THE Lee-Wiener Legacy

A History of the Statistical Theory of Communication

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The Research Laboratory of Electronics (RLE) at the Massachusetts Institute of Technology (MIT) was the home of many famous scientists and engineers in the days following World War II. One of the lesser known heroes of this era was Yuk Wing Lee (Fig. 1), a student of Norbert Wiener in the late 1920s, who led an important research group in statistical communication theory. Lee's own contributions, although significant, are not well known. However, his contributions to teaching and bringing the ideas of Wiener into electrical engineering are without precedent. This article describes the activities of Lee, Wiener, and the many well-known students of Lee who contributed and shaped the area of statistical signal processing and communication. While volumes have been written about Wiener, almost nothing has been written about Lee. (An exception is [3] published in China.) Nonetheless, without the efforts of Lee and his students many of Wiener's ideas may not have made the transition to electrical engineering.

▲ 1.Y.W. Lee. (Courtesy of the MIT Museum.)

Research Laboratory of Electronics

The MIT RLE evolved from the renowned MIT Radiation Laboratory that existed during World War II [1], [2]. The Radiation Lab brought together a group of engineers and scientists in a critical mass focused on a set of technological problems that would ultimately help the United States and its Allies win the war. When the war effort ended, to not lose the critical advantage that existed with this group of people, many of the resources of the Radiation Lab were "rolled over" into the new laboratory (RLE). RLE was funded by the Joint Services with the notion that as long as these people were kept together, they could be mobilized on shorter notice to the service of their country in the event that they were again needed. RLE, which remained on campus in the building that originally housed the Radiation Lab, had an open-ended research policy to explore a variety of problems in physics, electronics, and other areas that could in the long run be beneficial to the national security.

It was this environment that Y.W. Lee found when he joined the MIT Department of Electrical Engineering in 1946. Lee joined RLE with a charter—to bring into the electrical engineering community the ideas of his mentor, former thesis supervisor, and famous mathematician Norbert Wiener; and to develop their practical applications. In so doing, Lee himself became mentor and advisor to many students who did important work and became famous themselves. As a result, we have not only a body of mathematical knowledge as a legacy of Lee and Wiener but also a long "family tree" of electrical engineers that are, in this sense, descended directly from these individuals.

Lee's Education

Yuk Wing Lee (known as "Yuwing" or "Y.W." to his friends) was born in Macao, China, on 14 April 1904. He first came to MIT as a student in 1924 and received the bachelor's and master's degrees in electrical engineering in 1927 and 1928. Continuing in the graduate program, he performed his doctoral work under the supervision of Wiener and was granted the Ph.D. in 1930. Although Wiener held a position in the Mathematics Department, he had significant interest in electrical engineering and understood it well.

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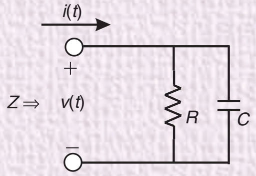
Hilbert Transform Relations in Network Synthesis

The response of any linear passive network or circuit to an impulse in voltage or current at $t=0$ must be purely one sided. That is, the impulse response $h(t)$ must satisfy

$$h(t)=0, \qquad t<0.$$

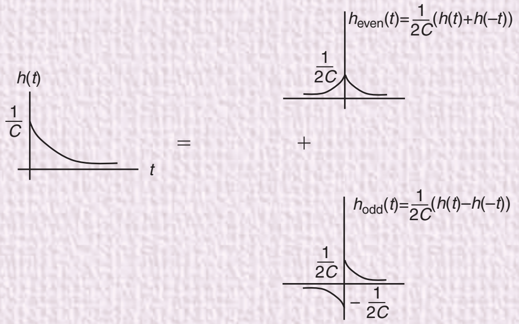
As a result, the impulse response can be written mathematically as a sum of even and odd parts, as illustrated in the following example:



$$Z(\omega)=\frac{\frac{1}{C}}{j\omega+\frac{1}{RC}}$$

For $i(t)=\delta(t)$, a unit impulse, the response is

$$v(t)=h(t)=\begin{cases}\frac{1}{C}e^{-t/RC} & t\geq 0\\ 0 & t<0\end{cases}$$



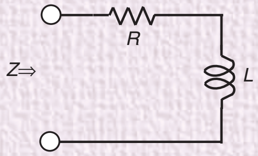
The Fourier transforms of $h_{\text{even}}(t)$ and $h_{\text{odd}}(t)$ can be shown to be the real and imaginary parts of $Z(\omega)$. Thus $\text{Re}(Z(\omega))$ and $\text{Im}(Z(\omega))$ are *related.* (They cannot be specified independently.)

The relation between the real and imaginary parts of the impedance (or any passive circuit transfer function, for that matter) are provided formally through the Hilbert transform. If $Z(\omega)=R'(\omega)+jX(\omega)$, then

$$R'(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{\omega-\lambda}d\lambda \text{ and } X(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{R'(\lambda)}{\omega-\lambda}d\lambda.$$

That is, $R'(\omega)$and $X(\omega)$ are said to form a *Hilbert transform pair* [30].

The formal relations preclude any singularities in $h(t)$ at $t=0$. Thus the impedance function for the circuit



$$Z(\omega)=R+j\omega L$$

appears to violate the principle that real and imaginary parts are related (although the corresponding admittance function does not). The answer lies in the fact that while $Z(\omega)=R+j\omega L$ is a convenient engineering representation for the circuit, the Fourier transform of $h(t)$ does not exist in the usual sense because of the derivative operation. (Note that the response to a unit impulse in current produces a "doublet" in voltage.)

This apparent counterexample was brought up to Lee during his Ph.D. defense and caused the faculty, who had never heard of the Hilbert transform, to disbelieve the results. It was Wiener's endorsement, however, that finally carried the day.

Lee's work while a graduate student at MIT was not without significance. His topic was on the application of Laguerre functions to the physical realization of various network immittance functions. The idea (proposed by Wiener) was that if a given impedance or admittance function could be expanded in a series involving Laguerre functions (as Wiener had shown), then there should be a way to physically realize any such immittance by combining simple circuits, each of which realized one of these functions. Lee referred to such realizations as network "synthesis." He appears to have been the first one to use that term, which was so extensively employed later by Guillemin and others to describe their work (e.g., [4]). The word appears in the title of the thesis "Synthesis of Electric Networks by Means of the Fourier Transforms of Laguerre's Functions" [5]. Fig. 2 shows Wiener, Lee, and Amar Bose (at a much later time in Lee's career) discussing the Lee-Wiener networks based on the Laguerre functions.

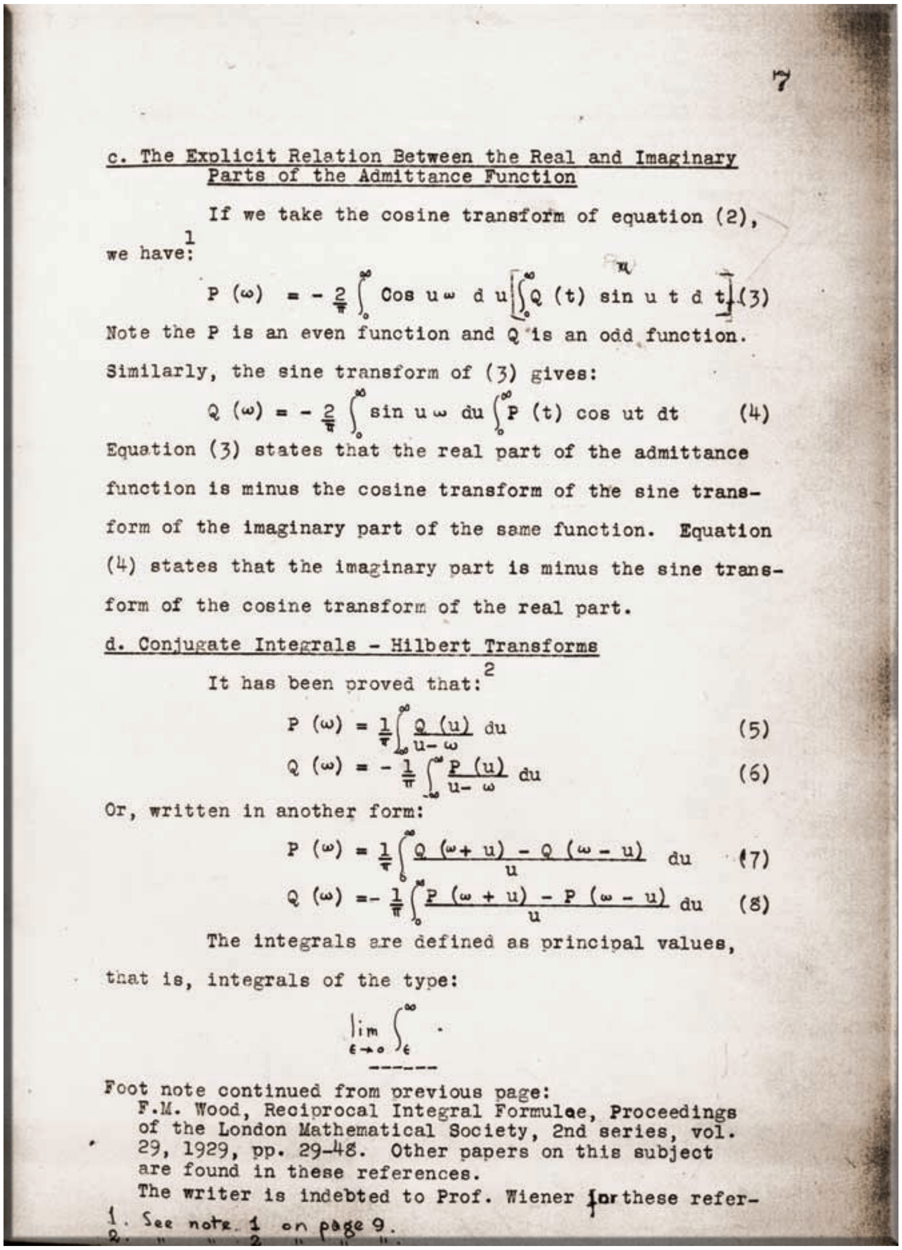
The period when Lee was studying for the Ph.D. is not well documented, but colleagues that knew Lee tell the following story. When first beginning his doctoral research, Lee's synthesis procedures were not working as he had expected. When he reported the difficulties to his advisor, Wiener thought for a moment and referred Lee to a paper in the mathematical literature of the day by E.C. Titchmarsh [6] that explained the Hilbert transform relations. After studying the paper for an extended period of time, Lee finally realized that the reason his synthesis procedures were not working was that he was not taking into account the relation between the real and imaginary parts of the immittance function. These two functions could not be specified independently because they were related through the

2. Lee, Bose, and Wiener discussing networks based on the Laguerre functions in the mid 1950s. (Courtesy of the MIT Museum.)

Hilbert transform (see “Hilbert Transform Relations in Network Synthesis”). With this discovery, the synthesis procedures started to produce the desired results. The Hilbert transform was not familiar to electrical engineers at the time however, and Lee, not being a dynamic or persuasive speaker, had a difficult time convincing the senior MIT faculty at his doctoral defense of the validity of the relation. Ultimately it was Wiener’s endorsement of the concept that allowed the work to pass and Lee to receive his degree. Although the Hilbert transform was not explained well even in the written thesis document (which may have contributed to the faculty’s reluctance to accept the idea), it was the key to the success of the work and appears to be the first time the Hilbert transform had been used in electrical engineering. A page of the original thesis document is shown in Fig. 3.

Besides the introduction of the Hilbert transform, Lee’s work on network synthesis was itself of considerable importance at the time. The network based on the Laguerre functions became known as the Lee-Wiener network, and Lee and Wiener obtained a series of three patents on their inventions from 1935 to 1938 [7]-[9]. The sales of the rights to AT&T provided the funds that later allowed Lee to return from China to the United States.

After his graduation from MIT, Lee spent a year as a development engineer for United Research Corporation (later Warner Brothers) in Long Island City where he continued to work on the Lee-Wiener network. In 1932 Lee returned to his homeland to work as an electrical engineer for China Electric Company in Shanghai, and in 1934 he became a Professor of Electrical Engineering at National Tsing Hua University in Peiping. During this time he was married (in 1933) to Elizabeth, a Canadian citizen, whom he had known in the United States. Three years later, during their return from a visit with his parents in Hangchow, the Japanese invaded Northern China and Lee and his wife were stranded in Shanghai. The Lees lost almost everything they owned in their house at the university and had to open an antique store in Shanghai to scratch out a living. Since the hope of returning to the



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c. The Explicit Relation Between the Real and Imaginary Parts of the Admittance Function

If we take the cosine transform of equation (2), we have:[1]

$$P(\omega) = -\frac{2}{\pi}\int_0^\infty \cos u\omega \, du\left[\int_0^\infty Q(t)\sin ut\, dt\right] \quad (3)$$

Note the P is an even function and Q is an odd function.

Similarly, the sine transform of (3) gives:

$$Q(\omega) = -\frac{2}{\pi}\int_0^\infty \sin u\omega \, du\int_0^\infty P(t)\cos ut\, dt \quad (4)$$

Equation (3) states that the real part of the admittance function is minus the cosine transform of the sine transform of the imaginary part of the same function. Equation (4) states that the imaginary part is minus the sine transform of the cosine transform of the real part.

d. Conjugate Integrals - Hilbert Transforms

It has been proved that:[2]

$$P(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{Q(u)}{u-\omega}\,du \quad (5)$$

$$Q(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{P(u)}{u-\omega}\,du \quad (6)$$

Or, written in another form:

$$P(\omega) = \frac{1}{\pi}\int_0^\infty \frac{Q(\omega+u)-Q(\omega-u)}{u}\,du \quad (7)$$

$$Q(\omega) = -\frac{1}{\pi}\int_0^\infty \frac{P(\omega+u)-P(\omega-u)}{u}\,du \quad (8)$$

The integrals are defined as principal values, that is, integrals of the type:

$$\lim_{\epsilon\to 0}\int_\epsilon^\infty .$$

Foot note continued from previous page:
F.M. Wood, Reciprocal Integral Formulae, Proceedings of the London Mathematical Society, 2nd series, vol. 29, 1929, pp. 29-48. Other papers on this subject are found in these references.
The writer is indebted to Prof. Wiener for these refer-

1. See note 1 on page 9.
2. " " 2 " " .

3. The page of Lee’s Ph.D. dissertation where the Hilbert transform is introduced. (Courtesy of the MIT Archives.)

Lee's contributions to teaching and bringing the ideas of Wiener into electrical engineering are without precedent.

university in China appeared dim, Lee wrote to Wiener about finding a position in the United States and was subsequently offered a position as assistant professor at MIT. This offer came in 1941 and although Lee had managed to purchase a pair of ship tickets from Hong Kong to America, the bombing of Pearl Harbor and the subsequent entry of the United States into the war in the Pacific complicated matters. His plans to leave China were thwarted by a combination of the escalation of the war and some bureaucratic errors in Washington involving a paper of minor significance that nonetheless prevented his departure. Thus Lee and his wife had to remain in Shanghai under Japanese control until the end of the war. He was fortunate to obtain some teaching positions during this time at Ta Tung University and St. John University, but the difficulties of life during the Japanese occupation were considerable. Elizabeth Lee published an interesting account of these years while waiting out the end of the war [10].

Lee's Work in the Early Years

When Lee finally arrived at MIT in 1946, RLE was just beginning. Wiener had recently developed his theory for optimal linear signal estimation and had written it up in the famous report *Extrapolation, Interpolation, and Smoothing of Stationary Time Series* [11], which became known as the "Yellow Peril" because of its yellow cover and difficult mathematics. Lee immediately set to work on the explanation of Wiener's theory of optimum linear systems and the tackling of various engineering problems related to the practical application of the work. In an early RLE quarterly report [12] Lee writes:

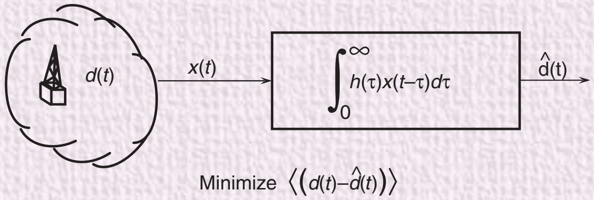
> In order that Prof. N. Wiener's new communication engineering theory be made available to engineers, an exposition of this theory has been under preparation. This is in great need since the theory is based upon new ideas and certain mathematical techniques which are unfamiliar to most engineers. A part of the expository writing has been finished.

He also lost no time in establishing a graduate course in the Department of Electrical Engineering titled "Optimum Linear Systems," first offered in October 1947. The title of the course was later changed to "Statistical Theory of Communication" (MIT course 6.571) and was taught for many years before Lee's book of the same name was published in 1960 [13]. By that time several other more "modern" courses had been offered dealing with statistics and communications, and the book seemed somewhat dated in its approach and notation. Lee, by that time, was offering a second course (6.572) dealing with the Wiener nonlinear theory. The two courses, which were taught by several faculty members, filled a niche in the curriculum and continued until Lee's retirement.

The essence of Wiener's theory of optimal linear systems is now well known to electrical engineers, especially those involved in communications or signal processing

Wiener Optimal Linear Filtering

Wiener's theory of optimal linear systems dealt with estimation of a desired signal $d(t)$ from some related observed signal $x(t)$ (see figure).



The problem was to find a *causal* linear filter with impulse response $h(t)$ whose output would be the desired signal estimate $\hat{d}(t)$ and which would be optimal in the mean-square sense. Specifically, if the error in the estimate were defined as $\varepsilon(t) = d(t) - \hat{d}(t)$, then the filter would minimize the quantity

$$\langle \varepsilon^2(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \varepsilon^2(t)dt.$$

Wiener had shown that the optimal filter was the solution to the integral equation

$$\int_0^{\infty} h(\xi)\varphi_{xx}(\tau-\xi)d\xi = \varphi_{xd}(\tau); \quad 0 < \tau < \infty,$$

today known as the Wiener-Hopf equation, where $\varphi_{xx}(\tau)$ is the autocorrelation function for the observed signal and $\varphi_{xd}(\tau)$ is the cross-correlation function between the desired and the observed signals defined as

$$\varphi_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)dt$$

and

$$\varphi_{xd}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)d(t+\tau)dt.$$

Because of the causality constraint on the filter, the solution of the Wiener-Hopf equation is not straightforward. Wiener, however, had encountered an equation of the same form in previous work with the German mathematician Eberhard Hopf and had developed the procedure known as "spectral factorization" for solving this equation. Wiener's work was not well understood by engineers at his time, however, and Lee's first mission upon returning to the United States was to teach others to understand and apply this theory.

(see “Wiener Optimal Linear Filtering”). The problem was to find a *causal* linear filter that would provide an estimate of some desired signal given only a second moment statistical description of the desired signal and the related observation. The set of problems to which this theory applied was very broad and could include estimation of future as well as current values of the signal (i.e., prediction). The observations could be nonlinearly related to the desired signal although the *estimate* would always be linear. The solution to this problem is not trivial because of the *causality* constraint on the filter; however, Wiener had encountered an equation of the same form during studies in physics with the German mathematician Eberhard Hopf and had developed the mathematics for solving this equation. Hence the complete solution of the estimation problem was in hand.

Since the application of Wiener’s theory involves knowledge of the autocorrelation $\varphi_{xx}(\tau)$ and cross-correlation functions $\varphi_{xd}(\tau)$ for the signals of interest, much of Lee’s early research with his graduate students was centered on studying and computing these functions. In the course of these years Lee’s group built a number of machines to estimate correlation functions of signals. Fig. 4 shows the correlation machine built by Lee’s first doctoral student Thomas P. Cheatham, Jr. as part of his dissertation. This first correlation machine was patented by Lee, Wiesner, and Cheatham and so resulted in Lee’s fourth patent [14].

The machine developed by Lee and Cheatham actually performed the correlation in discrete time by sampling the signal. However, the multiplications and additions to compute the correlation function were accomplished by analog methods. In a follow-on thesis, student Henry E. Singleton (later founder of Teledyne Corp.) built an all-digital correlator which was demonstrated by the group in 1949.

The machines built by the group supported basic research in optimal filtering, detection, and properties of correlation. In early investigations the group demonstrated an ability to detect very low-level periodic signals in noise through correlation [15], a fact that is now well known to all electrical engineers. Their studies of correlation began with various types of random noise but later included such signals as speech and brainwaves and were quite advanced for the time.

▲ *4. Jerome Wiesner (RLE director), Lee, student T.P. Cheatham, and Wiener (holding the ever-present cigar) gathered around the first analog correlator machine built in the late 1940s. (Courtesy of the MIT Museum.)*

Statistical Theory of Communication Group

The Middle Years

Although the name “Statistical Theory of Communication” for a group in RLE first appears in the July 1948 Quarterly Progress Report [16], 1954 is the first year when Lee clearly emerges as the leader. In this period, Lee and others conducted summer sessions on the new methods of communication theory that were emerging. Fig. 5

▲ *5. Photograph from a 1954 summer session in communication theory. Lee, Wiener, Shannon, and Fano are standing in the center of the front row. (Courtesy of the MIT Museum.)*

Lee's work on network synthesis was itself of considerable importance.

shows the class from a summer course taught on statistical communication at MIT. It should be noted that while Lee's group later participated in a variety of activities, Lee's own interest was primarily in the areas of what today would be called statistical signal processing and system theory. Shannon, Fano, and others within RLE were working in information theory, coding theory, and detection theory, which contributed importantly to communications as well but was separate from the work of Lee's group (see "Author's Note").

The early 1950s was the time at which the group began to study the Wiener theory of *nonlinear* systems, with Lee's new doctoral student Amar G. Bose, who joined in the fall of 1953. The story of this new work is related eloquently by Bose in a presentation given at MIT in celebration of Wiener's 100th birthday [17], [18]. Bose had returned from a summer job in The Netherlands in the fall of 1953. Before leaving for Europe he had arranged to do a Ph.D. dissertation with Prof. Ernst Guillemin in the area of conformal mapping as applied to network theory and had a research assistantship lined up in the RLE. When he returned to RLE, he found a note asking him to see Prof. Jerome Wiesner and Prof. Henry Zimmermann, the director and assistant director of the laboratory. Bose was asked by these men to work on a new topic from Wiener. Bose explained that he had already arranged to do a thesis with Guillemin, but they insisted and he had no choice but to acquiesce. That is how Bose became involved in the statistical theory of communication.

Author's Note

The information in this article is based on historical research performed during a sabbatical at MIT in the Research Laboratory of Electronics in the Fall of 1997. It is derived from documents and reports found at MIT and elsewhere and personal interviews conducted with former students and others who knew Lee and Wiener. I have tried to indicate how the information was gathered and where it is anecdotal.

In publishing an article of this sort, it would be negligent not to note that MIT and particularly RLE was the incubator for many other activities in the development of modern communication theory. For example, while Lee and Wiener focused on the continuous side of the theory, others, such as Shannon, Fano, Elias, and their students, were developing the discrete side (i.e., information theory and coding). One could say that this had an at least equivalent impact on communications. The decision to focus on the side of the work described in this article was out of personal interest and in no way was meant to ignore other important work in the area at MIT and elsewhere.

Bose was sent to Lee, his new advisor, who gave him a stack of 50 or so pages typed from various notes written by Wiener. Bose studied the package for a period of almost ten months without successfully understanding what was written. Lee could not help him because he himself did not understand the new theory, and Bose did not attempt to see Wiener because he felt he could not formulate an appropriate question. During this time Bose would return to Lee periodically and tell him of his frustration, to which Lee would always reply "Keep at it; it will come." This period, however difficult, is regarded by Bose as the most important learning experience in his life. The learning had to do with an attitude toward success in problems encountered in his life that he did not want to become involved in and carried over to Bose's significant achievements in all areas (technical, financial, managerial) of building a large company (the Bose Corporation).

Bose's progress in understanding the Wiener nonlinear theory may have been hastened by the following event. After Bose had been studying the papers for about six months, Lee announced to him that he had been scheduled to speak on the theory at an international conference in mathematics to be held at MIT. Bose, having never presented a paper before and not yet fully understanding the topic he was studying, was horrified. He continued to study the work even more diligently still without success until two weeks before the presentation, when it "all came together." The presentation went well, and even Wiener, who asked a question during the presentation, was apparently impressed with Bose's grasp of the new theory. Later when the conference had ended, Lee walked into Bose's office one day and quietly told him that the people attending the conference had voted and decided that the presentation of Wiener's theory was the best presentation of the entire conference.

Bose's explanation of the theory was subsequently written up in an RLE quarterly progress report [19]. Wiener's work dealt with the modeling of nonlinear systems through an extension of the methods used in the Volterra representation of functionals. In the Volterra representation, the output of a nonlinear system is represented by a power series of the form

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau_1) x(t-\tau_1) d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 \tau_2 + \cdots$$

where $x(t)$ is the input, and $h_i(\tau_1,\ldots,\tau_i)$ for $i=1,2,\ldots$ are known as the "kernels" that define the system. Each of the terms in the above equation is a homogeneous functional of a particular order. Wiener showed how to reformulate

Volterra Representation:

$$y(t) = \mathrm{H}_0[x(t)] + \mathrm{H}_1[x(t)] + \mathrm{H}_2[x(t)] + \cdots + \mathrm{H}_p[x(t)] +$$

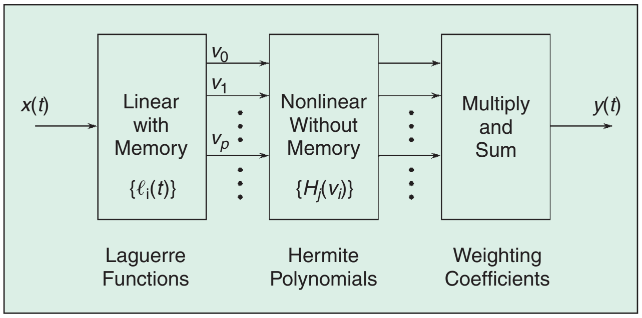
$$\mathrm{H}_p[x(t)] = \int_0^\infty \cdots \int_0^\infty h_p(\zeta_1,\ldots,\zeta_p)x(t-\zeta_1)\cdots x(t-\zeta_p)d\zeta_1\ldots d\zeta_p$$

Wiener Representation:

$$y(t) = \mathrm{G}_0[x(t)] + \mathrm{G}_1[x(t)] + \mathrm{G}_2[x(t)] + \cdots + \mathrm{G}_p[x(t)] +$$

$$E\{\mathrm{H}_m\,[x(t)]\mathrm{G}_p[x(t)]\} = 0 \quad \text{for} \;\; x(t) \sim N(0,\sigma^2), \quad m < p$$

▲ *6. A comparison of Volterra and Wiener representations of a nonlinear system.*



▲ *7. Wiener canonical representation of a nonlinear system.*

the problem in terms of a different set of functionals that are orthogonal when the input is white noise (see Fig. 6).

The terms can further be expanded in terms of sets of Laguerre and Hermite polynomials to put the system in a canonical form shown in Fig. 7. Because of mode of convergence of this orthogonal series, the class of systems represented is actually larger than that for the Volterra series [20].

Bose's entrance into the Statistical Theory of Communication Group was a significant turning point as it set the direction for the group's exploration of the Wiener nonlinear theory that was to continue for years afterward. Following Bose's presentation at the mathematics conference, Wiener, who used to visit Lee's group on occasion, started visiting the group daily to talk with Bose. Rather than advise Bose about his thesis, he used Bose as a sounding board for his own ideas (as he often did with others). This involvement continued, and Wiener had great respect for Bose.

After Bose's work on the Wiener nonlinear theory, several other students continued to work and develop the area. Fig. 8 shows Lee and a group of his graduate students discussing the symmetry properties of second-order correlation functions. These functions were defined as

$$\varphi_{xxx}(\tau_1,\tau_2) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau_1)x(t+\tau_2)dt$$

and are essential to the study of second-order (quadratic) nonlinear systems.The group had considerable interest in second-order correlation and even built a machine to compute these functions. Their properties were explored in depth in the Ph.D. dissertation by Hayase, and they were applied to problems outside of the field of nonlinear systems such as source location [21].

Bose completed his thesis and, after a one-year excursion to India as a Fulbright scholar, joined the MIT faculty and the Statistical Theory of Communication Group as an assistant professor in 1956. The group grew in size with many other students while Wiener continued to interact with Bose and others on a regular basis. These interactions led to the following interesting series of events [17]. Wiener had been coming into Bose's office almost daily, writing equations on the board, usually erasing one as soon as it was written to write another equation in its place, assuming that Bose had understood completely. One day, after one of these sessions, he wrote a final equation on the board and exclaimed "That's it, Bose; write it up!" It was difficult for Bose to explain to Wiener that he had understood only a fraction of what Wiener had been speaking about because Wiener had an unusually hard time understanding that other people were not on his level. It was at this point that Bose and Lee had the idea to invite Wiener for a series of lectures to the group which they would write up for him in the form of a monograph. This is how Wiener's book *Nonlinear Problems in Random Theory* [22] came to be published.

The lectures by Wiener were attended by Lee and Bose and doctoral students in the Statistical Theory of Communication Group for several weeks in 1958. The students listed by Wiener as participating in this project are Don Chesler, Don George, Irwin Jacobs, Al Nuttall, Charles Wernlein, although others such as Martin Schetzen, Don Tufts, and Ken Jordan attended for at least some of the time. Photographs of the blackboard were taken by Lee and Jordan, and Wiener's voice was recorded on reel-to-reel tape. Fig. 9 shows contact strips

▲ *8. Lee and his students discuss an aspect of nonlinear system theory in 1954. Bose is in the center in the white jacket. Other students that have been identified in the photograph are J. Hayase, K.L. Jordan, K.H. Powers, W. Smith, R.E. Wernikoff, and possibly S.G. Margolis and M. Schetzen (nearest to Lee). (Courtesy of the MIT Museum.)*

▲ *9. Contact print: photographs of the blackboard taken during Wiener's lectures (1958) leading to the book* Nonlinear Problems in Random Theory. *(Courtesy of the MIT Museum.)*

found in the MIT museum that may have been some of the ones taken for these historic series of lectures. The original negatives could not be found, and the contact prints are not all from the same roll of film. However many of the equations and figures appearing in Wiener's book can be identified in proper sequence in these frames. In interviews, some of these students recalled how they studied the equations at night and would find errors in the intermediate steps that Wiener had made. However, the errors were often inconsistent, in that an error made in one step was corrected in the next and a new unrelated error was sometimes introduced. While this kept the students "on their toes" and caused them to be elated when they found these errors in Wiener's work, they never found an error in any of the final results. Apparently Wiener knew exactly where he was going in these lectures and only became careless in the less significant intermediate steps. Occasionally, however, he would recognize these errors himself as he was lecturing, and if too many of these occurred he became despondent, put down his chalk, and left the room. Lee and Bose would then visit him a few days later and convince him to resume the talks.

Although Bose was the first to explain the nonlinear theory of Wiener [19], his thesis took a slightly different direction to expand the methods [23]. Schetzen, who joined the Statistical Theory of Communication Group after Bose, also contributed significantly to the body of knowledge produced by the group in this area [24], and like Lee, his interests remained closest to the original work of Wiener. A significant publication was the paper by Lee and Schetzen [25], which provided a practical method for measurement of the parameters in the Wiener nonlinear model. Schetzen's book [20], published in 1980, is still the principal reference for the Wiener nonlinear theory today.

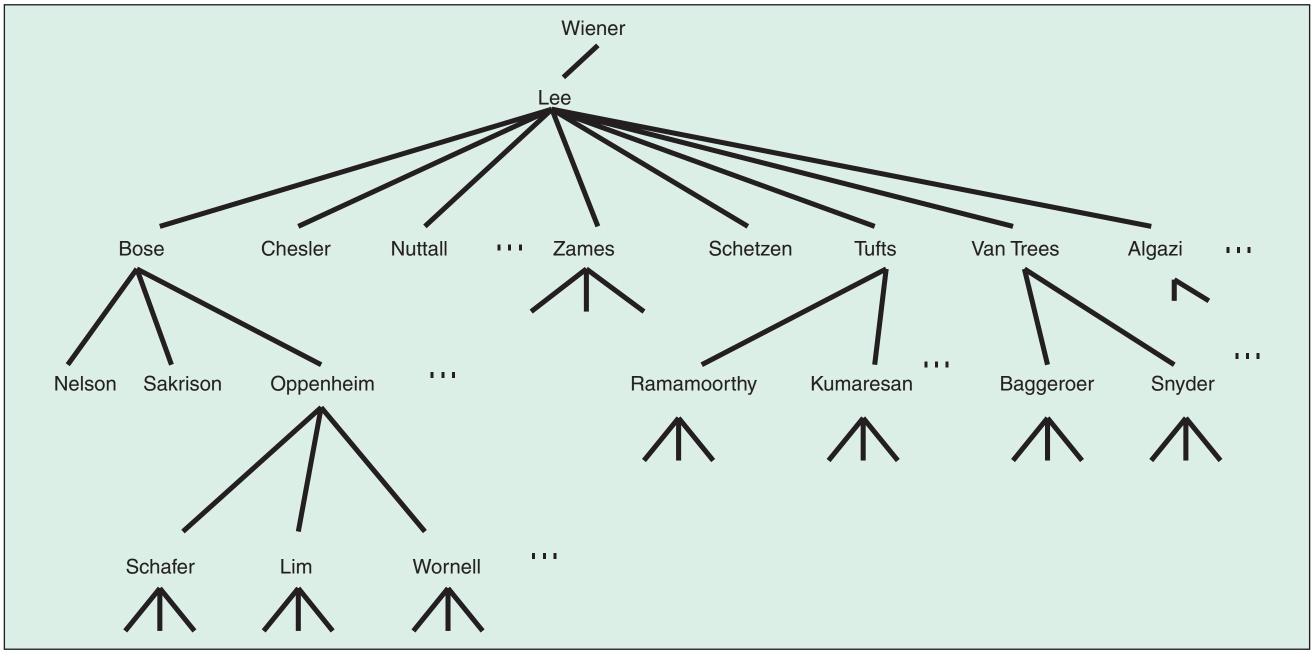
The Later Years

The Statistical Theory of Communication Group continued to work in areas to expand the Wiener theory and in many other areas as well. A partial list of reports published by the group shown in Table 1 hints about the breadth of activity. It also shows the names of many well-known researchers associated with the group as students and faculty.

Bose was a key member of the group, and his work with his students proceeded along at least two lines. First, he was persuaded by Wiener to become project manager for a joint effort of Harvard, MIT, and Massachusetts General Hospital to develop an electromechanical prosthetic device based on Wiener's theories of cybernetics [26]. This device would connect to the nerve ends in the stump of the amputee and respond to the electrical signals that are produced by the nervous system when a person desires to move a limb. The first such experimental device, known as the "Boston arm," was indeed successful and formed the basis for Ph.D. work for at least one member of the group (R. Alter, 1966) [27]. Second, Bose was working to develop his ideas in sound reproduction. Although none of the work seems to have appeared either in RLE reports or Ph.D. dissertations, the work was definitely innovative and led to the founding of the research-oriented Bose Corporation, one of the most successful firms in the audio and acoustics industry today.

Perhaps one of Lee's most famous students was Harry L. Van Trees. Van Trees' three volumes, *Detection, Estimation, and Modulation Theory* [28], written while he was at MIT are legendary, as are his classes and short courses. (The long-awaited fourth volume on array processing has recently been released by Wiley. In Van Trees' typical manner to thoroughly cover a topic, this work has grown from an originally intended short monograph to a volume of some 1,400 pages.) Van Trees, the late David Sakrison, and others joined the group as students and became faculty members after graduation. They helped to take the statistical theory of communication in new directions that could be applied to radar, sonar, and other related areas. This added fire to the activity and in the years from 1961 through 1968 a tremendous number of new students and faculty had joined the group. Projects that fell under the umbrella of the group varied widely, and by the mid-1960s the group's activities had expanded considerably beyond Lee's original focus. Wiener, who died in 1964, was no longer a significant force. However, the new activities were taking the group into important new directions. For example, Van Trees, whose thesis dealt with the application of the Volterra-Wiener methods in nonlinear control [29], became more interested in detection and estimation theory and began to develop those interests. This eventually led to the birth of a new research group. In the world at large, the fast Fourier transform was being discovered and the seeds were planted for digital signal processing. Alan Oppenheim joined as a student and later as a professor and became a mentor and force in his area much like Lee had been in his own. Fig. 10 shows a partial "family tree" of students and advisors

Lee's interest was primarily in the areas of what today would be called statistical signal processing and system theory.



▲ 10. "Family tree" for the Statistical Theory of Communication Group showing students and advisors. (Courtesy of the MIT Museum.)

Table 1. Technical Reports Published by the RLE Group on Statistical Theory of Communication (1954-1969).

Report Number	Author	Title (Year)
TR-309	A.G. Bose	A Theory of Nonlinear Systems (1956)
TR-310	R.E. Wernikoff	Outline of Lebesgue Theory: A Heuristic Introduction (1957)
TR-311	K.H. Powers	A Unified Theory of Information (1956)
TR-317	C.S. Lorens	Theory and Applications of Flow Graphs (1956)
TR-326	H.E. White	An Analog Probability Density Analyzer (1957)
TR-330	J.Y. Hayase	Properties of Second-Order Correlation Functions (1957)
TR-331	R.E. Wernikoff	A Theory of Signals (1958)
TR-343	A.H. Nuttall	Theory and Application of the Separable Class of Random Processes (1958)
TR-345	M.B. Brilliant	Theory of The Analysis of Nonlinear Systems (1958)
TR-355	D.A. George	Continuous Nonlinear Systems (1959)
TR-356	I.M. Jacobs	Connectivity in Probabilistic Graphs (1959)
TR-366	D.A. Chesler	Nonlinear Systems With Gaussian Inputs (1960)
TR-368	D.W. Tufts	Design Problems in Pulse Transmission (1960)
TR-370	G. Zames	Nonlinear Operators for System Analysis (1960)
TR-371	A.D. Hause	Nonlinear Least-Squares Filtering and Frequency Modulation (1960)
TR-378	K.L. Jordan, Jr.	Discrete Representations of Random Signals (1961)
TR-390	M. Schetzen	Some Problems in Nonlinear Theory (1962)
TR-391	D.J. Sakrison	Application of Stochastic Approximation Methods to System Optimization (1962)
TR-420	V.R. Algazi	A Study of the Performance of Linear and Nonlinear Filters (1964)
TR-429	J.D. Bruce	Optimum Quantization (1965)
TR-432	A.V. Oppenheim	Superposition in a Class of Nonlinear Systems (1965)
TR-441	A.M. Bush	Some Techniques for the Synthesis of Nonlinear Systems (1966)
TR-444	R.B. Parente	Functional Analysis of Systems Characterized by Nonlinear Differential Equations (1966)
TR-445	T.G. Kincaid	Time-Domain Analysis of Impulse-Response Trains (1967)
TR-446	R. Alter	Bioelectric Control of Prostheses (1966)
TR-456	D.E. Nelsen	Statistics of Switching-Time Jitter for a Tunnel Diode Threshold-Crossing Detector (1967)
TR-459	A.B. Baggeroer	A State-Variable Approach to the Solution of Fredholm Integral Equations (1967)
TR-461	M.E. Austin	Decision Feedback Equalization for Digital Communication over Dispersive Channels (1967)
TR-466	R.W. Schafer	Echo Removal by Discrete Generalized Linear Filtering (1969)

"descended" from Wiener. Wiener of course had other students, but none had such impact on the course of electrical engineering as Lee and his students.

Throughout the period covered in this article, there were others at MIT and within RLE doing important work in communication theory who were not members of the group headed by Lee. This included many well-known researchers such as Wilbur Davenport (although Davenport had some early association with the group) and others (see "Author's Note"). Our study has not included these other groups in order to focus on the special relation that Lee's group had with Wiener. This dispersion of activities within RLE, however, as well as the broadening of activities within the Statistical Theory of Communication Group naturally led to a need for reorganization. During the late 1960s several prominent members left Lee's group for other areas so that after Lee's retirement in 1969 the Statistical Theory of Communication Group ceased to exist. Its importance in the early years as well as its function as a incubator for many new activities cannot be underestimated however.

Conclusion

Although Lee was clear in his purpose of developing Wiener's ideas, he probably could not have anticipated the effect that the group would have on the field of electrical engineering. The effect of the group was so broad that a large fraction of the persons now working in signal processing or communication engineering can trace their intellectual genealogy to someone who was a member of the group. In other words, many of those who are not students of a member of the group are students of students … of students of Lee and in this sense are directly descended from Wiener.

Although Lee is probably most remembered as a great teacher and mentor of graduate students, some of his contributions to research both as a leader and through personal accomplishments are

- ▲ development of the "Lee-Wiener" network and introduction of the Hilbert transform to electrical engineers
- ▲ practical development of the Wiener theory of optimal linear systems, exposition of the theory for electrical engineers, and application to engineering problems
- ▲ development of machines for calculation of the first- and second-order correlation functions for signals and exposition of properties of correlation
- ▲ application of correlation to detection of low-level periodic signals in noise
- ▲ recognition of the Wiener theory of nonlinear systems as an important contribution and practical development of the theory.

These contributions, and others, will not be forgotten.

Lee retired from MIT in 1969 and moved to Belmont, CA, where he remained until his death from leukemia in 1989. Although there were a few significant papers, his book on the statistical theory of communication remains as one of his most enduring contributions. After Lee's retirement, Schetzen wanted him to collaborate on the book on the nonlinear theory [20] but Lee declined. Lee and his wife had no children, so there are no descendants. Lee was soft spoken and not willing to indulge in the politics that are so often required for success in promoting good ideas. As such, his name is on the verge of being lost in history. No biography exists and the written information pertaining to his personal life is very scarce. Still students and colleagues that knew him recall him as a great teacher and mentor. He did not engage in giving great technical advice to his students. Rather, he created an environment where students could learn and develop confidence in their own abilities. Bose recalls when (as a young faculty member) he had been trying to license his ideas of sound reproduction to some prominent audio companies without success. When discussing that with Lee, Lee told him of his days as an antique dealer in China. He said, "The dream of every antique dealer is that someday something extremely valuable will come into his hands, and when it does, he hopes that he will recognize it and not give it away." Although Lee said nothing more, Bose knew that the thing of extreme value that Lee was referring to were the ideas that he had developed in acoustics. He knew then that he should stop trying to license it to others and form his own company. The rest, as we say, is history.

Bose's entrance into the Statistical Theory of Communication Group was a significant turning point as it set the direction for the group's exploration of the Wiener nonlinear theory that was to continue for years afterward.

Acknowledgments

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