Quiz: Convolutions

This quiz is designed to test your knowledge of convolutions of 2π-periodic functions.

In this entire quiz, the expression $f * g$ denotes convolution of f and g, while the expression fg denotes the pointwise product of f and g. The expression $\hat{f}$ denotes the Fourier transform of f, thus $\hat{f}(n)$ is the n^{th} Fourier coefficient of f.

Discuss this quiz

(Key: correct, incorrect, partially correct.)

1. Let f and g be continuously differentiable 2π-periodic functions. The derivative $(f * g)'$ of the convolution $f * g$ is given by
   - A. $(f') * g$
   - B. $f * (g') + (f') * g$
   - C. $(f') * (g')$
   - D. $(g') * (f')$
   - E. $f * (g')$
   - F. In general, there is no simple formula available.

2. Let f and g be continuously differentiable 2π-periodic functions, and let n be an integer. The n^{th} Fourier coefficient $\widehat{f * g}(n)$ of the convolution $f * g$ is given by
   - A. $\hat{f} * g(n)$
   - B. $\hat{f}(n)\hat{g}(n)$
   - C. $\hat{f} * \hat{g}(n)$
   - D. $\hat{f}(n) + \hat{g}(n)$
   - E. $\hat{f}(n)g + f\hat{g}(n)$
   - F. In general, there is no simple formula available.

3. Let f and g be continuously differentiable 2π-periodic functions. The average value of $f * g$ is equal to
   - A. The difference between the average value of f and the average value of g.
   - B. The average of the average value of f and the average value of g.
   - C. The convolution of the average value of f and the average value of g.
   - D. The product of the average value of f and the average value of g.
   - E. The sum of the average value of f and the average value of g.
   - F. In general, there is no simple formula available.

4. Let f, g, h be continuous 2π-periodic functions. The expression $f * (g + h)$ can also be written as
   - A. $(f + g) * h$
   - B. $f * h + g * h$
   - C. $g * f + f * h$
   - D. $f * (g * h)$

E. $g * (f + h)$
F. None of the above.

5. Let f, g, h be continuous 2π-periodic functions. The expression $(f + 3h) * (2g)$ can also be written as
A. $2(f * g) + 6(h * g)$
B. $6 * f * g * h$
C. $2 * f * g + 3 * h * g$
D. $(2f) * g + (3h) * g$
E. $6 * h * g + 2 * f * g$
F. None of the above.

6. Let f, g, h be continuous 2π-periodic functions. The expression $f * (gh)$ can also be written as
A. $(fg) * h$
B. $(f * g)(f * h)$
C. $f * g + f * h$
D. $f(g + h)$
E. $f(g * h)$
F. None of the above.

7. Let f, g be 2π-periodic functions. If f is continuously differentiable, and g is twice continuously differentiable, then the best we can say about $f * g$ is that it is 2π-periodic and
A. Riemann integrable.
B. Piecewise continuous.
C. Continuous.
D. Continuously differentiable.
E. Twice continuously differentiable.
F. Three times continuously differentiable.
G. Infinitely differentiable.

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A. Riemann integrable.
B. Piecewise continuous.
C. Continuous.
D. Continously differentiable.
E. Twice continuously differentiable.
F. Three times continuously differentiable.
G. Infinitely differentiable.

9. Let f, g be 2π-periodic functions. If f is continuously differentiable, and g is twice continuously differentiable, then the best we can say about fg is that it is 2π-periodic and
A. Riemann integrable.
B. Piecewise continuous.
C. Continuous.

D. Continously differentiable.
E. Twice continuously differentiable.
F. Three times continuously differentiable.
G. Infinitely differentiable.

10. Let f, g be 2π-periodic functions. If f and g are Riemann integrable, then the best we can say about $f * g$ is that it is 2π-periodic and
    A. Bounded.
    B. Riemann integrable.
    C. Piecewise continuous.
    D. Continuous.
    E. Continously differentiable.
    F. Twice continuously differentiable.
    G. Infinitely differentiable.

11. Let f, g be 2π-periodic functions. If f and g are Riemann integrable, then the best we can say about fg is that it is 2π-periodic and
    A. Bounded.
    B. Riemann integrable.
    C. Piecewise continuous.
    D. Continuous.
    E. Continously differentiable.
    F. Twice continuously differentiable.
    G. Infinitely differentiable.

12. Let f be a 2π-periodic function, and let 1 be the constant function 1. Then $f * 1$ is
    A. The constant function with value equal to the mean of f.
    B. The value of $f(x)$ at the point $x = 0$.
    C. The constant function with value equal to $f(1)$.
    D. 0.
    E. The same function as f.
    F. The constant function 1.

13. Let f be a continuous 2π-periodic function, and let K_n be a family of approximations to the identity (a.k.a. good kernels). Which of the following statements is true?
    A. The functions $f * K_n$ converge to zero as n goes to infinity.
    B. For each x, $K_n(x)$ converges to $f(x)$ as n goes to infinity.
    C. For each n, $f * K_n(x)$ converges to $f(x)$ as x goes to infinity.
    D. For each x and each n, we have $f * K_n(x) = f(x)$.
    E. For each x, $f * K_n(x)$ converges to $f(x)$ as n goes to infinity.
    F. For each x, $f * K_n(x)$ converges to 1 as n goes to infinity.

Score: 0/130

Expand all answers