+L2}| = V_x^{RMS}(19.3)$

$\downarrow$

- $V_x^{RMS} = 210.3 \text{ V}$

$S_\phi^{L2} = S_\phi^{L1+L2} - S_\phi^{L1}$

$\longrightarrow S_\phi^{L1} = 888 + j\,187$

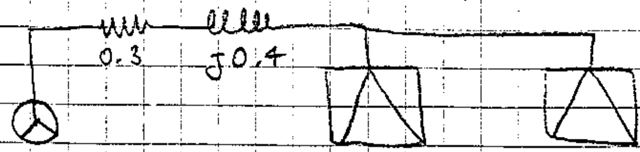


- $V_2 = V^{\text{line to line}}_{L2,RMS} = \sqrt{3}\, V_x = 364 \text{ V (RMS)}$
- $|S_\phi^{L2}| = |I_{\phi,2}^{RMS}|\, V_x^{RMS} \longrightarrow I_{2\phi}^{RMS} = \frac{907}{210.3} \longrightarrow \sqrt{888^2 + 187^2}$

$I_{2\phi}^{RMS} = 4.3 \text{ A}$

$I_2 = 4.3 \text{ A}$

Solution 2 : Single line diagram



$|S_{3\phi}^{gen}| = \sqrt{3}\, V_{line}^{gen} \cdot I_{line}^{gen}$

(formula valid for both Y & Δ connection)

$9000\sqrt{2} = \sqrt{3}\ 380\ I^{gen}_{line} \longrightarrow I^{gen}_{line} \longrightarrow I^{gen}_{line} = 19.3\ A$

$S^{line}_{3\phi} = 3(19.3)^2(0.3 + j0.4) = 336 + j447$

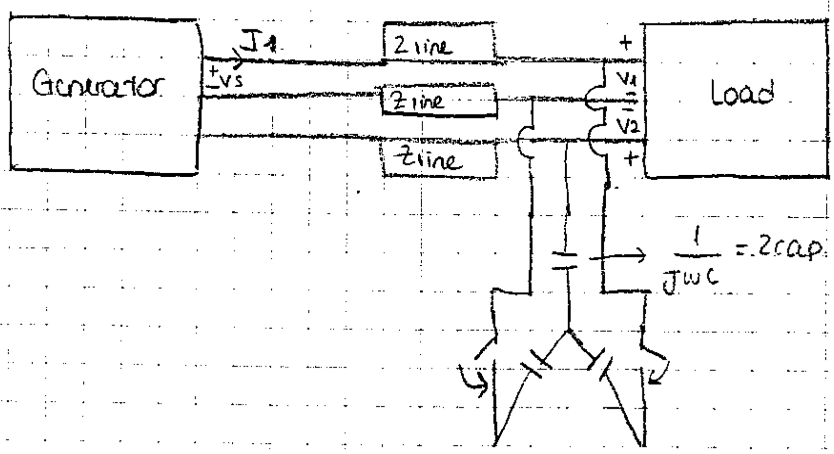
$S^{L1+L2}_{3\phi} = S^{gen}_{3\phi} - S^{line}_{3\phi} = 8664 + j8553$

$|S^{L1+L2}_{3\phi}| = \sqrt{3}\ V^{L1+L2,\,RMS}_{line-line}\ I^{L1+L2,\,RMS}_{line-line}$

$V^{L1+L2}_{line-line} = 364\ V \longrightarrow V_2$

$S^{L2}_{3\phi} = S^{L1+L2}_{3\phi} - S^{L1}_{3\phi} = 2664 + j561$ * Find I_2 !

Example



A balanced 3ϕ load 400 kW p.f. $1/\sqrt{2}$ lagging

$V_1 = 400\angle 30°$ Vrms Zline = $0.07 + j0.16$

$V_{1,RMS}$

$V_2 = 400\angle -30°$ Vrms

a.) Switches open find I_1, V_S the complex power supplied by source and efficiency.

$P_{load} = 400\,kW = \sqrt{3}\ V^{Load}_{line}\ I^{load}_{line}\ \cos(\theta_{load})$

$I^{load}_{line} = \sqrt{\frac{2}{3}}\ 1000\ A$ (RMS)

$= 816.5$ (RMS)

I_1

$S_{3\phi}^{line-loss} = 3(816.5)^2(0.07 + j0.16) = 140 + j320$ kVA

$S_{3\phi}^{supplied} = (400 + j400) + (140 + j320) = 540 + j720$ kVA

Eff: $\frac{P^{load}}{P^{supplied}} = \frac{400\,kW}{540\,kW} = 74\,\%$

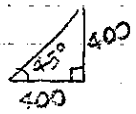
$\frac{watts}{watts}$

b.) Switches are closed and pf of the compensated load becomes 0.8 lagging.

Find the susceptance of capacitors in the bank, Vs, complex power supplied, efficiency

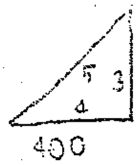
Before

$S_{3\phi} = 400 + j400$

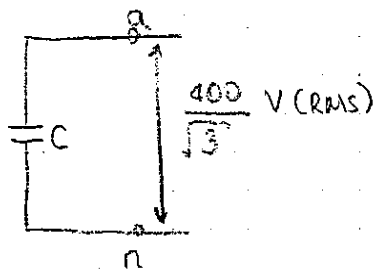


After

$S_{3\phi}^{desired} = 400 + j300$



$S_{3\phi}^{caps\,bank} = -j100^{k} \longrightarrow S_{\phi}^{cap} = -j\frac{100}{3} = \frac{(V_{cap}^{rms})^2}{Z_{cap}^*}$



$\longrightarrow Z_{cap}^* = j1600/10^3$

$Z_{cap} = -j1600/10^3 = -\frac{16j}{10}$

$P_{Load}^{after} = \sqrt{3}\, V_{line}^{load}\, I_{line}^{load} \underbrace{\cos(\theta_{load}^{after})}_{0.8}$

$400 \cdot 10^3 = \sqrt{3} \cdot 400 \cdot I_{line}^{load} \cdot 0.8$

$I_{line}^{Load} = 721.6$ A (RMS)

$S_{loss}^{Line} = 3(721.6)^2(0.07 + j0.16) = 109 + j250$

$S_{supplied} = S_{load}^{after} + (109 + j250) = (509 + j550)10^3$

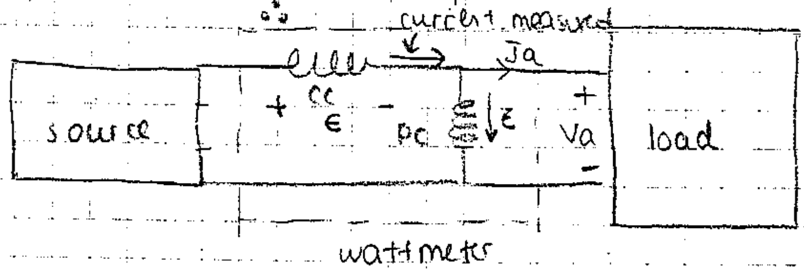
HW # 3 PS 3 : 2, 4, 6, 9
PS 4 : 5, 7
Due date : 27 April

Eff : $\frac{400}{509}$ = 78 %   * In p.f. compensation problems, the load voltage is assumed to be fixed!

That, $V_i^{RMS} = 400\,V$ before and after compensation.

3ϕ Power: (cont'd)

3ϕ power measurement by two Watt meter



CC : Current coil (measures Ja) [Current coil has low impedance]

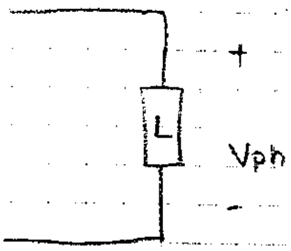
PC : Potential coil (measures Va) [Potential coil has high impedance]

$S_{Load} = V_a I_a^* \longrightarrow P_{AV} = \mathrm{Re}\{V_a I_a^*\}$ Everything expressed after rms scaling

wattmeter measures

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(\tau)\,d\tau}$: effective value is another name for rms

real number



Vph : phasor ⟵ complex power

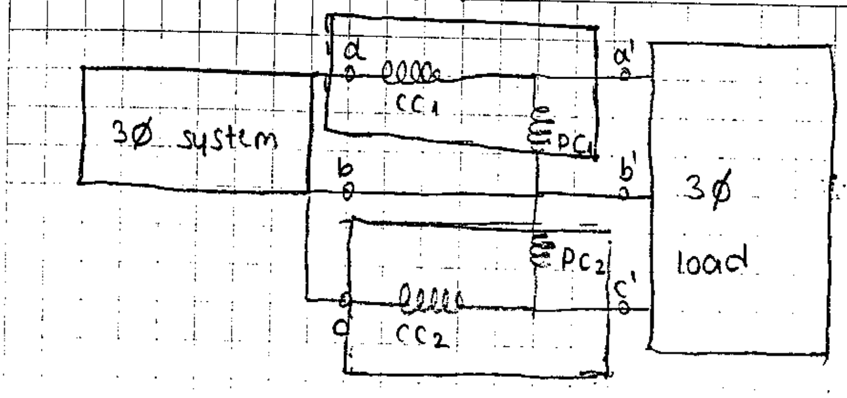
$V_{ph} = V_m \angle\theta_L\ V$ ⟵ conventional phasor

$V_{ph}^{(2)} = V_m^{(2)} \angle\theta_L^{(2)}\ V_{rms}$ ⟵ rms refers to the scaling of the amplitude by $1/\sqrt{2}$

The value after scaling by $\frac{1}{\sqrt{2}}$

So $V_m^{(2)}$ is not the peak value of the phasor but its rms value.

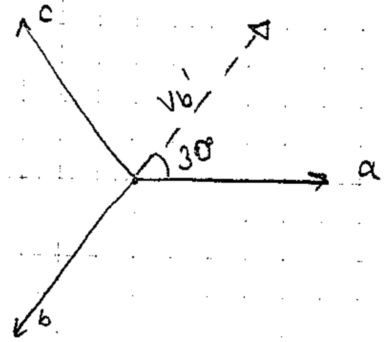
$S = \frac{1}{2} V_L I_L^*$ $S = V_L I_L^*$ if V_L and I_L are expressed in RMS



Power measured by wattmeter 1 : $Re\{V_{a'b'} I_{a'}^*\}$ ∴

Wattmeter 2 : $Re\{V_{c'b'} I_{c'}^*\}$ ∴

- $V_{a'b'} = V_{line} \angle 30°$ ⟵ why? since we have assumed positive sequence for a-b-c lines.



⟵ positive sequence

- $V_{c'b'} = V_{line} \angle 90°$

$I_{a'} = I_{line} \angle -\theta$

$I_{c'} = I_{line} \angle -\theta + 120°$

∴ $W_1 = Re\{V_{a'b'} I_{a'}^*\} = Re\{V_{line} \angle 30° \; I_{line} \angle \theta\}$

$W_2 = Re\{V_{c'b'} I_{c'}^*\} = Re\{V_{line} \angle 90° \; I_{line} \angle \theta - 120\}$

$W_1 = V_{line} I_{line} \cos(\theta + 30)$

$W_2 = V_{line} I_{line} \cos(\theta - 30)$

$W_1 + W_2 = V_{line} I_{line} \{ \text{———} - \cos(\theta + 30) + \cos(\theta - 30) \}$

$= \sqrt{3} V_{line} I_{line} \cos(\theta)$

$P_{3\phi}$ consumed by load ;

Brief Review of Laplace Transform and s-domain Circuit Analysis:

$(D^2+3D+2)x(t) = f(t)$ $\quad x(0^-) = x_0$
$\quad\quad \dot{x}(0^-) = \dot{x}_0$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix}_{2\times 2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \ \\ \ \end{bmatrix} f(t)$$

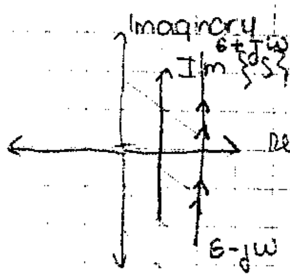
$x(t) = x_2(t)$ or
$x(t) = x_1(t) - x_2(t)$

$f(t) \xleftrightarrow[\text{Laplace}]{} F(s)$ $\quad\Big|\quad F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$ $\quad s \in R.O.C.$ (region of convergence)

a signal

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s)\, e^{st}\, ds$$

Transform Domain:



$u(t) \longleftrightarrow \frac{1}{s}$ $\quad Re\{s\} > 0$

$\cos(\alpha t) \longleftrightarrow \frac{s}{s^2+\alpha^2}$ $\quad Re\{s\} > 0$

Example: $\mathcal{L}\{u(t)\} = \int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt = \int_{0}^{\infty} 1 \cdot e^{-st}\, dt = \frac{e^{-st}}{-s}\Bigg|_{t=0}^{t=\infty}$

laplace transform

$= \frac{1}{s}\left[e^{-st}\right]_{t=\infty}^{t=0} = \frac{1}{s}\left[1 - e^{-st}\right]\Big|_{t=\infty}$;

$$= \begin{cases} \frac{1}{s}[1-0] & Re\{s\} > 0 \\ -\infty & Re\{s\} < 0 \end{cases}$$

$= \frac{1}{s}$ for $s \in R.O.C$

R.O.C $Re\{s\} > 0$

$u(t) \longleftrightarrow \frac{1}{s}$ $\quad$ ROC : $Re\{s\} > 0$

$e^{-\alpha t} u(t) \xleftrightarrow[\text{Lap}]{} \frac{1}{s+\alpha}$ $\quad$ ROC : $Re\{s\} > -\alpha$

$$\int_0^\infty e^{-\alpha t} e^{-st} dt = \frac{1}{s+\alpha} e^{-(s+\alpha)t} \Big|_{t=0}^{t=\infty} \longrightarrow Re\{s+\alpha\} > 0$$

So that $e^{-(s+\alpha)t}\Big|_{t=\infty}$ will be zero.

$\downarrow$

$Re\{s\} > -\alpha$

Types of Laplace Transforms

① Unilateral (one sided)

$$F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$$

Suitable for initial value problems

$(D^2+3D+2)\,x(t) = f(t)$

② Bilateral (two sided)

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

Suitable for general analysis

Unilateral Transform

$\mathcal{L}\{\dot f(t)\} = sF(s) - f(0^-)$;

$$\mathcal{L}\{\dot f(t)\} = \int_{0^-}^{\infty} \underset{dv}{\dot f(t)}\, \underset{u}{e^{-st}}\, dt$$

$$= f(t)\, e^{-st}\Big|_{t=0}^{t=\infty} + s\int_{0^-}^{\infty} f(t) e^{-st} dt$$

$= f(\infty)\, e^{-s\infty} - f(0^-) + sF(s)$ provided that $F(s)$ has R.O.C.

to eliminate $e^{-s\infty}$, $Re\{s\} > 0$

$= sF(s) - f(0^-)$, ROC for $\mathcal{L}\{\dot f(t)\} = (\text{ROC of } F(s)) \cap (Re\{s\} > 0)$

$\rightarrow \mathcal{L}\{\ddot f(t)\} = \mathcal{L}\{\dot g(t)\}$ $\qquad g(t) = \dot f(t)$

$= s\,G(s) - g(0) = s\,G(s) - f'(0) = s\,\mathcal{L}\{\dot f(t)\} - f'(0) = s\big(sF(s) - f(0)\big) - f'(0)$

$= s^2 F(s) - s f(0) - f'(0)$

$\rightarrow \mathcal{L}\{f^{4}(t)\} = s^4 F(s) - s^3 f(0) - s^2 f'(0) - s f''(0) - f'''(0)$

Example: $(D^2+3D+2)\,V_c(t) = f(t)$

$V_c(0^-) = V_0$

$\dot V_c(0^-) = \dot V_0$

Assume a solution exist $V_c(t)$

Then $\mathcal{L}\{V_c(t)\} = V_c(s)$ assume a ROC also

$$\mathcal{L}\left\{\frac{d}{dt}V_c(t)\right\} = sV_c(s) - V_0$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}V_c(t)\right\} = s^2V_c(s) - sV_0 - \dot{V}_0$$

- Then let's take Laplace transform of $(D^2+3D+2)V_c(t) = f(t)$

$$\mathcal{L}\{D^2 V_c(t)\} + \mathcal{L}\{3DV_c(t)\} + \mathcal{L}\{2V_c(t)\} = F(s)$$

$$s^2V_c(s) - sV_0 - \dot{V}_0 + 3sV_c(s) - 3V_0 + 2V_c(s) = F(s)$$

$$V_c(s)\left[s^2+3s+2\right] + V_0(-s-3) - \dot{V}_0 = F(s)$$

$$V_c(s) = \frac{(s+3)V_0 + \dot{V}_0}{s^2+3s+2} + \frac{F(s)}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{F(s)}{s^2+3s+2}$$

$$V_c(s) = \frac{2V_0+\dot{V}_0}{s+1} + \frac{-V_0-\dot{V}_0}{s+2} + \frac{Fs}{s^2+3s+2}$$

① $V_c^{zs}(s) = \dfrac{F(s)}{s^2+3s+2}$ $\qquad V_c^{zi}(s) = \dfrac{2V_0+\dot{V}_0}{s+1} + \dfrac{-V_0-\dot{V}_0}{s+2}$

② $V_c(t) = \mathcal{L}^{-1}\{V_c(s)\} = (2V_0+\dot{V}_0)e^{-t}u(t) - (V_0+\dot{V}_0)e^{-2t}u(t) + \mathcal{L}^{-1}\left\{\dfrac{F(s)}{s^2+3s+2}\right\}$

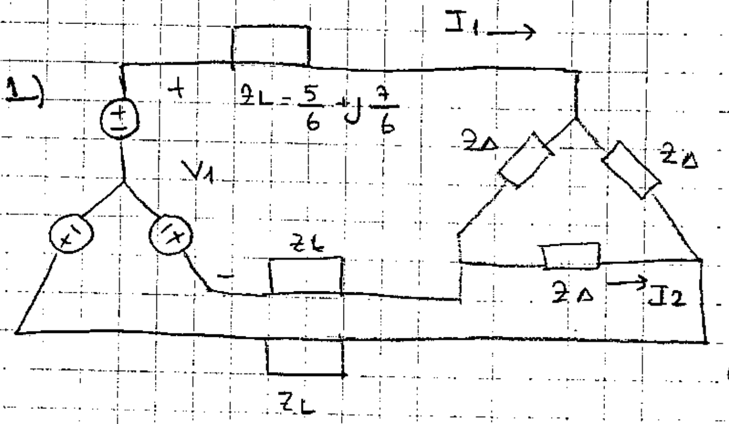
$$V_c^{zi}(t) = \left[(2V_0+\dot{V}_0)e^{-t} - (V_0+\dot{V}_0)e^{-2t}\right]u(t)$$

to excite $\lambda = -2$ mode in $V_c^{zi}(t)$ then $V_0+\dot{V}_0 = 0$ or

$$\begin{bmatrix} V_0 \\ \dot{V}_0 \end{bmatrix} \propto \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Recitation :

1)



$Z_\Delta = ?$ $S_{source} = ?$

$|V_1| = 150\sqrt{6}$ Vrms

$|I_2| = 20\sqrt{3}$ Arms

The total real power consumed by the inductive load is 18 kW

$|I_1| = \sqrt{3}\,|I_2| = 60$ Arms

$|S_{source}| = \sqrt{3}\,|V_L||I_L| = \sqrt{3}\,|V_L||I_L| = 27000\sqrt{2}$ VA

$\underset{150\sqrt{6}}{|V_L|}\ \underset{60}{|I_L|}$

$S_{source} = S_{line} + S_{load}$

$S_{line} = 3|I_1|^2 Z_{line} = 3 \cdot 60^2 \cdot \left(\frac{5}{6} + j\frac{7}{6}\right) = 9000 + j\,12600$ VA

$S_{load} = 18000 + jQ_L$ $Q_L > 0$

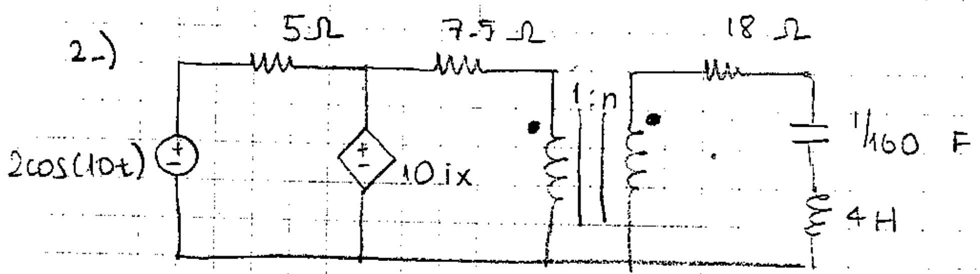
$S_{source} = 27000 + j(Q_L + 12600)$ $Q_L + 12600 = 27000$

$Q_L = 14400$ VAR $S_{source} = 27000 + j\,27000$ VA

$S_{load} = 18000 + j\,14400$ VA. $S_{load,\phi} = 6000 + j\,4800$

$$Z_\Delta = \frac{S_{load,\phi}}{|I_2|^2} = 5 + j4$$

2.)



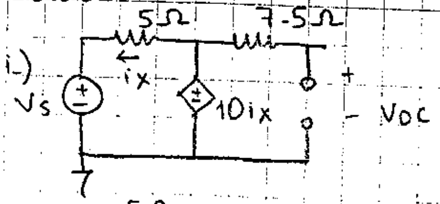
i.) Obtain the thevenin equivalent of the left side

ii.) Find n such that max average power is transferred to the load

iii.) Find the value of average power delivered to the load for the n value in part ii)

Solution:

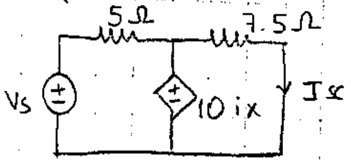
i)



$V_s = 5 i_x$

$V_{oc} = 10 i_x$

$V_{oc} = 2 V_s$



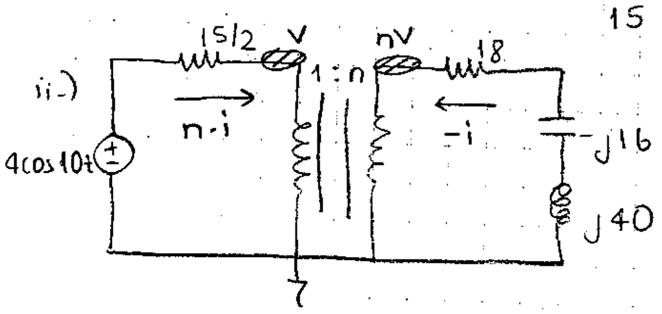
$V_s = 5 i_x$

$10 i_x = 7.5 I_{sc}$

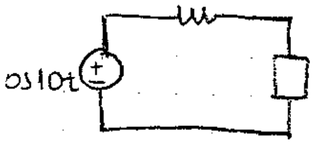
$I_{sc} = \frac{4}{15} V_s$

$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{15}{2}\ \Omega$

ii)



$\frac{nV}{i} = 18 + j24 \longrightarrow \frac{V}{n \cdot i} = \frac{18 + j24}{n^2}$



$Z = \frac{18 + j24}{n^2} = R_L + j X_L$

$I = \frac{V_{th}}{R_{th} + R_L + j X_L}$ $\qquad P_L = \frac{1}{2} |I|^2 \, Re\{R_L + j X_L\}$

$P_L = \frac{1}{2} \frac{V_{th}^2}{(R_{th} + R_L)^2 + X_L^2} \cdot R_L$, $\frac{R_L}{(R_L + R_{th})^2 + X_L^2}$ must be maximum

$X_L = \frac{4}{3} R_L \longrightarrow f(R_L) = \frac{R_L}{(R_{th} + R_L)^2 + \frac{16}{9} R_L^2}$ must be max

$f'(R_L) = 0 = (R_{th} + R_L)^2 + \frac{16}{9} R_L^2 - R_L \left[\frac{32}{9} R_L + 2(R_{th} + R_L) \right]$

$R_{th}^2 - \frac{50}{9} R_L^2 + \frac{25}{9} R_L^2 = 0 \qquad R_{th} = \frac{5}{3} R_L$

$R_L = \frac{3}{5} \cdot \frac{15}{2} = \frac{9}{2} \qquad \frac{18}{n^2} = \frac{9}{2} \quad ; \quad \boxed{n = 2}$

iii) $P_L = \frac{1}{2} \frac{V_{th}^2}{(R_{th} + R_L)^2 + X_L^2} \cdot R_L$

$= \frac{1}{2} \frac{16}{\left(\frac{15}{2} + \frac{9}{2}\right)^2 + 6^2} \cdot \frac{9}{2} = \frac{1}{5}\ W$

Inverse Laplace Transforms

Partial Fraction Methods

$1/s \longrightarrow u(t)$

$1/(s+a) \longrightarrow e^{-at}\, u(t)$

$1/(s+a)^2 \longrightarrow t\, e^{-at}\, u(t)$

$K/(s+a)^r \longrightarrow \dfrac{K t^{r-1} e^{-at}}{(r-1)!}\, u(t)$

$$\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}$$

$$2|K|\, e^{-\alpha t} \cos(\beta t+\theta)\, u(t)$$

where $K = |K|\, e^{j\theta}$

$K^* = |K|\, e^{-j\theta}$

1) Distinct Real roots of D(s)

Example: $F(s) = \dfrac{96(s+5)(s+12)}{s(s+8)(s+6)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+8} + \dfrac{K_3}{s+6}$

$K_1 = 120$, $K_2 = -72$, $K_3 = 48$

$L^{-1}\{F(s)\} = 120\, u(t) - 72\, e^{-8t} u(t) + 48\, e^{-6t} u(t)$

2) Distinct Complex Roots of D(s)

$$F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)} = \frac{K_1}{s+6} + \frac{K_2}{s+3+4j} + \frac{K_3}{s+3-4j}$$

$F(s)(s+6)\Big|_{s=-6} = K_1 = -12$ $\qquad$ $F(s)(s+3+4j)\Big|_{s=-3-4j} = K_2 = 6+8j$

$F(s)(s+3-4j)\Big|_{s=4j-3} = 6-8j = K_3$

$$L^{-1}\{F(s)\} = -12\, e^{-6t} u(t) + \underbrace{20}_{2|K|}\, e^{\overset{\alpha}{-3}t} \cos(\overset{\beta}{4}t \overbrace{-\tan^{-1}(4/3)}^{\theta})\, u(t)$$

3. Repeated Real Roots of D(s)

$$F(s) = \frac{100(s+25)}{s(s+5)^3} = \frac{K_1}{s} + \frac{K_2}{(s+5)^3} + \frac{K_3}{(s+5)^2} + \frac{K_4}{(s+5)}$$

$F(s)\, s\Big|_{s=0} = K_1 = 20$ $\qquad$ $K_3 = \dfrac{d}{ds}\left[F(s)(s+5)^3\right]\Big|_{s=-5} = -100$

$F(s)(s+5)^3\Big|_{s=-5} = K_2 = -400$ $\qquad$ $2K_4 = \dfrac{d^2}{ds^2}\left[F(s)(s+5)^3\right] = -40$

$K_4 = -20$

$$F(s) = \frac{20}{s} - \frac{400}{(s+5)^3} - \frac{100}{(s+5)^2} - \frac{20}{s+5}$$

$$L^{-1}\{F(s)\} = 20u(t) - \frac{400t^2 e^{-5t}}{2!}u(t) - \frac{100 t e^{-5t}}{1!}u(t) - 20e^{-5t}u(t)$$

$$= 20u(t) - 200t^2 e^{-5t}u(t) - 100t e^{-5t}u(t) - 20e^{-5t}u(t)$$

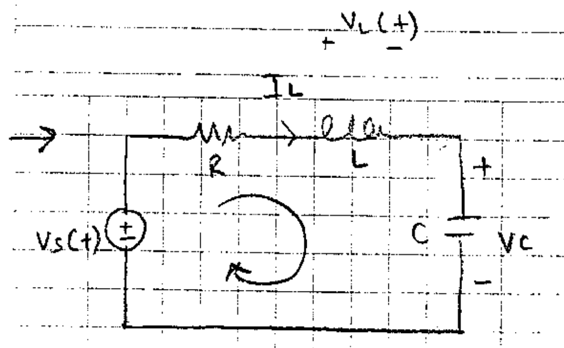
4- Repeated complex Roots

$$F(s) = \frac{768}{(s^2+6s+25)^2} = \frac{K_1}{(s+3-4j)^2} + \frac{K_2}{s+3-4j} + \frac{K_1^*}{(s+3+4j)^2} + \frac{K_2^*}{s+3+4j}$$

$$K_1 = F(s)(s+3-4j)^2\Big|_{s=-3+4j} = -12 \qquad K_1^* = -12$$

$$K_2 = \frac{d}{ds}\Big[F(s)(s+3-4j)^2\Big]_{s=-3+4j} = -3j \qquad K_2^* = +3j$$

$$F(s) = -12\left[\frac{1}{(s+3-4j)^2} + \frac{1}{(s+3+4j)^2}\right] + \frac{3\angle{-90^\circ}}{s+3-4j} + \frac{3\angle{90^\circ}}{s+3+4j}$$



$V_C(0^-) = V_0$

$I_L(0^-) = I_0$

KVL : $-V_s(t) + \underbrace{R i_L(t)}_{V_R(t)} + \underbrace{L \frac{d i_L(t)}{dt}}_{V_L(t) \;\; t \geq 0} + \underbrace{V_C(t)u(t) + \frac{1}{C}\int_{0^-}^{t} i_L(\tau)\,d\tau}_{V_C(t) \;\; t > 0} = 0$

Laplace Transform

$\mathcal{L}\{D i_L(t)\}$ $\quad \mathcal{L}\{u(t)\}$ $\quad \mathcal{L}\{D^{-1} i_L(t)\}$

$$-V_s(s) + R I_L(s) + L\left(s I_L(s) - \underbrace{i_L(0^-)}_{I_0}\right) + \underbrace{V_C(0^-)}_{V_0}\cdot\frac{1}{s} + \frac{1}{C}\,\frac{I_L(s)}{s} = 0$$

$$V_s(s) = \mathcal{L}\{V_s(t)u(t)\} = \int_0^{t} V_s(t)u(t)\,e^{-st}\,dt$$

- $$\mathcal{L}\left\{\int_{0^-}^{t} x(t')\,dt'\right\} = \int_{t=0^-}^{\infty}\underbrace{\left(\int_{0^-}^{t} x(t')\,dt'\right)}_{u}\underbrace{e^{-st}\,dt}_{dv}$$ (Integration by parts)

$$= \left(\int_{0^-}^{t} x(t')\,dt'\right)\frac{e^{-st}}{-s}\Bigg|_{t=0^-}^{t=\infty} - \int_{0^-} x(t)\,\frac{e^{-st}}{-s}\,dt$$

$$= 0 - 0 + \frac{1}{s}X(s) \quad (s \in ROC)$$

$$= \frac{X(s)}{s}$$

- $\mathcal{L}\{x'(t)\} = sX(s) - x(0^-)$

$\mathcal{L}\{D^{-1}x(t)\} = \frac{X(s)}{s}$

$$\rightarrow I_L(s) = \frac{C s V_s(s) + sLC I_0 - C V_0}{s^2 LC + RCs + 1}$$

$$I_L(s) = \underbrace{\frac{\frac{s}{L}V_s(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}_{\text{due to external input: zero state response}} + \frac{s I_0 - V_0/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

→ response due to initial cond. } zero-input response

complete response in s domain

a- Find unit-step response: (zero state response for unit step input)

$$V_s(s) = \mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$i_L^{\text{unit step}}(t) = \mathcal{L}^{-1}\left\{\frac{S/L\, V_s(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\right\}\Big\downarrow_{V_s(s) = 1/s}$$

Let $L = 1H$, $R = 6\,\Omega$, $C = \frac{4}{100}F$, $I_0 = 5A$, $V_0 = 1V$

$$= \mathcal{L}^{-1}\left\{\frac{1}{\underbrace{s^2 + 6s + 25}_{(s+3)^2 + 16}}\right\} = e^{-3t}\frac{\sin 4t}{4}$$

- $\mathcal{L}^{-1}\left\{\frac{1}{4}\frac{4}{(s+3)^2 + 4^2}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{4}{(s+3)^2 + 4^2}\right\}$
- $\mathcal{L}^{-1}\left\{\frac{b}{(s+a)^2 + b^2}\right\} = e^{-at}\sin(bt)u(t)$
- $\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2 + b^2}\right\} = e^{-at}\cos(bt)u(t)$

b- Impulse Response (Remember all responses are calculated at zero state (zero initial energy) (required due to linearity)

$$h(t) = i_L^{\text{impulse}}(t) = \mathcal{L}^{-1}\left\{\frac{\frac{S}{L}V_s(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\right\}\Big\downarrow_{V_s(s) = \mathcal{L}\{\delta(t)\} = 1}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+3-3}{(s+3)^2 + 4^2}\right\} = e^{-3t}\cos 4t - \frac{3}{4}e^{-3t}\sin 4t$$

$$= \frac{1}{4}e^{-3t}(4\cos 4t - 3\sin 4t)u(t)$$

$$= \frac{5}{4}e^{-3t}\cos\left(4t + \tan^{-1}(3/4)\right)u(t)$$

c- Doublet Response

$$\delta'(t) = \text{Doublet} = \frac{d}{dt}\delta(t)$$

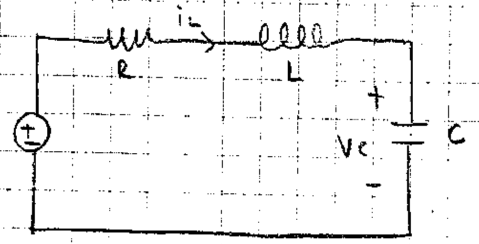
$$\mathcal{L}\{\delta'(t)\} = s$$

$$i_L^{\text{doublet}}(t) = \mathcal{L}^{-1}\left\{\frac{s^2}{(s+3)^2+4^2}\right\} = \mathcal{L}^{-1}\left\{1 + \frac{-6s-25}{(s+3)^2+4^2}\right\}$$

$$= \delta(t) - \mathcal{L}^{-1}\left\{\frac{6s+25}{(s+3)^2+4^2}\right\} = \delta(t) - \mathcal{L}^{-1}\left\{\frac{6(s+3)}{(s+3)^2+4^2} + \frac{7}{(s+3)^2+4^2}\right\}$$

$$= \left(\delta(t) - 6e^{-3t}\cos 4t - \frac{7}{4}e^{-3t}\sin 4t\right)u(t)$$

→ (cont'd)



$$\begin{bmatrix} \dot{v}_c(t) \\ \dot{I}_L(t) \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s(t)$$

Laplace Transform

$$\begin{bmatrix} sV_c(s) - V_c(0^-) \\ sI_L(s) - i_L(0^-) \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s(s)$$

$$= s\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} - \begin{bmatrix} V_c(0^-) \\ I_L(0^-) \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} \end{bmatrix} \begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} + \begin{bmatrix} \underline{b} \end{bmatrix} V_s(s)$$

$$(s\underline{\underline{I}} - \underline{\underline{A}})\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} = \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} + \begin{bmatrix} \underline{b} \end{bmatrix} V_s(s)$$

$$\underbrace{\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix}}_{\text{Complete response in s domain}} = \underbrace{(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\begin{bmatrix} V_0 \\ I_0 \end{bmatrix}}_{\text{zero input}} + \underbrace{(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{b}\, V_s(s)}_{\text{zero state}}$$

← general form of solution

- Let's find step response from state eqn system

step response $\longrightarrow$ $V_o = 0$, $I_o = 0$ $\quad V_s(s) = 1/s$

$$\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} = (sI - A)^{-1} \underline{b}\, \underbrace{V_s(s)}_{1/s}$$

$$A = \begin{bmatrix} 0 & 25 \\ -1 & -6 \end{bmatrix}; \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad L = 1 \quad R = 6 \quad C = \frac{4}{100}$$

$$\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} = \frac{1}{s}\begin{bmatrix} s & -25 \\ 1 & s+6 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s\Delta}\begin{bmatrix} s+6 & 25 \\ -1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta = s^2 + 6s + 25$$

$$\begin{bmatrix} V_c(s) \\ I_L(s) \end{bmatrix} = \frac{1}{s(s^2+6s+25)}\begin{bmatrix} 25 \\ s \end{bmatrix}$$

$$V_c^{step}(s) = \frac{25/s}{s^2+6s+25}$$

$$I_L^{step}(s) = \frac{1}{s^2+6s+25}$$

Circuit Components in s Domain

<u>Capacitor</u> $\qquad$ <u>time domain</u> $\qquad$ <u>s domain</u>

$+$ $\quad i_c(t)$

$v_c(t)$

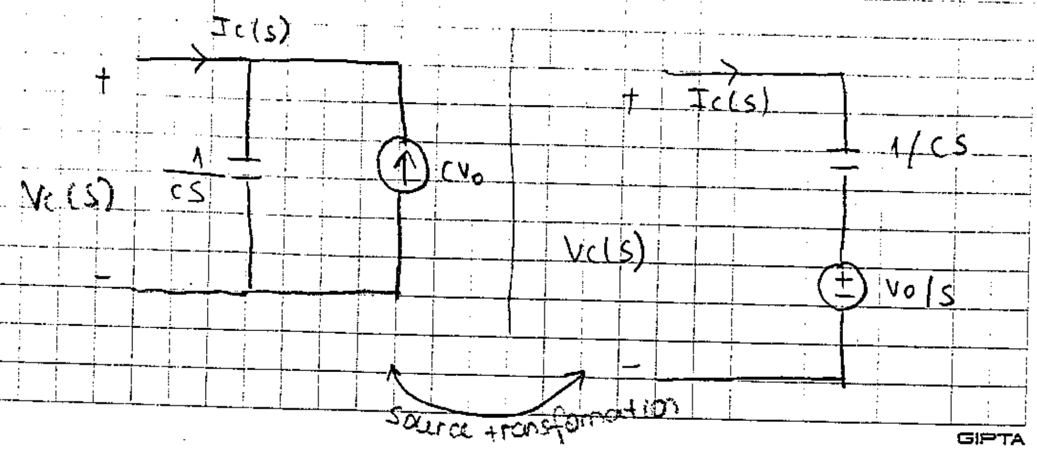
$-$

$$i_c(t) = C\frac{d}{dt}v_c(t) \longrightarrow i_c(s) = C[sV_c(s) - V_c(0^-)]$$

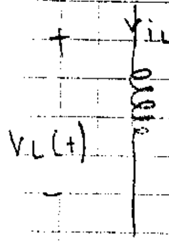
$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_c(\tau)d\tau \longrightarrow V_c(s) = \frac{V_c(0^-)}{s} + \frac{I_c(s)}{Cs}$$

$$t \geq 0$$

$V_c(0^-) = V_o$



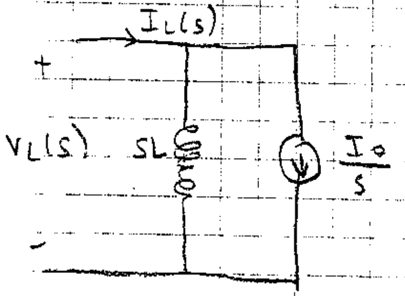
Inductor: Time domain / Laplace Domain



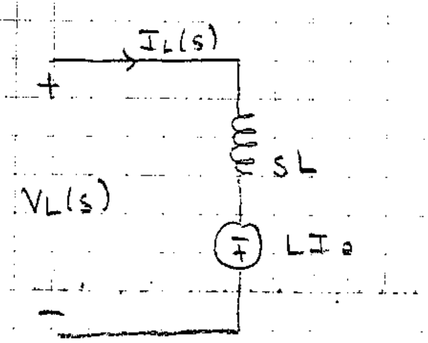
$$V_L(t) = L \frac{d}{dt} i_L(t)$$

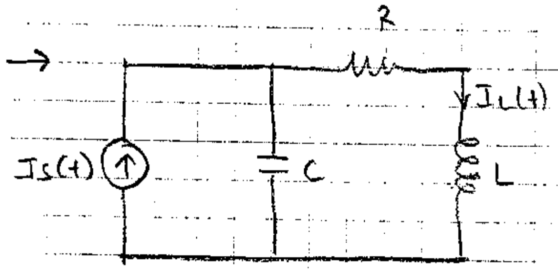
$$i_L(0^-) = I_o$$

$$V(s) = L(sI_L(s) - I_o)$$



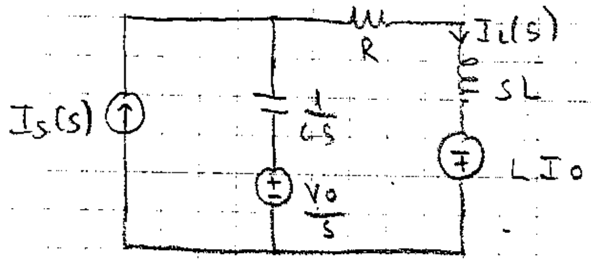
source transformation





$V_c(0^-) = V_o$ Find $I_L(t)$

$I_L(0^-) = I_o$



$$I_L(s) = \underbrace{I_s(s) \frac{1/cs}{1/cs + R + SL}}_{\text{zero state}} + \underbrace{\frac{V_o}{s} \frac{1}{R + SL + 1/cs} + LI_o \frac{1}{R + SL + 1/sC}}_{\text{zero input}}$$

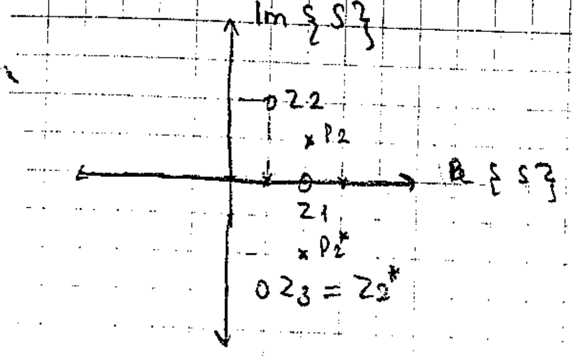
$$V_x(s) = (R + sL)\, I_L(s) - LI_o$$

Note $V_x(s)$ is the branch of the capacitor so it is the cap voltage

Poles and zeros in S domain

$$X(s) = K \frac{\prod_{i=1}^{z} (s - z_i)}{\prod_{i=1}^{p} (s - p_i)} \qquad X(z_i) = 0$$

$$X(p_i) \to \infty$$



→ pole-zero diagram

$\boxed{K=10}$

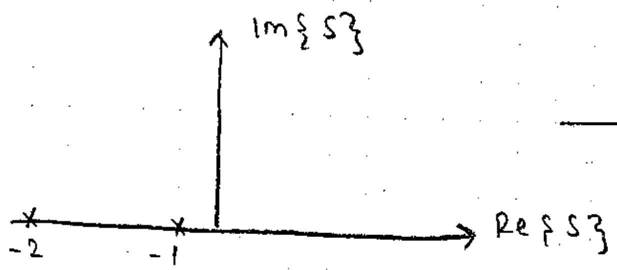
$$P(s)=s^2+3s+2=0$$

$$P(z_1)=0 \rightarrow P(z_1)^*=0^*=0$$

$$(z_1^2+3z_1+2)^*=0$$

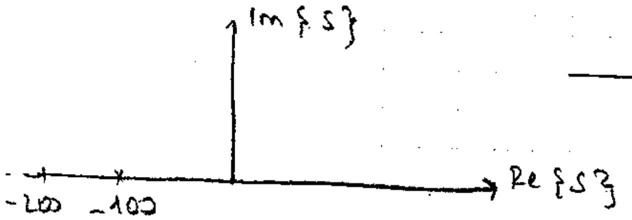
$$P(z_1^*)=0 \longleftarrow (z_1^*)^2+3(z_1)^*+2=0$$

Interpretation of Pole-Zero diagrams

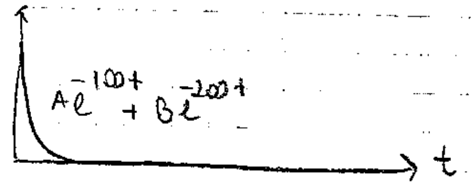


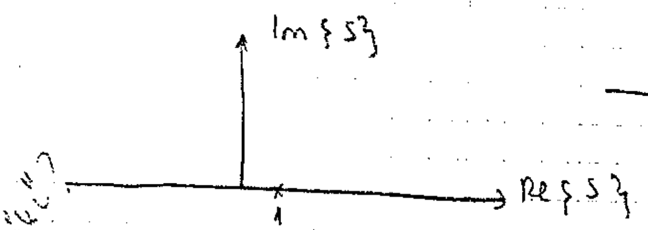
$$\xrightarrow{L^{-1}\{\}} x(t)=(Ae^{-t}+Be^{-2t})u(t)$$

$$X(S)=K\frac{1}{(S+1)(S+2)}$$

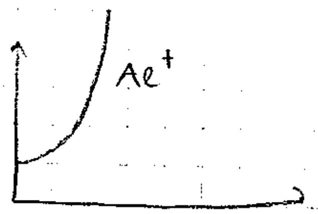


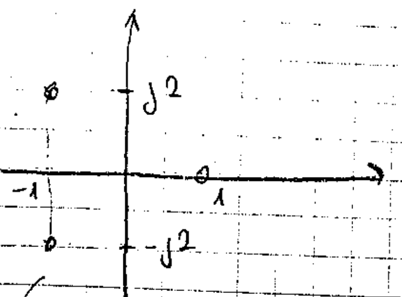
$\longrightarrow$





$\longrightarrow$



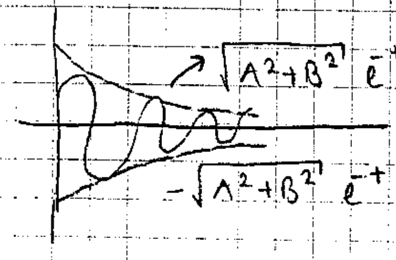


$$X(s)=K\frac{1}{[s-(-1+j2)][s-(-1-j2)]}$$

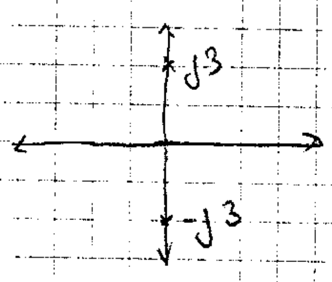
$$=\frac{K}{(s+1)^2+4}$$

$$x(t)=Ae^{-t}\cos 2t+Be^{-t}\sin 2t$$

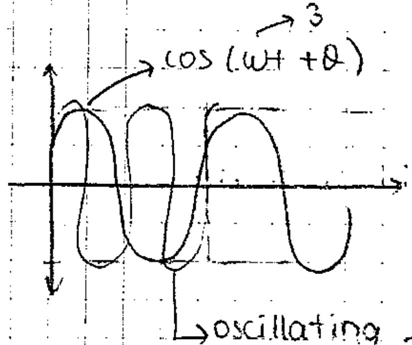
$1, \; -1+2j$



$$x(t) = \sqrt{A^2+B^2}\; e^{-t}\cos\left(2t - \tan^{-1}\frac{B}{A}\right)$$

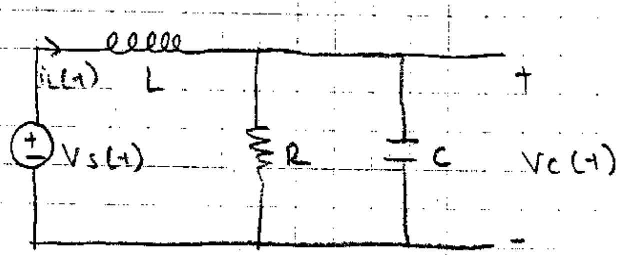


$\xrightarrow{\mathcal{L}^{-1}\{\,\}}$



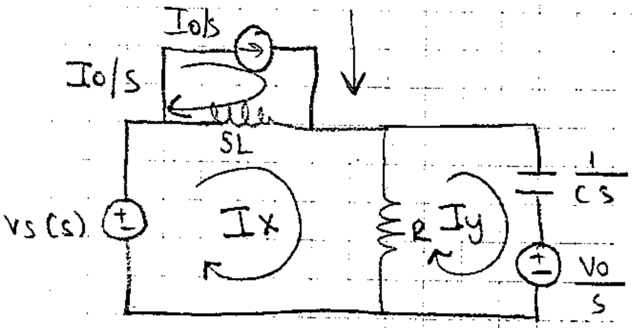
$$X(s) = k\,\frac{1}{s^2+9}$$

→ Mesh analysis in S-domain



$i_L(0^-) = I_0$

$V_c(0^-) = V_0$



$$\begin{bmatrix} sL+R & -R \\ -R & R+\frac{1}{sC} \end{bmatrix}\begin{bmatrix} I_x(s) \\ I_y(s) \end{bmatrix} = \begin{bmatrix} I_0L + V_s(s) \\ -\frac{V_0}{s} \end{bmatrix}$$

$$\underbrace{I_x(s)}_{\text{complete response}} = \underbrace{\frac{\left(\frac{1}{RCL} + \frac{s}{L}\right)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\,V_s(s)}_{\text{zero-state}} + \underbrace{\frac{\frac{I_0}{RC} - \frac{V_0}{L} + sI_0}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}}_{\text{zero-input}}$$

Note: $I_x(s) = I_L(s)$ of charged inductor

$I_x(s)$: with initial enugy

zero state solution; $I_L(s) = \dfrac{\frac{1}{RCL} + \frac{s}{L}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} V_s(s)$

$$\Longrightarrow \left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right) I_L(s) = \left(\frac{1}{RCL} + \frac{s}{L}\right) V_s(s)$$

$$\rightarrow L^{-1}\{\ \} \quad \left(D^2 + 1/RC\,D + 1/LC\right) I_L(t) = \frac{1}{RCL} V_s(t) + \frac{1}{L} D V_s(t)$$

Powerla ilgili hatırlatma

$V_s(t) = 2\cos(\omega t + 45)$ V $\longrightarrow 2\angle 45 \longrightarrow \sqrt{2} + j\sqrt{2}$ V

$V_s(t) = \frac{2}{\sqrt{2}}\cos(\omega t + 45)$ Vrms $\longrightarrow \frac{2}{\sqrt{2}}\angle 45$

$$S = \frac{1}{2} V I^* = V_{RMS} I^*_{RMS} = V_{RMS} I^*_{RMS}$$

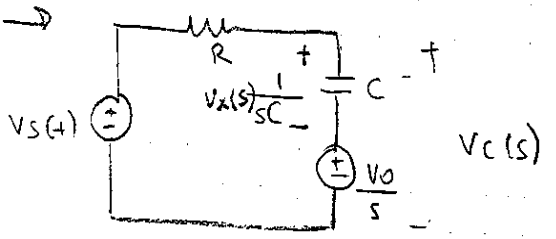
Initial Value, Final Value Theorem

Initial Value; $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$ $\quad s\to\infty$ s should be in ROC

Final Value; $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$ $\quad s = 0$ s should be ROC

$\hookrightarrow$ provided that this limit exists.

$\rightarrow$



$$V_c(s) = \frac{V_0}{s} + V_x(s) = \frac{V_0}{s} + \left(V_s(s) - \frac{V_0}{s}\right)\frac{1}{sCR + 1} \quad \nearrow 1/sC \quad \searrow R + 1/sC$$

$$V_c(s) = \underbrace{\frac{V_0}{s}\left(1 - \frac{1}{1+sRC}\right)}_{} + \underbrace{\frac{1}{1+sRC} V_s(s)}_{V_c^{zs}}$$

$$V_c^{zi}(s) = \frac{V_0}{s}\left(1 - \frac{1}{1+sRC}\right) \quad ; \quad V_c^{zs}(s) = \frac{1}{1+sRC} V_s(s)$$

$$\lim_{s\to\infty} sV_c^{zi} = V_0 \Longleftarrow V_c^{zi}(0^+)$$

$$V_c^{zs}(t) = \frac{1}{1+sRC} V_s(s) \quad ; \quad V_s(t) = u(t) \longrightarrow V_s(s) = \frac{1}{s}$$

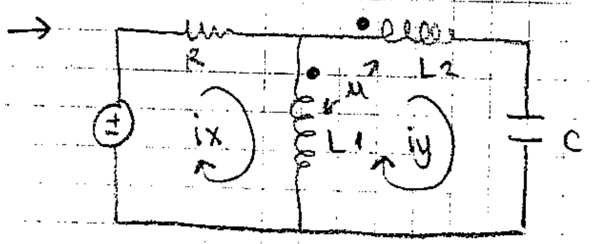
$$\lim_{s\to\infty} V_c^{zi}(s) = V_0 \Longleftarrow V_c^{zi}(0^+)$$

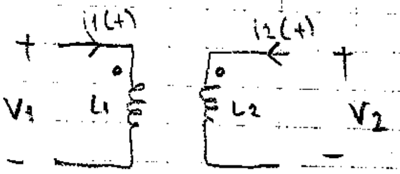
GIPTA

$$\underbrace{\mathcal{L}\{e^{-t}\}}_{f(t)} = \underbrace{\frac{1}{s+1}}_{F(s)} \longrightarrow f(0^+) = 1 \qquad \longrightarrow \lim_{s\to\infty} sF(s) = 1$$

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1} \longrightarrow f(0^+) = 1 \qquad \lim_{s\to\infty} sF(s) = 1$$

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1} \longrightarrow f(\infty) = ? \qquad \lim sF(s) = 0$$





$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} d/dt\ i_1(t) \\ d/dt\ i_2(t) \end{bmatrix}$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} sI_1(s) - I_1(0^-) \\ sI_2(s) - i_2(0^-) \end{bmatrix}$$

$I_1(s) \rightarrow ix - iy$ s domain

$I_2(s) \rightarrow iy(s)$

KVL ix; $-V_s(s) + R\,ix(s) + V_1(s) = 0$

iy; $\frac{1}{sC}\, iy(s) - V_1(s) + V_2(s) = 0$

Network Functions:

Linear time invariant

$x(t) \longrightarrow$ LTI $\longrightarrow y(t)$

$$H(s) = \frac{\mathcal{L}\{y^{zs}(t)\}}{\mathcal{L}\{x(t)\}}$$

$zs \rightarrow$ zerostate

Network function

Transfer function

$\longrightarrow y^{zs}(t) \xrightarrow[H(s)\text{ given}]{} y^{zs}(t) = \mathcal{L}^{-1}\{H(s)X(s)\}$

$X(s) \leftarrow \mathcal{L}\{x(t)\} \leftarrow$ input

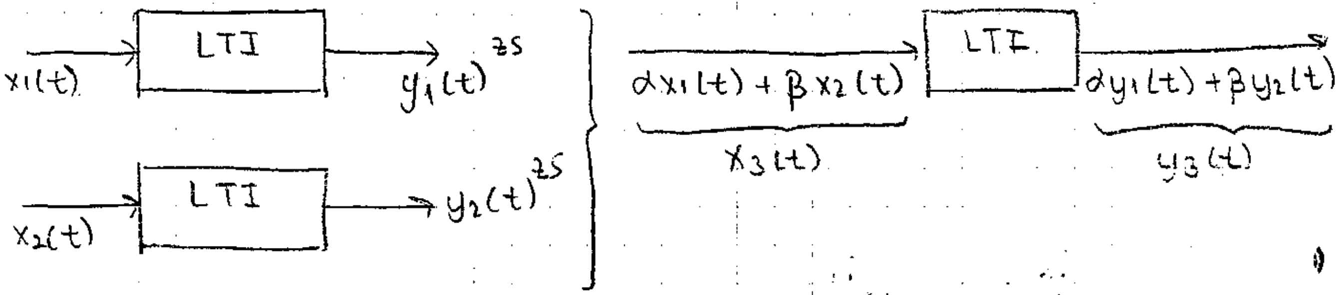
* zero state responses

$$H(s) = \frac{I_L^{zs}(s)}{V_s(s)} = \frac{s}{s+1} \text{ is given}$$

$$(s+1) I_L^{zs}(s) = s V_s(s)$$

$$\Big\downarrow \mathcal{L}^{-1}\{\ \}$$

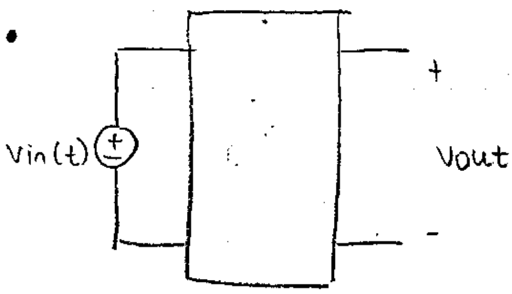
$$\frac{d}{dt} I_L^{zs}(t) + I_L^{zs}(t) = \frac{d}{dt} V_s(t) \longrightarrow \text{Note that : ICs are zero.}$$



$$Y_3^{zs}(s) = \underbrace{H(s)}_{\text{proportionality "constant"}} X_3(s)$$

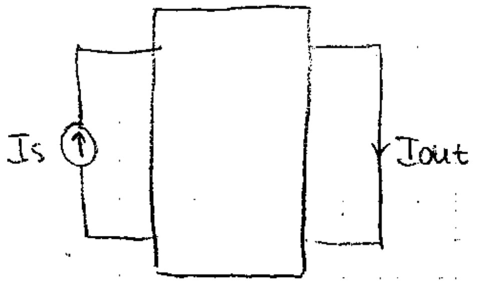
$$= H(s) \left[\alpha X_1(s) + \beta X_2(s) \right] = \alpha Y_1^{zs}(s) + \beta Y_2^{zs}(s)$$

"Superposition is only valid in zero state responses"

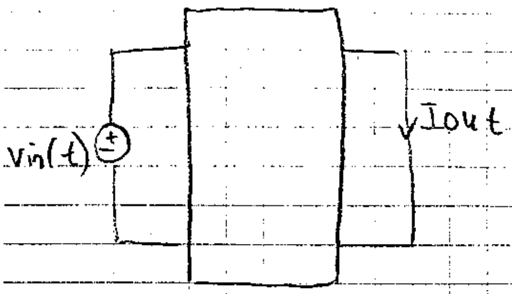
- 

$$H_v(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

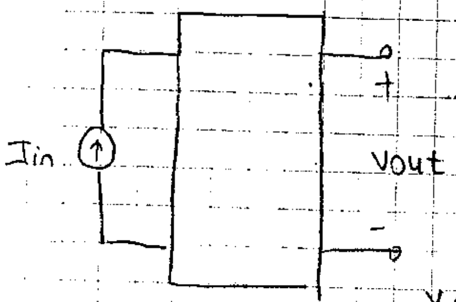
voltage transfer function



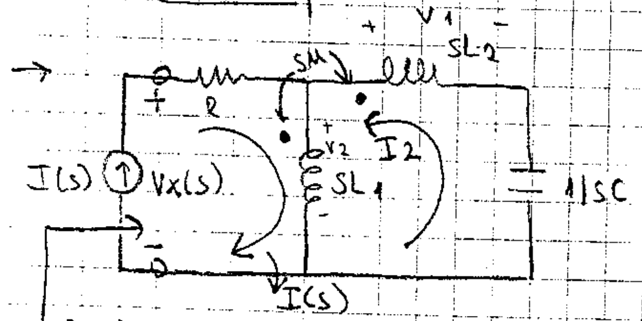
$$\frac{I_{out}(s)}{I_{in}(s)}$$ current transfer function



$$\frac{I_{out}(s)}{V_{in}(s)}$$: transfer admittance



$\frac{V_{out}(s)}{I_{in}(s)}$: transfer impedance



$Z(s)$: transfer impedance (All initial conditions are zero) demek

$$Z(s) = \frac{V_x(s)}{I_{in}(s)}$$

KVL around I_2 loop;

$$I_2 \frac{1}{sC} - V_1(s) + V_2(s) = 0$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix} \begin{bmatrix} -I_2(s) \\ I_2(s) + I_{in}(s) \end{bmatrix}$$

$V_2(s) - V_1(s) =$

$-sM I_2(s) + sL_2 [I_2(s) + I_{in}(s)] + sL_1 I_2(s) - sM [I_2(s) + I_{in}(s)]$

$= s[I_2(s) + I_{in}(s)](L_2 - M) + I_2(s)\, s\, (L_1 - M)$

$= I_2(s) [s(L_1 + L_2 - 2M)] + I_{in}(s) \cdot s(L_2 - M)$

Putting KVL equation;

$$I_2(s)\left[\frac{1}{sC} + s(L_1 + L_2 - 2M)\right] + I_{in}(s) \cdot s(L_2 - M) = 0$$

$$\boxed{I_2(s) = \frac{s(M - L_2) I_{in}(s)}{\frac{1}{sC} + s(L_1 + L_2 - M)}}$$

$\frac{s(L_2 - M) I_2}{I_{in}}$ $s(M - L^{?})$

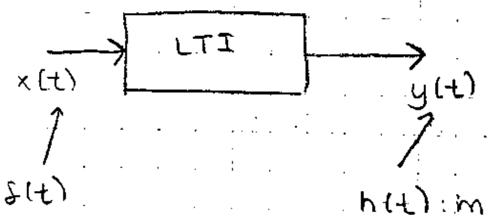
$V_x(s) = I_{in}(s) R + V_2(s)$

$= I_{in}(s) R + s I_2(s)(L_2 - M) + sL_2 I_{in}(s)$

GIPTA

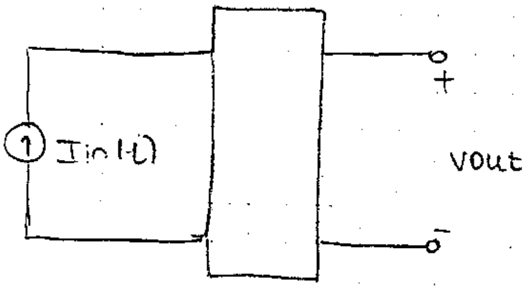
$$\frac{V_x(s)}{I_{in}(s)} = R + sL_2 - \frac{s^2 (L_2 - M)^2}{1/sC + s(L_1 + L_2 - 2M)}$$

Network Function and Impulse Response



$$\frac{Y^{zs}(s)}{X(s)} = H(s) = \frac{\mathcal{L}\{y^{zs}(t)\}}{\mathcal{L}\{\underbrace{x(t)}_{\delta(t)}\}} = \frac{\mathcal{L}\{h(t)\}}{1}$$

• $h(t) = \mathcal{L}^{-1}\{H(s)\}$



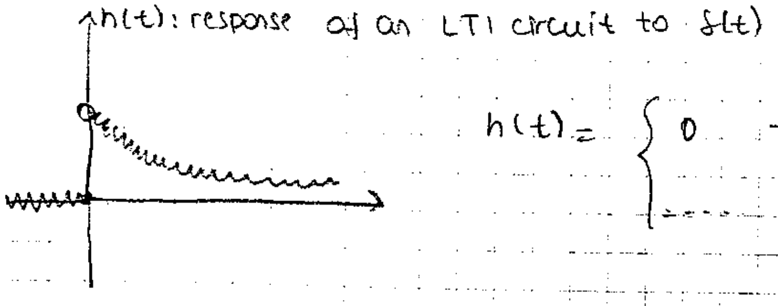
$$V_{out}(s) = H(s) \cdot I_{in}(s) \xrightarrow{\mathcal{L}^{-1}\{\}}$$

$$\underbrace{V_{out}(t) = \int_{-\infty}^{\infty} h(\tau) I_{in}(t-\tau)\, d\tau}$$

convolution operator
convolution integral

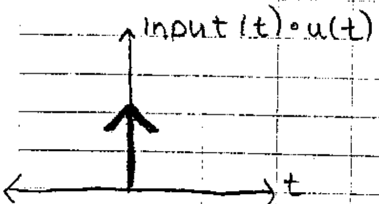
$h(t) * I_{in}(t)$ ⟶ convolution operator

About convolution Integral;



$$h(t) = \begin{cases} 0 & t<0 \\ \dots & t>0 \end{cases}$$

Systems with $h(t) = 0$ for $t>0$ are called causal systems.



output at $0^- = 0$

GIPTA

$$V_{out}(t) = \left(\int_{-\infty}^{0} + \int_{0}^{t} + \int_{t}^{\infty} \right)$$

$$\int_{-\infty}^{0} \cancel{h(\tau)}\, I_{in}(t-\tau)\, d\tau = 0$$

0 (causality)

$$\int_{t}^{\infty} h(\tau) \boxed{I_{in}(t-\tau)}\, d\tau =$$

I_{in} (negative argument) or zero

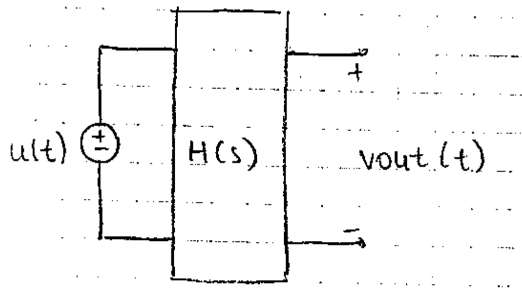
$I_{in}(t) = f(t)\,u(t)$

$$\int_{t}^{\infty} h(\tau)\, I_{in}(t-\tau)\, u(\cancel{t-\tau})\, d\tau = 0$$

0

$$\boxed{V_{out}(t) = \int_{0}^{t} h(\tau)\, I_{in}(t-\tau)\, d\tau}$$ for causal systems

Step Response:

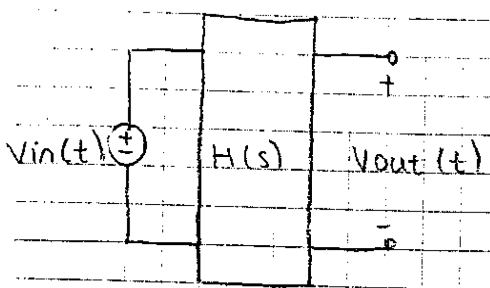


$$V_{out}^{step}(s) = V_{out}(s) = H(s)\,\frac{1}{s}$$

$$V_{out}^{step}(t) = \int_{-\infty}^{t} h(\tau)\, d\tau = \int_{0}^{t} h(\tau)\, d\tau$$

$V_{out}^{step}(t)$ (convolution) $= \int_{0}^{t} h(\tau) \underbrace{V_{in}(t-\tau)}_{u(t-\tau)=1} d\tau = \int_{0}^{t} h(\tau)\, d\tau$

Sinousidal Steady-State Response and H(s):



$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} \longrightarrow V_{out}(s) = H(s)\, I_{in}(s)$$

$V_{out} = ?$

(I am only interested in the steady state part of $V_{out}^{ss}(t)$)

$V_{in}(t) = A\cos(\omega t + \phi)$

$$L\{v_{in}(t)\} = L\left\{ A(\cos\omega t \cos\phi - \sin\omega t \sin\phi) \right\}$$

$$= A \, \frac{s\cos\phi - \omega\sin\phi}{s^2+\omega^2}$$

$$V_{out}(s) = H(s)\, A \, \frac{s\cos\phi - \omega\sin\phi}{s^2+\omega^2}$$

$$= \underbrace{\left(\frac{K}{s-j\omega} + \frac{K^*}{s+j\omega} \right)}_{\text{steady state response due to cosine}} + \left(\frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2} + \cdots \frac{A_n}{s-\lambda_n} \right)$$

(natural frequency of poles of $H(s)$

$Re\{\lambda\} < 0$

$$K = H(s)\, A \, \frac{s\cos\phi - \omega\sin\phi}{(s^2+\omega^2)\,(s+j\omega)} \, \cancel{(s-j\omega)} \Bigg|_{s=j\omega}$$

$$K = H(j\omega) \, \frac{j\cancel{\omega}\cos\phi - \cancel{\omega}\sin\phi}{2j\cancel{\omega}} \, A$$

$$K = H(j\omega)\, \frac{A}{2} \left(\cos\phi + j\sin\phi \right) = H(j\omega)\, \frac{A}{2}\, e^{j\phi}$$

$$V_{out}^{ss}(t) = L^{-1}\left\{ \frac{K}{s-j\omega} + \frac{K^*}{s+j\omega} \right\}$$

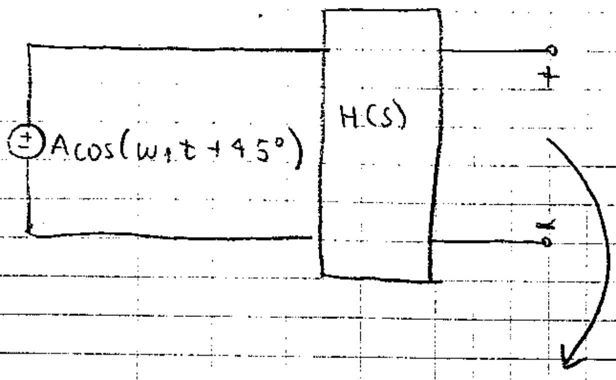
$$= K e^{j\omega t} + K^* e^{-j\omega t}$$

$$= 2\, Re\left\{ K e^{j\omega t} \right\}$$

$$= 2\, Re\left\{ |K|\, e^{j\angle K}\, e^{j\omega t} \right\} = \cancel{2}\, Re\left\{ \|H(j\omega)\| \frac{A}{\cancel{2}}\, \underbrace{e^{j(\omega t + \angle H(j\omega) + \phi)}}_{e^{j\omega t}\, e^{j\angle K}} \right\}$$

$$V_{out}^{ss}(t) = Re\left\{ \|H(j\omega)\|\, A\, e^{j(\omega t + \angle H(j\omega) + \phi)} \right\}$$

If I have;

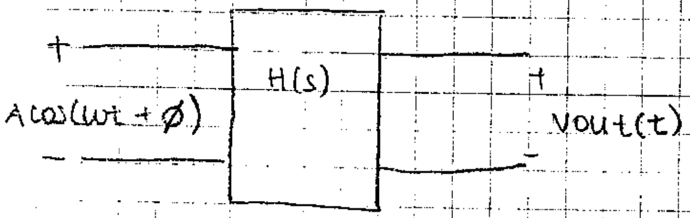


$$H(s) = \frac{1}{s+1} \Bigg|_{H(j\omega)}$$

$$= \frac{1}{j\omega_1 + 1}$$

$$A\|H(j\omega)\| \cos(\omega_1 t + 45 + \angle H(j\omega))$$

Frequency Response:



$$V_{out} = \underbrace{A\,|H(j\omega)|}_{\text{amplitude of the output}} \cos(\omega t + \phi + \underbrace{\measuredangle H(j\omega)}_{\text{phase of the output}})$$

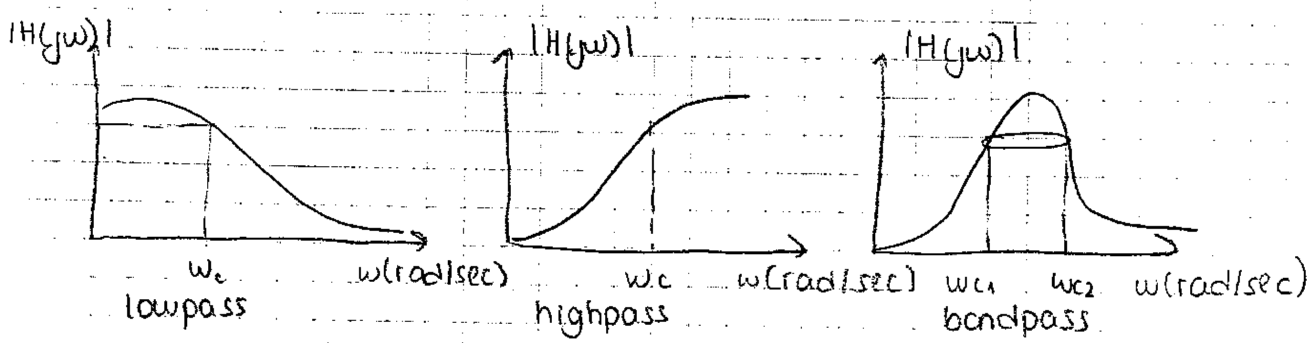
Gain $= |H(j\omega)|$

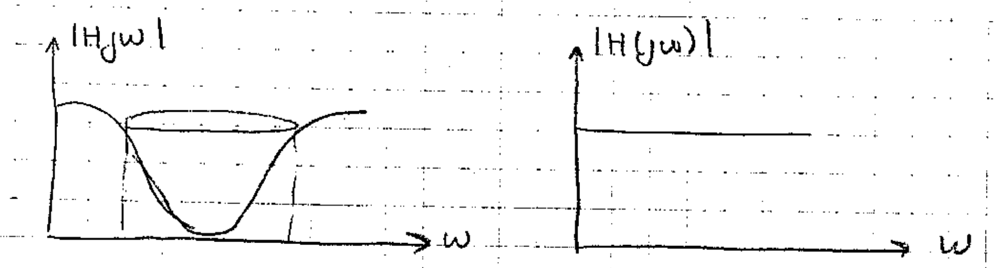
↳ amplitude gain/loss

$\measuredangle H(j\omega)$ = phase response

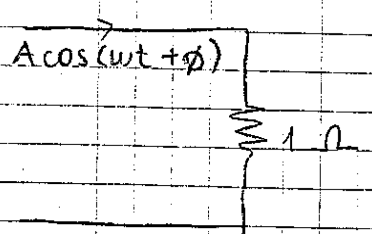
Types of Filters:

$|H(j\omega)|$: magnitude response

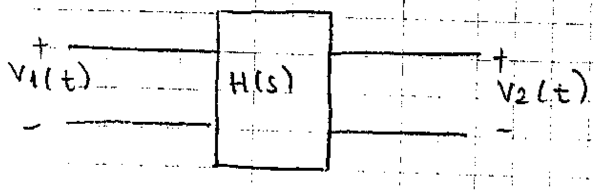




Decibel:



$$P^{av}_{1\Omega} = \frac{A^2}{2} R \rightarrow 1\,\Omega$$



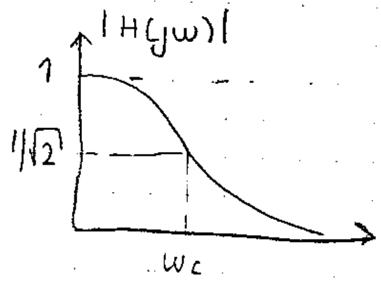
Power at the input and output is at interest

Decibel: $10\log_{10} \frac{(\text{Power at the input})}{(\text{Power at the output})} = 10\log|H(j\omega)|^2$

can be written as $20 \log |H(j\omega)|$

Decibel is a relation between powers at input and output

(Decibel is about power)

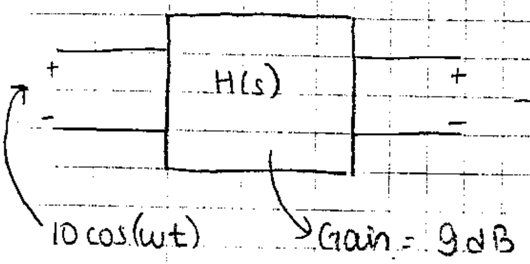


Lowpass critical frequency, half power freq, cut off frequency
3dB frequency

$\|H(j\omega)\|^2$	$10\log_{10}\|H(j\omega)\|^2$
1	0
2	3 dB
3	4,77 dB
4	6 dB
5	7 dB
6	7,77
7	8,45
8	9 dB
9	9,54
10	10

$10\log 2 = 3$

$10\log(1/2) = -3$ dB



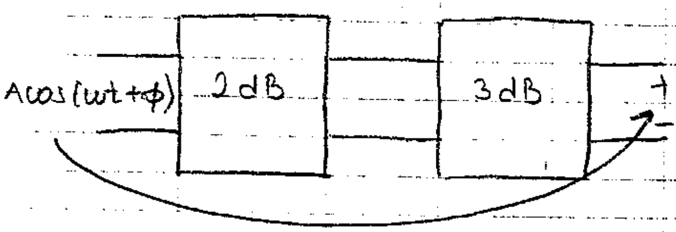
$\longrightarrow 10\,|H(j\omega)|\cos(\omega t + \phi + \measuredangle H(j\omega))$

$P_{out} = \dfrac{10^2\,|H(j\omega)|^2}{2R}$

$|H(j\omega)|^2 = 5\text{dB} = 3.5 \qquad |H(j\omega)| = \sqrt{3.5}$

$P_{in} = \dfrac{10^2}{2R} = 50 \text{ watts} = 17 \text{ dB}$

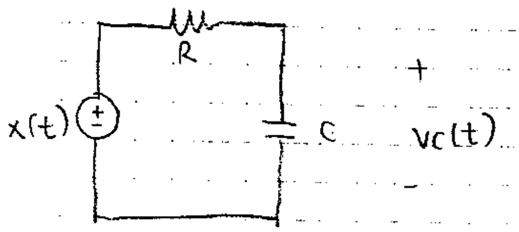
$(P_{out})_{dB} = (P_{in})_{dB} + (\text{gain})_{dB}$



Output power increase by 5dB or (3.5 fold)

Frequency Response of 1st Order Systems

a.) Lowpass



$\longrightarrow H(s) = \dfrac{K}{s+\alpha}$

$H(s) = \dfrac{1/sC}{1/sC + R} = \dfrac{1}{1+sCR} = \dfrac{1/RC}{s + 1/RC}$

where $K = 1/RC$ and $\alpha = 1/RC$

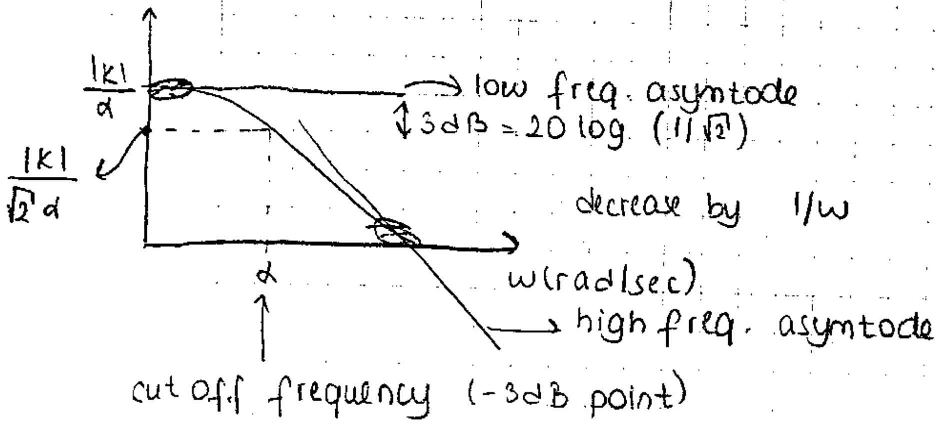
$|H(j\omega)| = ? \longrightarrow |H(j\omega)| = \dfrac{|K|}{\sqrt{\alpha^2+\omega^2}}$

$\measuredangle H(j\omega) \longrightarrow \measuredangle K - \tan^{-1}(\omega/\alpha)$

when $K < 0 \longrightarrow \measuredangle K = 180$

$K > 0 \longrightarrow \measuredangle K = 0$

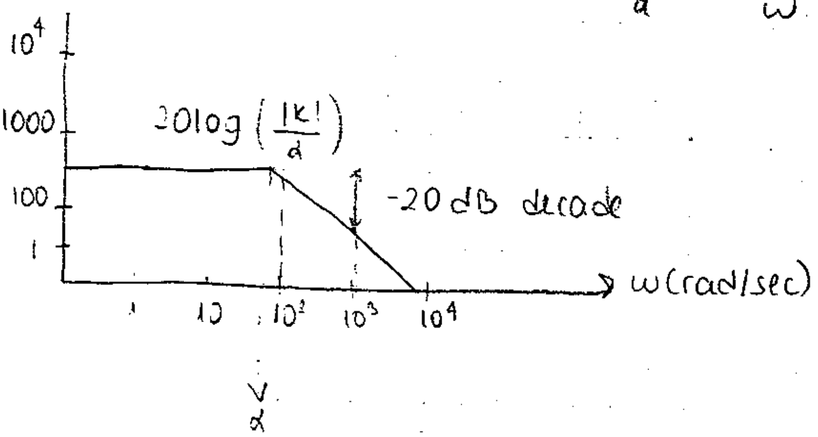
$$|H(j\omega)| = \frac{|K|}{\omega\sqrt{\left(\frac{\alpha}{\omega}\right)^2+1}} = \frac{|K|}{\alpha\sqrt{1+\left(\frac{\omega}{\alpha}\right)^2}}$$



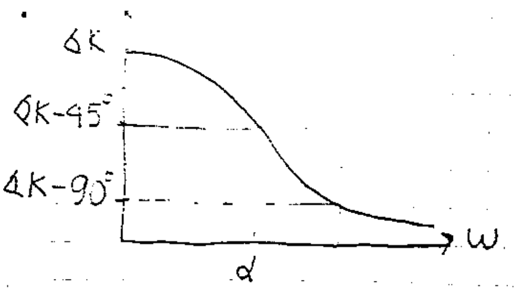
i.) $\frac{\omega}{\alpha} \ll 1 \longrightarrow (\omega \ll \alpha)$

ii.) $\frac{\alpha}{\omega} \ll 1 \longrightarrow |H(j\omega)| \simeq \frac{|K|}{\omega}$

Two asymptodes meet at $\frac{|K|}{\alpha} = \frac{|K|}{\omega} \longrightarrow \omega = \alpha$



$\measuredangle H(j\omega) \Rightarrow \quad \measuredangle K - \tan^{-1} \omega/\alpha$



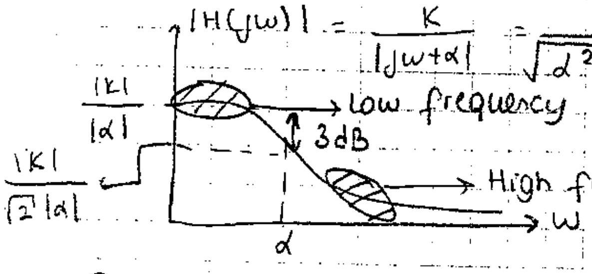
$K > 0 \quad \measuredangle K = 0$

$K < 0 \quad \measuredangle K = \mp 180°$

Review: $H(s) = \frac{K}{s+\alpha}$

$A\cos(\omega t) \rightarrow \boxed{H(s)} \rightarrow A|H(j\omega)|\cos(\omega t + \measuredangle H(j\omega))$

$|H(j\omega)| = \frac{K}{|j\omega+\alpha|} = \frac{|K|}{\sqrt{\alpha^2+\omega^2}}$



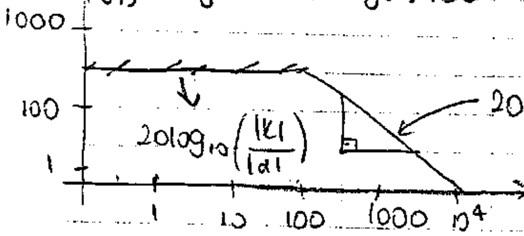
High frequency $\rightarrow \omega \gg \alpha \rightarrow |H(j\omega)| \cong \frac{K}{\omega}$

Two asymptotes meet at $\frac{|K|}{|\alpha|} = \frac{|K|}{\omega} \longrightarrow \omega = \alpha$

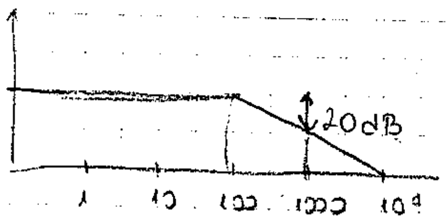
$\updownarrow -3dB = 20\log_{10}(1/\sqrt{2})$

$20\log_{10}|H(j\omega)| = |H(j\omega)|_{dB}$

Matlab notation (loglog notation)



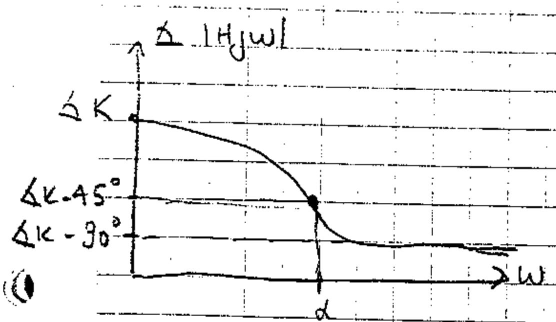
ω (rad/sec) $\rightarrow$ also logaritmic



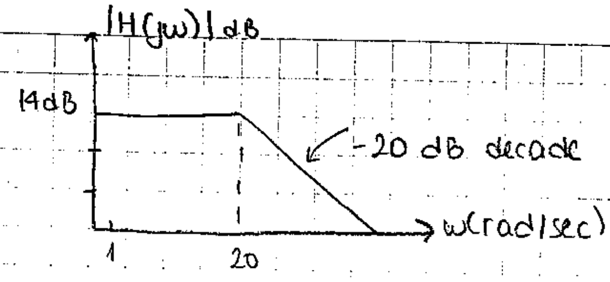
let's cover $\measuredangle|H(j\omega)| = ? \longrightarrow \measuredangle K - \tan^{-1}\left(\frac{\alpha}{j\omega}\right)$ $\quad \omega \rightarrow \infty \quad \tan^{-1}\frac{\alpha}{j\omega} = \pi/2$

$K > 0 \longrightarrow \measuredangle K = 0$ $\quad \omega \rightarrow 0 \quad \tan^{-1}\frac{\alpha}{j\omega} = 0$

$K < 0 \longrightarrow \measuredangle K = \mp 180°$



Example: $H(s) = \frac{100}{s+20}$



$$H(j\omega) = \frac{100}{j\omega + 20}$$

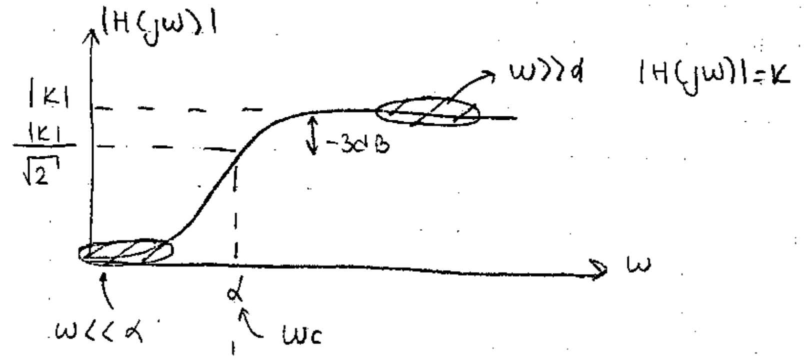
$$|H(j\omega)| = \frac{100}{\sqrt{20^2 + \omega^2}}$$

Highpass Filters:

$$H(s) = \frac{Ks}{s+\alpha} \qquad H(j\omega) = \frac{Kj\omega}{j\omega + \alpha}$$

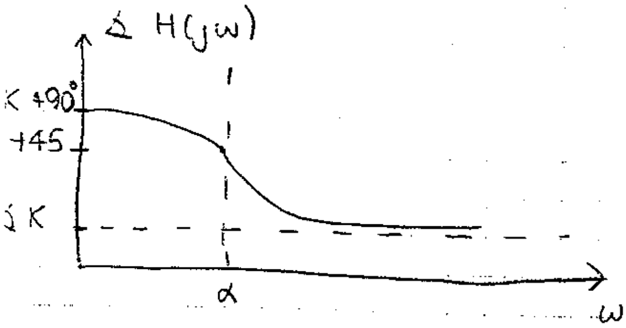
$$\lim_{\omega \to \infty} H(j\omega) = K$$

$$\lim_{\omega \to 0} H(j\omega) = 0$$

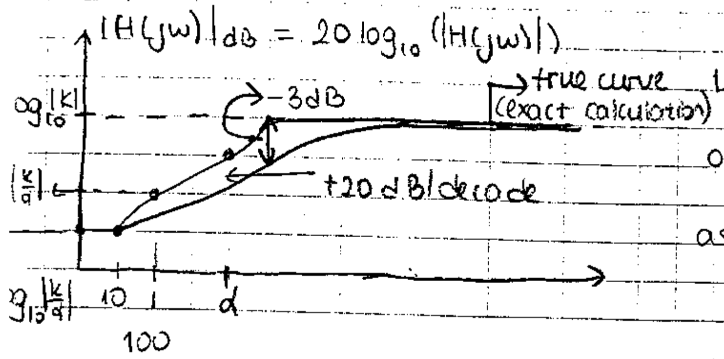


$$H(j\omega) \cong \frac{K}{\alpha} j\omega \longrightarrow |H(j\omega)| = \left|\frac{K}{\alpha}\right| \omega$$

$$\measuredangle H(j\omega) = \measuredangle K + 90^\circ - \tan^{-1}(\omega/\alpha)$$



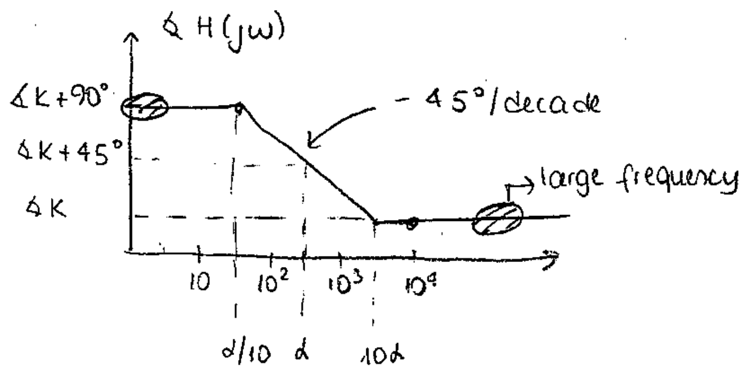
$$|H(j\omega)|_{dB} = 20\log_{10}(|H(j\omega)|)$$



Let's check the intersection point of low frequency and high-frequency assymptotes.

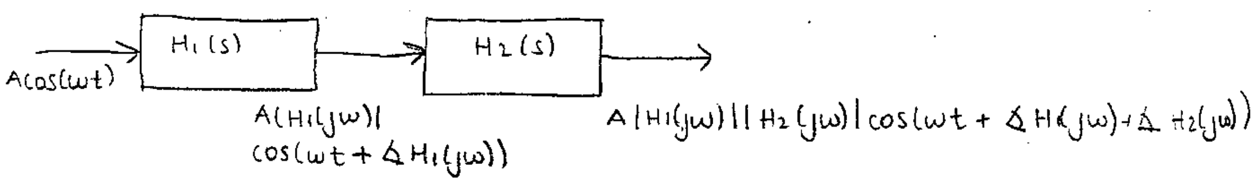
$$20\log_{10}|K| = 20\log_{10}\left(\frac{|K|}{\alpha}\omega\right)$$

high frequency   low frequency



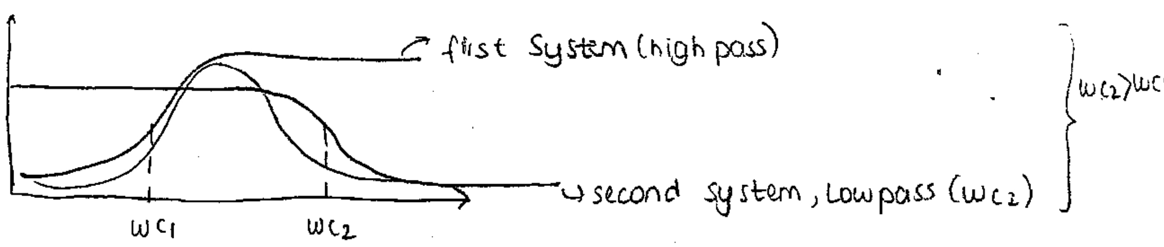
Bandpass - Bandstop Filters using 1st Order Circuits

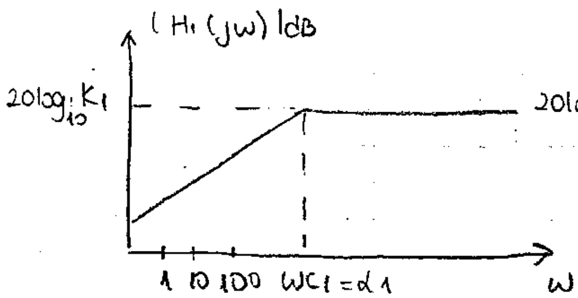
Bandpass system:

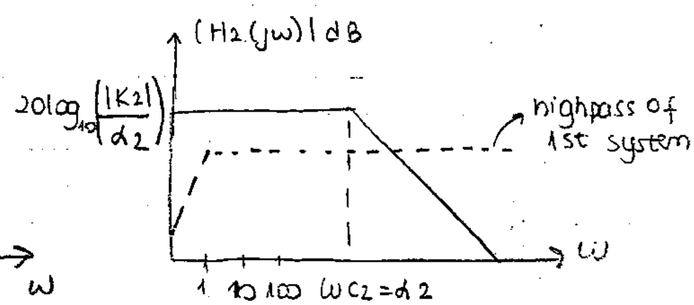


We note that first system does not have a loading effect on 2nd System

$H(s) = H_1(s)H_2(s)$



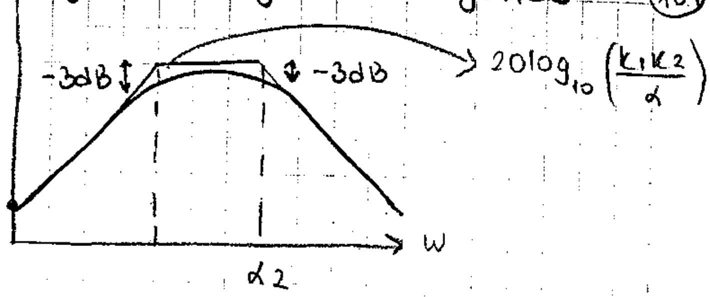




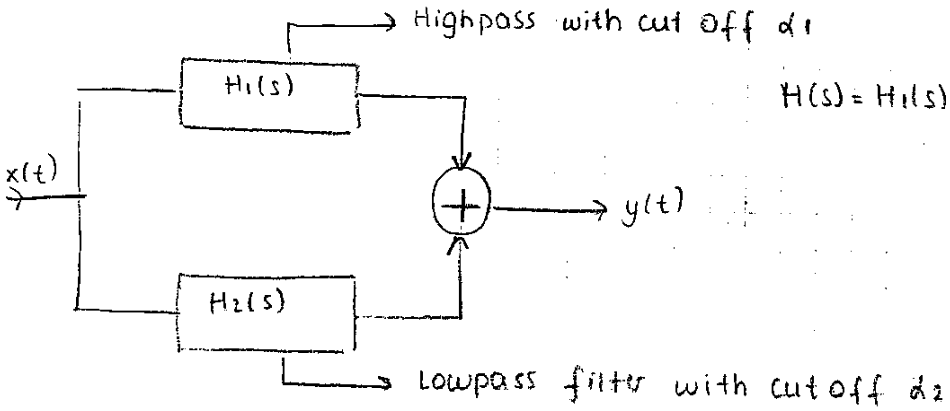
$H_1(s) = \frac{K_1 s}{\ldots}$] highpass $H_2(s) = \frac{K_2}{\ldots}$] lowpass

$$H(s) = \frac{k_1 k_2 \, s}{(s+\alpha_1)(s+\alpha_2)} \longrightarrow$$

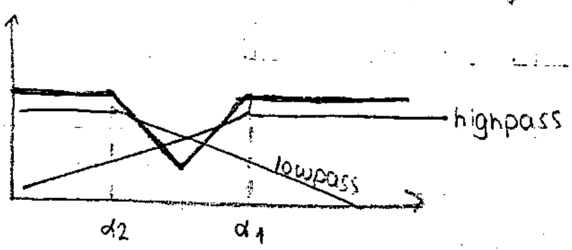
$$|H(j\omega)|_{dB} = |H_1(j\omega)|_{dB} + |H_2(j\omega)|_{dB}$$



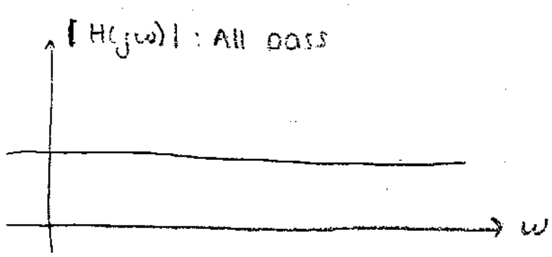
Bandstop Filter:



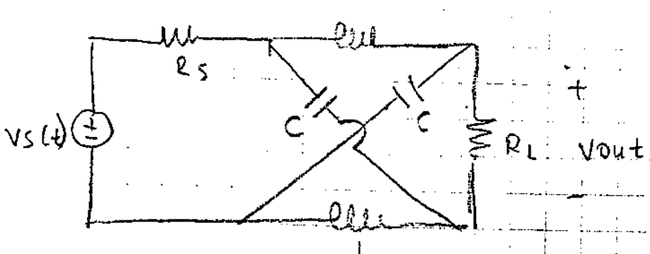
$H(s) = H_1(s) + H_2(s)$



All pass filter but $\alpha_2 > \alpha_1$



Example: All pass constructed from 1st Order Filters



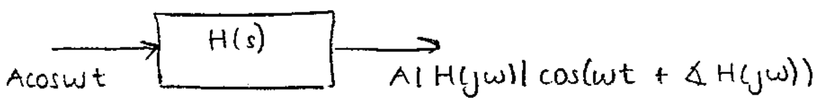
$$\frac{L}{C} = R_L^2 \longrightarrow H(j\omega) = \frac{R_L}{R_L+R_s} \cdot \frac{1-j\omega C R_L}{1+j\omega C R_L}$$

$$|H(j\omega)| = \frac{R_L}{R_L+R_s}$$

Second Order System

1 Bandpass Systems:

$$H(s) = \frac{Ks}{s^2 + 2\gamma\omega_0 s + \omega_0^2}$$



Pole locations: $s^2 + 2\gamma\omega_0 s + \omega_0^2 = 0 \longrightarrow (s+\gamma\omega_0)^2 = \omega_0^2(\gamma^2 - 1)$

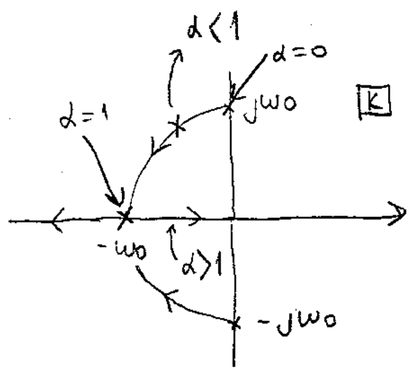
$$s_{1,2} = \left\{ -\gamma\omega_0 \mp \omega_0\sqrt{\gamma^2-1} \right\}$$

(A) $\gamma = 1 \longrightarrow s_{1,2} = \{-\omega_0\} \longrightarrow$ Double root at $s = -\omega_0 \rightarrow$ critically damped

(B) $\gamma > 1 \longrightarrow s_{1,2} \longrightarrow$ 2 distinct real roots $\rightarrow$ overdamped

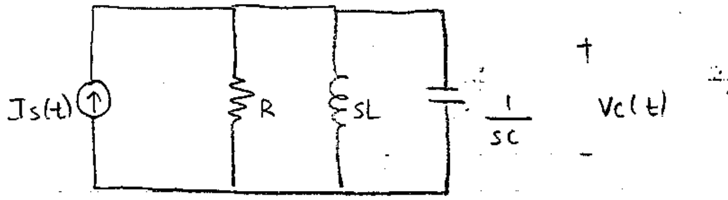
(C) $\gamma < 1 \longrightarrow s_{1,2} \longrightarrow$ complex conjugate roots $\rightarrow$ underdamped

$$s_{1,2} = \left\{ -\gamma\omega_0 \mp j\omega_0\sqrt{1-\gamma^2} \right\}$$



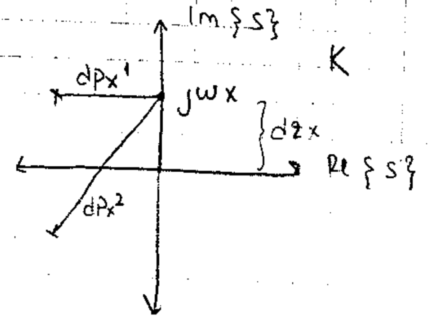
Pole - zero diagram for H(s)

• Damping in electrical circuits is related to how long is "R" in the circuit

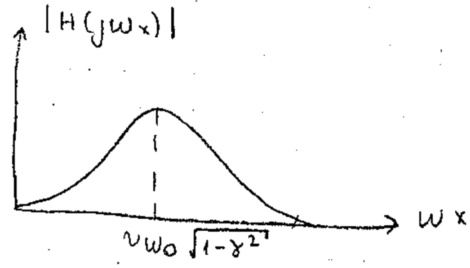


$$\frac{V_c(s)}{I_s(s)} = H(s) = \frac{1}{1/R + 1/sL + sC} = \frac{sLR}{sL + R + s^2 RCL} = \frac{s/C}{s^2 + \frac{1}{LC} + \frac{s}{RC}}$$

$$H(s) = \frac{Ks}{(s-p_1)(s-p_2)} \qquad |H(j\omega)| = \frac{|K|\,|j\omega|}{|j\omega - p_1|\,|j\omega - p_2|}$$



$K \qquad |H(j\omega_x)| = \dfrac{|K|\, d_{zx}}{d_{Px}^{(1)}\, d_{Px}^{(2)}}$



Algebraic Approach for $|H(j\omega)|$ and $\measuredangle H(j\omega)$

$$H(s) = \frac{Ks}{s^2 + 2\gamma\omega_0 s + \omega_0^2}$$

$$H(j\omega) = \frac{Kj\omega}{-\omega^2 + \omega_0^2 + j2\gamma\omega_0\omega}$$ divide by $j\omega$ both sides

$$= \frac{K}{j\dfrac{(\omega^2 - \omega_0^2)}{\omega} + 2\gamma\omega_0} = \frac{K}{\omega_0\left(2\gamma + j\left[\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right]\right)}$$

$$|H(j\omega)| = \frac{|K|}{\omega_0\left[(2\gamma)^2 + \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2\right]^{1/2}}$$

$$\measuredangle H(j\omega) = \measuredangle K - \tan^{-1}\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot \frac{1}{2\gamma}\right]$$

$|H(j\omega)| \longrightarrow \max|H(j\omega)| = ?$ 1

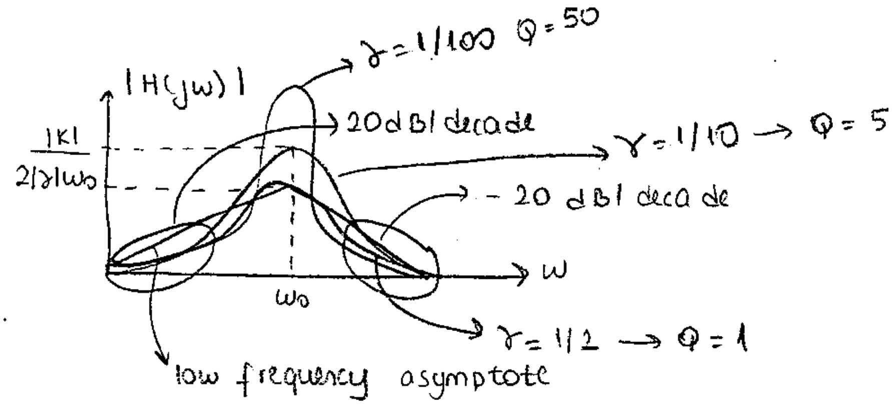
at $\omega = \omega_0 \longrightarrow |H(j\omega)| = \dfrac{|K|}{\omega_0[(2\gamma)^2 + 0^2]^{1/2}}$

$$= \frac{|K|}{\omega_0\, 2|\gamma|}$$

2 $\omega \ll \omega_0 \longrightarrow H(j\omega) \sim \dfrac{K}{\omega_0\left(2\gamma + j\left(0 - \dfrac{\omega_0}{\omega}\right)\right)}$

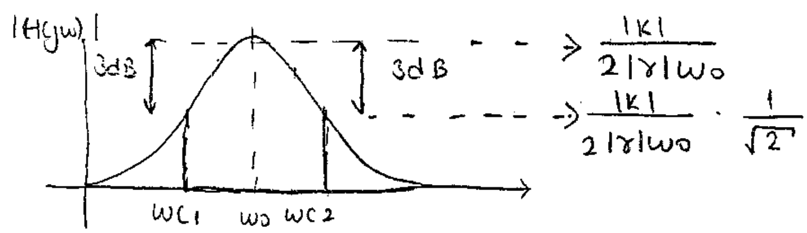
$|H(j\omega)| \sim \dfrac{|K|\,\omega}{\omega_0^2}$

3 $\omega \gg \omega_0 \longrightarrow |H(j\omega)| \sim \dfrac{|K|}{\omega}$



4-) Intersection of asymptotes:

$$\underbrace{|K| \frac{w}{w^2}}_{\text{low frequency}} = \underbrace{\frac{|K|}{w}}_{\text{high frequency}} \longrightarrow w = w_0$$



If $\left(\frac{w}{w_0} - \frac{w_0}{w}\right) = \mp 2\gamma \longrightarrow w$ is a cut-off frequency

For w_{c2} $(w_{c2} > w_0) \longrightarrow \left(\frac{w}{w_0} - \frac{w_0}{w}\right) = 2\gamma \longrightarrow$

$w_{c2}^2 - (2\gamma w_0) w_{c2} - w_0^2 = 0$

$w_{c2} = w_0 \left(\gamma + \sqrt{1+\gamma^2}\right) \longrightarrow w_0 \left(\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}\right)$

plus or minus
(we choose plus)

For w_{c1} $(w_{c1} < w_0)$ —— $\left(\frac{w_{c1}}{w_0} - \frac{w_0}{w_{c1}}\right) = -2\gamma \longrightarrow$

$w_{c1} = w_0 \left(-\gamma + \sqrt{1+\gamma^2}\right)$

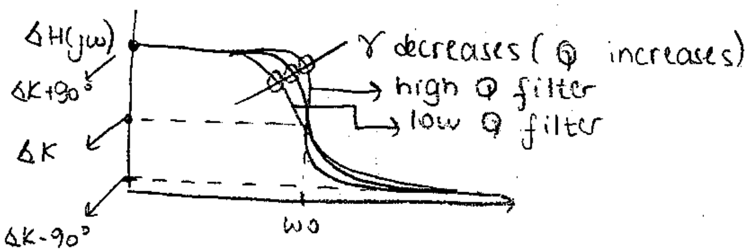
$BW = w_{c2} - w_{c1} = 2 w_0 \gamma$ (Bandwidth)

$$H(s) = \frac{Ks}{s^2 + \underbrace{(2\gamma w_0)}_{BW} s + w_0^2}$$ resonance frequency

$Q = \frac{w_0}{B_w}$ $w_0 \rightarrow$ center frequency, $B_w \rightarrow$ bandwidth $= \frac{1}{2\gamma}$ $\left[\begin{array}{l} Q = 1/2\gamma \\ \gamma = 1/2Q \end{array}\right]$

$\rightarrow \sqrt{\omega_{c1}\,\omega_{c2}} = \omega_0$

$$\measuredangle H(j\omega) = \measuredangle K - \tan^{-1}\left(\frac{(\omega/\omega_0) - (\omega_0/\omega)}{2\gamma}\right)$$



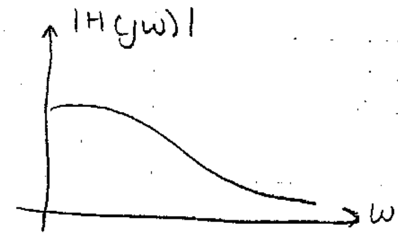
Second Order Lowpass Systems

$$H(s) = \frac{K}{s^2 + 2\gamma\omega_0 s + \omega_0^2}$$

$|H(j\omega)|$: Mag Response

$\measuredangle H(j\omega)$: Phase Response

$H(j\omega)\Big|_{\omega=0,\, s=0} = K/\omega_0^2$



$\lim_{\omega\to\infty} H(j\omega) = 0$ with K/s^2
$(s \to \infty)$

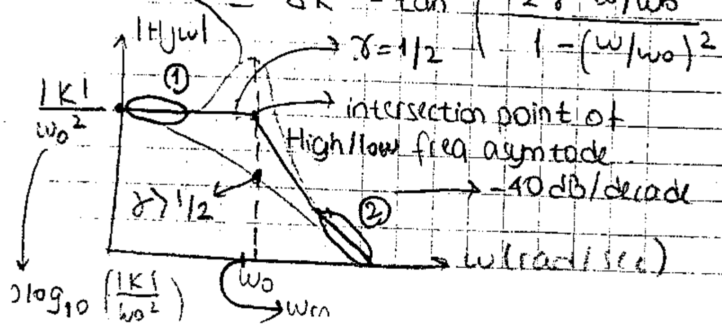
$$H(j\omega) = \frac{K}{\omega_0^2 - \omega^2 + j\,2\gamma\omega_0\omega}$$

$$|H(j\omega)| = \frac{|K|}{\left[(\omega_0^2-\omega^2)^2 + (2\gamma\omega_0\omega)^2\right]^{1/2}} = \frac{K}{\omega_0^2\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right) + j\,2\gamma\frac{\omega}{\omega_0}\right]}$$

$$= \frac{|K|}{\omega_0^2\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\gamma\frac{\omega}{\omega_0}\right)^2\right]^{1/2}}$$

$$\measuredangle H(j\omega) = \measuredangle K - \tan^{-1}\left(\frac{2\gamma\omega_0\omega}{\omega_0^2-\omega^2}\right)$$

$$= \measuredangle K - \tan^{-1}\left(\frac{2\gamma\,\omega/\omega_0}{1-(\omega/\omega_0)^2}\right)$$



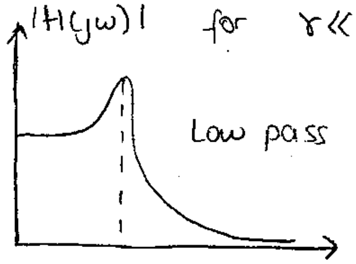
① $\omega \ll \omega_0 \longrightarrow H(j\omega)$

② $\omega \gg \omega_0 \longrightarrow H(j\omega) = \dfrac{K}{\omega_0^2\left[\frac{-\omega^2}{\omega_0^2}\right]} = \dfrac{-K}{\omega^2}$

$\underbrace{|H(j\omega)|}_{20\log_{10}|H(j\omega)|}{}_{dB} = -40\,dB/sec$

3- $H(j\omega) = \dfrac{K}{\omega_0^2\left(j2\gamma\frac{\omega_0}{\omega_0}\right)}$

$|H(j\omega)|$ for $\gamma \ll 1/2$



Low pass filter

• 2nd Order $\gamma > 1/2$ Overdamped
$\gamma < 1/2$ Underdamped $\longrightarrow$ Two complex conjugate poles

4- What is the peak value of $|H(j\omega)| = ?$

(Is the peak located at $\omega = \omega_0$) açıkça ilgisi yok

$\underset{\omega}{\mathrm{Arg\,max}} = |H(j\omega)| = \omega_{max} = \underset{\omega}{\mathrm{arg\,min}}\left(\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \left(2\gamma\frac{\omega}{\omega_0}\right)^2\right)$

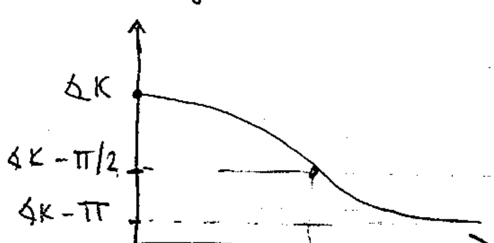
$f(x) = (1-x^2)^2 + (2\gamma x)^2$; $x = \dfrac{\omega}{\omega_0}$

$f'(x) = -4x(1-x) + 2(2\gamma)^2 x = 0$

$x = \sqrt{1-2\gamma^2}$ | $\omega_m = \omega_0 \underbrace{\sqrt{1-2\gamma^2}}_{x}$ $\gamma = 1/2Q$ where Q = quality factor

• $\omega_m = \omega_0\sqrt{1-\dfrac{1}{2Q^2}}$ provided that $Q > \dfrac{1}{\sqrt{2}}$ $\gamma < \dfrac{1}{\sqrt{2}}$

$\measuredangle H(j\omega)$



$H(j\omega) = \dfrac{K}{(\omega_0^2-\omega^2) + j(2\gamma\omega_0)\omega}$

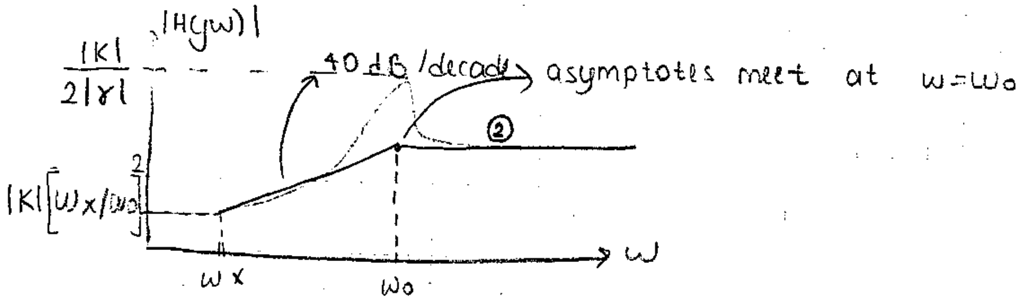
$= \dfrac{K\left((\omega_0^2-\omega^2) - j(2\gamma\omega_0\omega)\right)}{(\omega_0^2-\omega^2)^2 + (2\gamma\omega_0\omega)^2}$

Second Order High Pass System

$$H(s) = \frac{K s^2}{s^2 + 2\gamma \omega_0 s + \omega_0^2} \qquad H(j\omega) \underset{\omega \to \infty}{\simeq} K \qquad \underset{\omega \to 0}{} \sim 0$$

$$H(j\omega) = \frac{K\ -\omega^2}{\omega_0^2 - \omega^2 + j\, 2\gamma\omega_0\omega}$$

$$= -K \frac{(\omega/\omega_0)^2}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\, 2\gamma\left(\frac{\omega}{\omega_0}\right)}$$

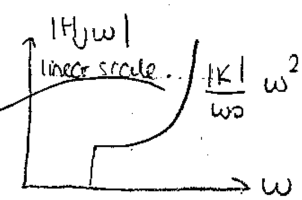


① $\omega/\omega_0 \ll 1$ $\qquad \omega_x/\omega_0 \ll 1 \quad (\omega_x = 10^{-6}\omega_0)$

$\longrightarrow H(j\omega) \sim -K\,(\omega/\omega_0)^2$

② $\omega/\omega_0 \gg 1$

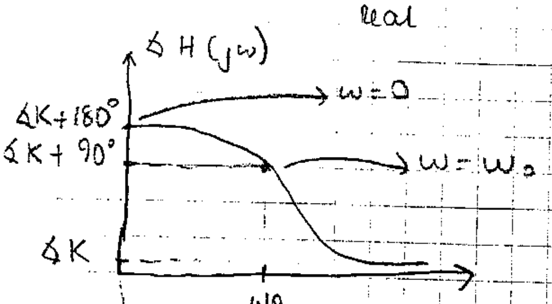
$\longrightarrow H(j\omega) \simeq +K$



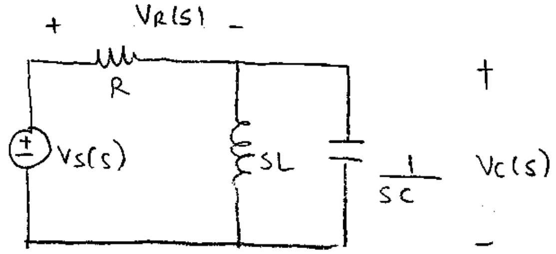
③ $H(j\omega_0) = \dfrac{-K}{j2\gamma}$

$$H(j\omega) = |H j\omega|\, e^{j\measuredangle Hj\omega}$$

$$H(j\omega) = \underbrace{\frac{-K(\omega/\omega_0)^2}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left[2\gamma\left(\frac{\omega}{\omega_0}\right)\right]^2}}_{\text{real}} \left(1 - \left(\frac{\omega}{\omega_0}\right)^2 - j\,2\gamma\left(\frac{\omega}{\omega_0}\right)\right)$$



Example:



$$H_1(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$H_2(s) = \frac{V_R(s)}{V_s(s)} = 1 - H_1(s)$$

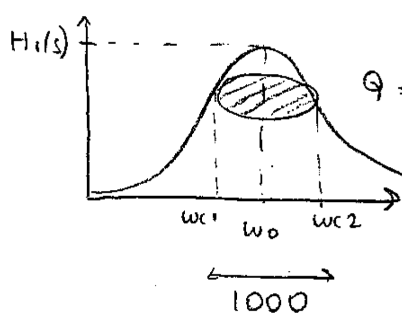
↑ Bandstop

$L = 1/4\ H \qquad R = 1k\Omega \qquad C = 1\mu F$

$$H_1(s) = \frac{s\,1000}{s^2 + 1000s + (2000)^2} = \frac{K\ s}{s^2 + 2\gamma\omega_0 s + \omega_0^2}$$

$K = 1000 \qquad \omega_0 = 2000$

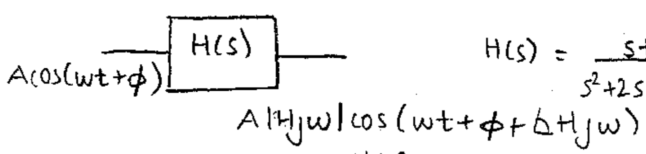
$$\gamma = \frac{1000}{2\omega_0} = 1/4 \longrightarrow Q = 2 \left(= \frac{1}{2\gamma}\right)$$



$$Q = \frac{\omega_0}{BW}$$

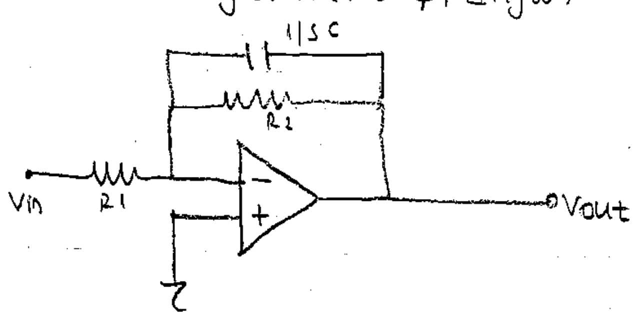
$$\omega_0 = \sqrt{\omega_{c1}\,\omega_{c2}}$$

Example: Active filters: Filters with active components



$$H(s) = \frac{s+1}{s^2+2s+5}$$

[$|H(j\omega)| > 1$ ise; active filters Adı üstünde amplif. var]



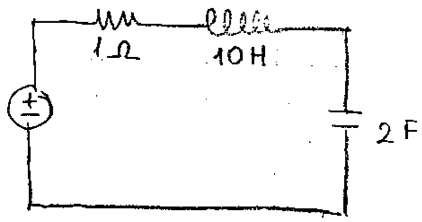
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2}{R_1}\,\frac{1}{1+sR_2C}$$

Operating in linear region
(ideal op-amp)

Scaling:

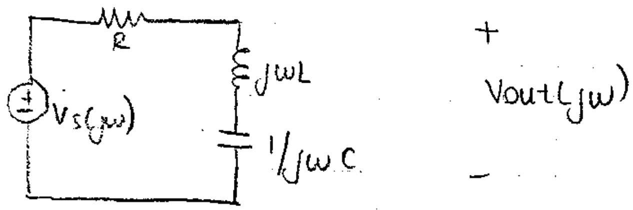
① Magnitude Scaling

② Frequency scaling



→ bigger than earth!

① Magnitude scaling:



$$H(j\omega) = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{Z_2(j\omega) + Z_3(j\omega)}{Z_1(j\omega) + Z_2(j\omega) + Z_3(j\omega)}$$

$R \to k_m R$ (replace) $\longrightarrow Z_1(j\omega)$ scaled by k_m

$L \to k_m L$ $\longrightarrow Z_2(j\omega)$ scaled by k_m

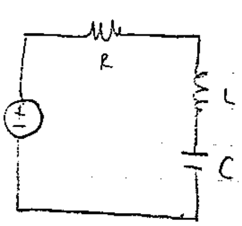
$C \to C/k_m$ $\longrightarrow Z_3(j\omega)$ scaled by k_m

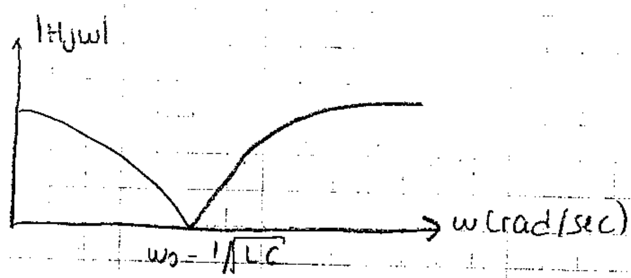
} $H(j\omega)$ remains the same

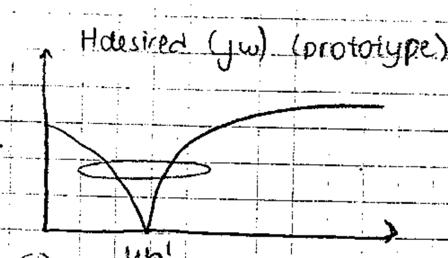
For example: $Z_1(j\omega)$ replaced by $k_m Z_1(j\omega)$

② Frequency Scaling:

Previous example:







$H(j\omega)_{desired} = H\left(j\omega \dfrac{\omega_0}{\omega_0'}\right)_{prototype}$ $\qquad \omega_0'$: desired null frequency

↳ scalar

$$H_{pro}(j\omega) = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C}$$

$R \longrightarrow R$

$L \longrightarrow L/k_f$

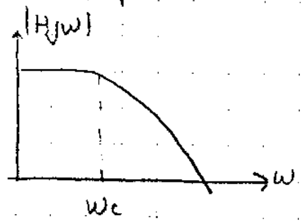
$C \longrightarrow C/k_f$

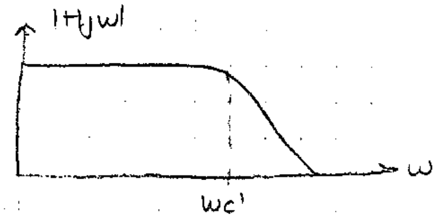
$H(j\omega)$ after k_f scaling $= \dfrac{j\omega L/k_f + k_f/jC\omega}{R + j\dfrac{\omega}{k_f}L + \dfrac{k_f}{jC\omega}}$

$= H_{pro}\left(\dfrac{j\omega}{k_f}\right)$

So to scale the null from $\omega_0 \longrightarrow \omega_0'$

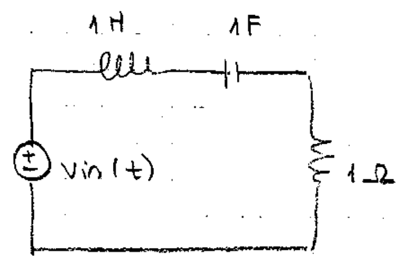
$H_{pro}\left(j\dfrac{\omega}{k_f}\right) = H\left(j\omega \dfrac{\omega_0}{\omega_0'}\right) \longrightarrow k_f = \dfrac{\omega_0'}{\omega_0}$ ← new frequency, → old frequency





→ lowpass example $k_f = \dfrac{\omega_c'}{\omega_c}$

<u>Example</u>:



Scale the circuit such that the resonant freq is not 500 Hz and use a $2\mu F$ cap.

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 1$ rad/sec

$Bw = 2\gamma\omega_0 = \dfrac{R}{L} = 1$ rad/sec

$Q = \dfrac{\omega_0}{Bw} = 1$

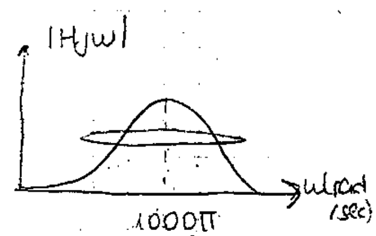
$\omega_0' = 2\pi\, 500 = 1000\pi$, Use $2\mu F$ cap

$R \longrightarrow Rk_m \longrightarrow Rk_m \longrightarrow 160\,\Omega$

$L \longrightarrow Lk_m \longrightarrow Lk_m/k_f \longrightarrow 50\,mH$

$C \longrightarrow C/k_m \longrightarrow C/k_m k_f \longrightarrow 2\mu F$

$k_f = \dfrac{\omega_0'}{\omega_0} = 1000\pi \qquad k_m = ?$



$\dfrac{\overset{1}{C}}{\underset{1000\pi}{k_m k_f}} = 2\mu F \longrightarrow k_m = \dfrac{1}{2\,10^{3}\pi} \sim 160$

Bw after: $\dfrac{R^{after}}{L^{after}} = k_f\, Bw^{before} = 1000\pi$

$Q = \frac{1000\pi}{1000\pi} = 1$ (remains the same)

Example: $H(s) = 12\,500 \dfrac{s+10}{(s+50)(s+500)}$

① Bring $H(s)$ into standard form: (all polynomials should be expressed as $(1 + \frac{s}{\alpha_1})$)

constant term = 1

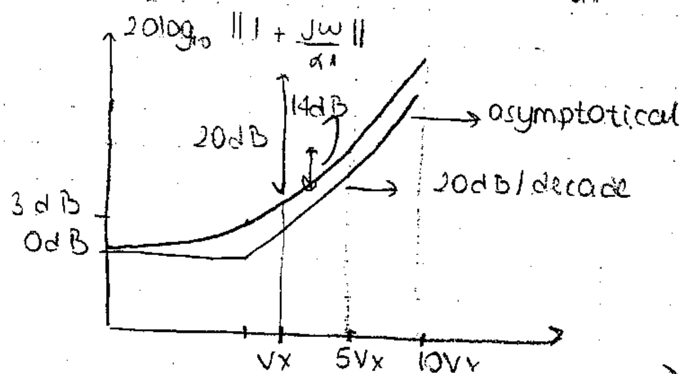
$$H(s) = 12\,5\cancel{0}0 \frac{\cancel{10}(1+s/10)}{50(1+s/50)\,5\cancel{0}0(1+s/500)} = 5\frac{(1+s/10)}{(1+s/50)(1+s/500)}$$

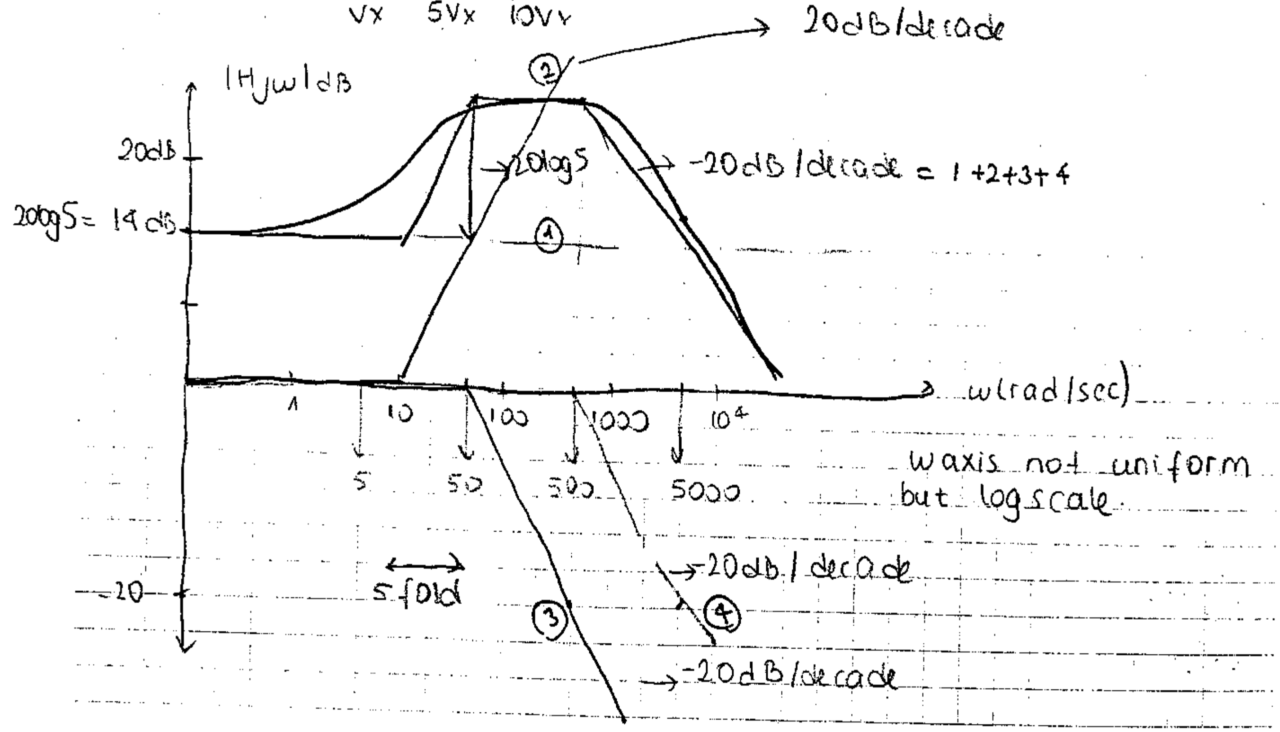
② Express $|H_{j\omega}|_{dB}$ & $\measuredangle H_{j\omega}$

$$H(j\omega) = 5 \frac{1+j\omega/10}{(1+j\omega/50)(1+j\omega/500)}$$

$$20\log_{10}|H_{j\omega}| = 20\log_{10}5 + 20\log_{10}\left\|1+\frac{j\omega}{10}\right\| - 20\log_{10}\left\|1+\frac{j\omega}{50}\right\| - 20\log_{10}\sqrt{1+\frac{\omega^2}{500^2}}$$

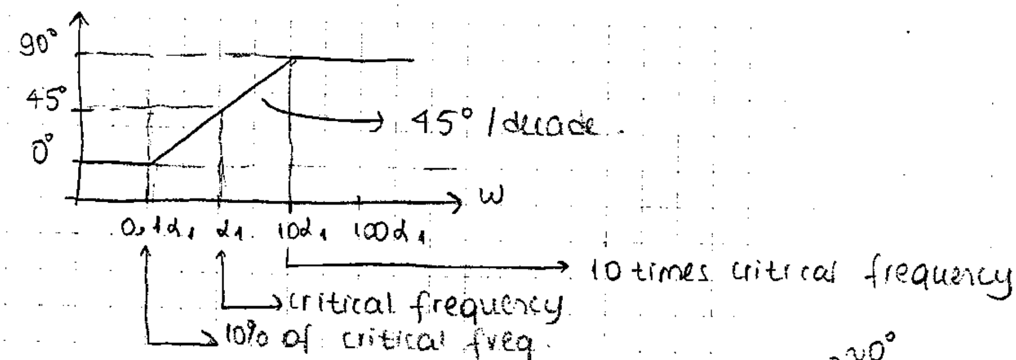
Let's focus on $20\log_{10}\left\|1+\frac{j\omega}{\alpha_1}\right\| = 20\log_{10}\sqrt{1+\frac{\omega^2}{\alpha_1^2}}$





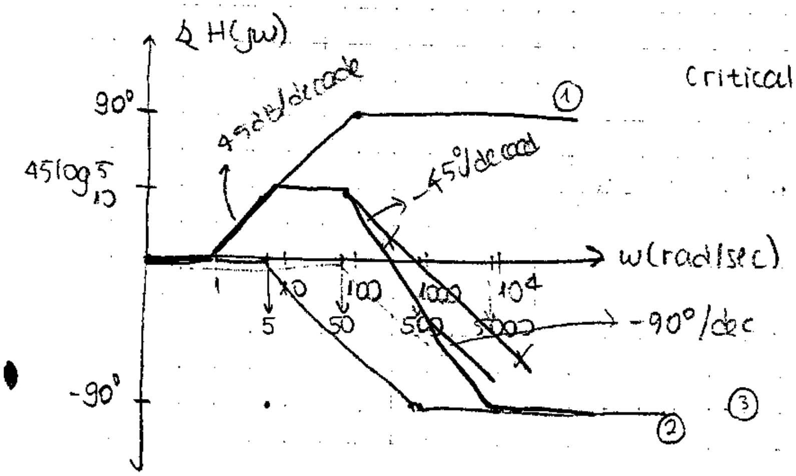
Note that: at every critical frequency $(1+\frac{s}{\alpha_1})$ such as α_1 there is an increasing or decreasing 20 dB/dec. slope.

Phase resp $\measuredangle(1+j\frac{\omega}{\alpha_1}) = \tan^{-1}(\omega/\alpha_1)$



$$H(j\omega) = 5\,\frac{(1+j\omega/10)}{(1+j\omega/50)(1+j\omega/50)} \longrightarrow \measuredangle H(j\omega) = \overset{\to 0^\circ}{\measuredangle 5} + \overset{④}{\measuredangle\left(1+\frac{j\omega}{10}\right)} \overset{②}{-\measuredangle\left(1+\frac{j\omega}{50}\right)} \overset{③}{-\measuredangle\left(1+\frac{j\omega}{500}\right)}$$

Critical freq: $\{10, 50, 500\}$



Bode Plots with 2nd Order system

$$H(s) = \frac{(s+1)}{(s+2)(s+3)(s^2+4s+5)}$$

↳ does not have a reel root

$A(s) = s^2 + 2\gamma\omega_0 s + \omega_0^2$ (divide by ω_0^2)

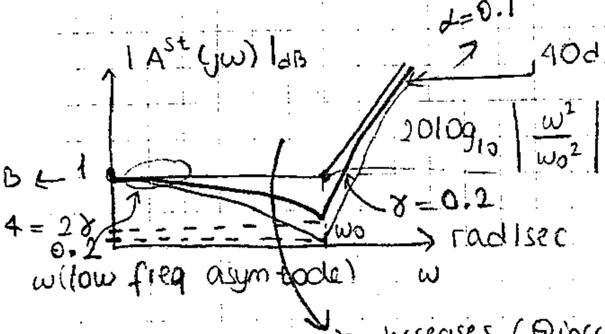
Bring the $A(s)$ into standard form $\longrightarrow \left(\frac{s}{\omega_0}\right)^2 + \frac{2\gamma}{\omega_0}s + 1 = A^{std}(s)$

the constant term

$$H(s) = \frac{(1+s)}{2(1+s/2)\,3(1+s/3)\,5(1+s\frac{4}{5}+\frac{s^2}{5})}$$

$$H(s) = \frac{1}{30}\,\frac{(1+s)}{(1+s/2)(1+s/3)(1+s\frac{4}{5}+s^2/5)}$$

$A^{st}(j\omega) = \left(-\frac{\omega^2}{\omega_0^2} + 1\right) + j\left(2\gamma\frac{\omega}{\omega_0}\right)$



← high freq asymtodes

exact curve passes from this point

at $2\gamma = 1 \rightarrow \gamma = 1/2$

bunlara dikkat et

$A^{std}(j\omega_0) = j2\gamma$

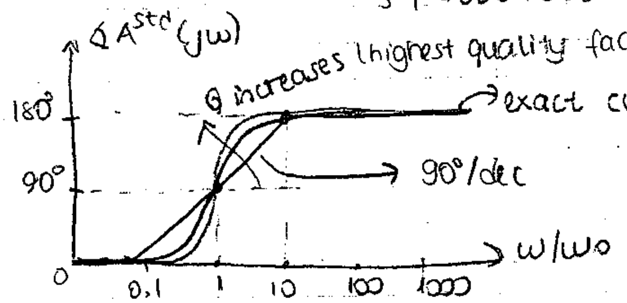
when $\gamma < 1$ $\underline{Q > 1/2} \rightarrow$ underdamped

$\gamma = 1$ $\underline{Q = 1/2} \rightarrow$ critically damped

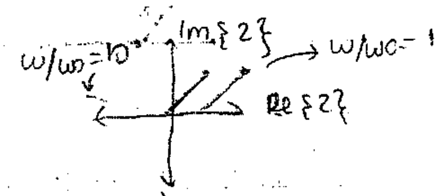
$\gamma > 1$ $\underline{Q < 1/2} \rightarrow$ overdamped → This case can be factored as multiplication of two first order $\left(1 + \frac{s}{\alpha_1}\right)\left(1 + \frac{s}{\alpha_2}\right)$ type polynomials

$Q = 1/2\gamma$ (quality factor)

Example: $H(s) = 5000\,\frac{(s+100)}{s^2 + 400s + 500^2}$



$A^{std}(j\omega) = \left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1\right] + j2\gamma\left(\frac{\omega}{\omega_0}\right)$

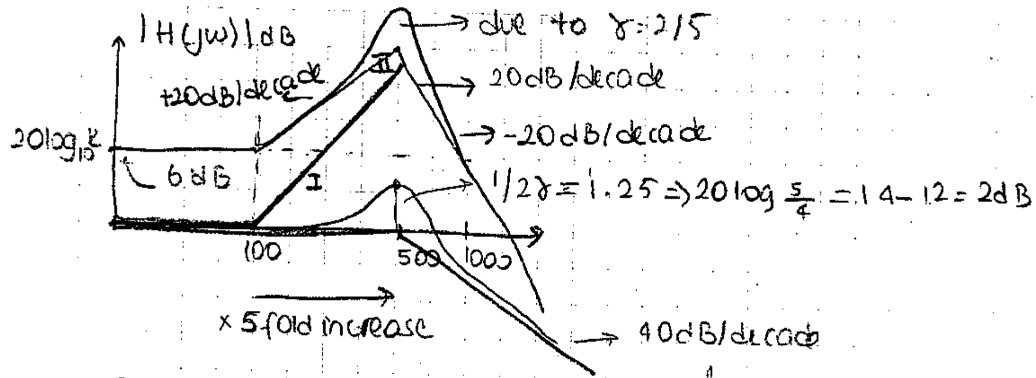


$H(s) = 5000\,\frac{(s+100)}{s^2 + 400s + 500^2} = 5000\,\frac{100(1 + s/100)}{500^2\left(1 + \frac{400}{500^2}s + s^2/500^2\right)}$

↑ two complex roots

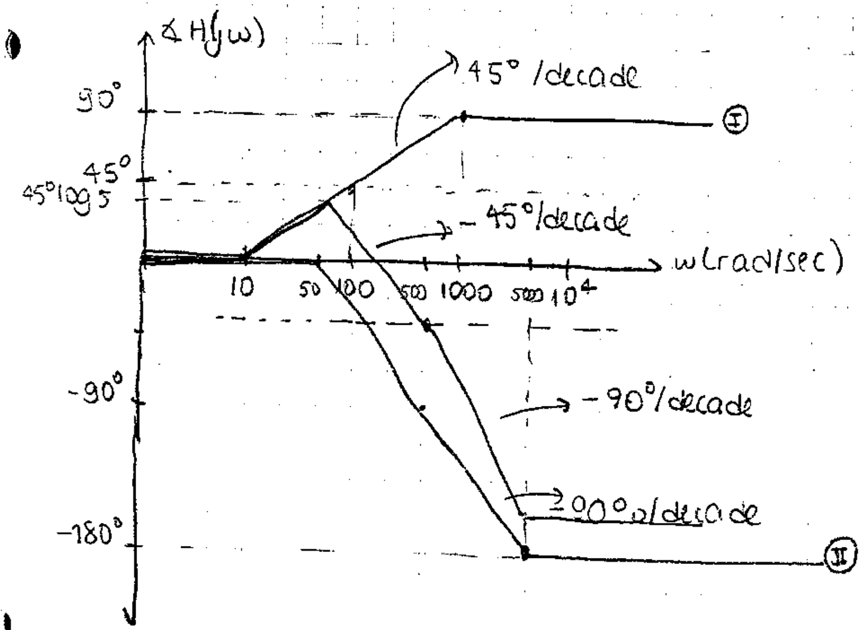
$H(s) = \frac{\overset{\mathrm{I}}{K}\,\overset{\mathrm{II}}{(1 + s/\alpha_1)}}{\underset{\mathrm{III}}{\left(1 + \frac{2\gamma}{\omega_0}s + \frac{s^2}{\omega_0^2}\right)}}$ $\omega_0 = 500$, $\gamma = 2/5$

Critical freq; $= \{100, 500\}$



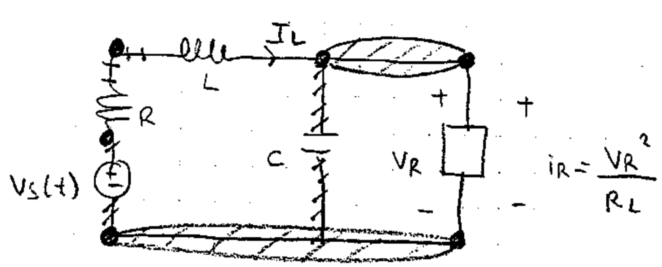
$\gamma \to 0$ olurken, peak ∞'da olur;

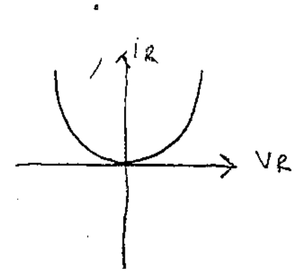
Phase Response:



State equation with Non-linear Elements:

Example:





$\{I_L(t), V_c(t)\}$

Fundamental Cut-set : $\rightarrow C\dot{V_c} = I_L - I_{NL} = I_L - \dfrac{V_c^2(t)}{R_L}$ ✓

Fundamental loop $\longrightarrow L\dot{I_L} = -I_L R + V_s(t) - V_c(t)$ ✓

[DC olduklarını kabul ediyoruz]

$$\dot{V_c}(t) = -\frac{V_c^2(t)}{R_L C} + \frac{J_L}{C}$$

$$\dot{J_L}(t) = -V_c(t)\cdot\frac{1}{L} - \frac{R}{L}J_L(t) + \frac{V_s(t)}{L}$$

Zero-input solution $V_c(0^-) = V_0$ $J_L(0^-) = I_0$ $V_s(t) = 0$

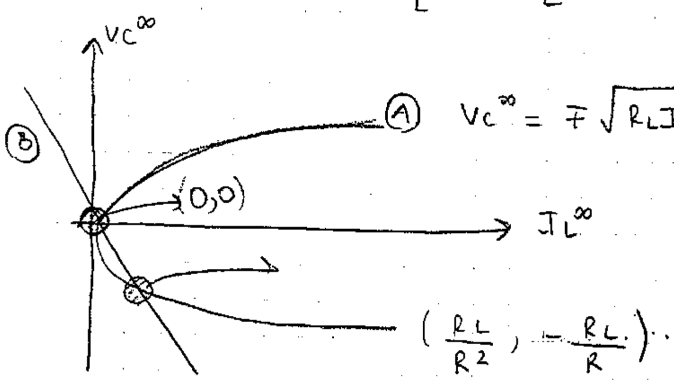
If there is a solution, then $\left.\begin{matrix}\dot{V_c}(t)=0\\ \dot{J_L}(t)=0\end{matrix}\right\}$ when we reach the steady state solution.

$t \to \infty$ $V_c(t) \longrightarrow V_c^{\infty}$

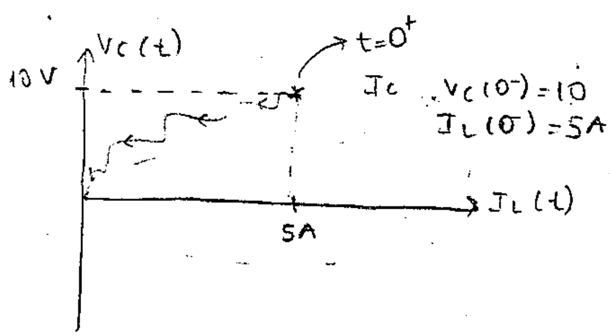
$J_L(t) \longrightarrow J_L^{\infty}$

As $t \to \infty$ $0 = -\frac{(V_c^{\infty})^2}{R_L C} + \frac{J_L^{\infty}}{C} \longrightarrow (V_c^{\infty})^2 = R_L \cdot J_L^{\infty}$

$0 = -\frac{V_c^{\infty}}{L} - \frac{R}{L} I_L^{\infty}$ $V_c^{\infty} = -R I_L^{\infty}$



$V_c^{\infty} = \mp\sqrt{R_L J_L^{\infty}}$



→ phase plane

→ phase portrait