



$$V_+ = \frac{R_3}{R_3 + R_4} V_0 \quad (1)$$

$$\frac{V_{in} - V_-}{R_1} = \frac{V_- - V_0}{R_2}$$

$$\Rightarrow V_- = \frac{R_2}{R_1 + R_2} V_{in} + \frac{R_1}{R_1 + R_2} V_0 \quad (2)$$

$$I = \frac{1}{R_1 + R_2} (V_{in} - V_0) \quad (3)$$

Define:  $\beta := \frac{R_3}{R_3 + R_4}$ ,  $\gamma := \frac{R_1}{R_1 + R_2}$  Note that both  $0 < \beta < 1$  and  $0 < \gamma < 1$

Linear region:  $V_+ = V_-$  and  $-E_s \leq V_0 \leq E_s$

from (1) and (2)  $\Rightarrow \beta V_0 = \gamma V_0 + (1 - \gamma) V_{in}$

$$\Rightarrow V_0 = \frac{1 - \gamma}{\beta - \gamma} V_{in}$$

Define  $\bar{V} := \left| \frac{\beta - \gamma}{1 - \gamma} \right| E_s \Rightarrow$  In linear region  $-\bar{V} \leq V_{in} \leq \bar{V}$

from (3)  $\Rightarrow I = \frac{1}{R_1 + R_2} \left( 1 - \frac{V_0}{V_{in}} \right) V_{in}$

$$\Rightarrow I = \frac{1}{R_1 + R_2} \left( 1 - \frac{1 - \gamma}{\beta - \gamma} \right) V_{in} = \frac{1}{R_1 + R_2} \frac{\beta - 1}{\beta - \gamma} V_{in}$$

$$\Rightarrow I = \frac{1}{R_1 + R_2} \frac{\beta - 1}{\beta - \gamma} V_{in}$$

Define  $G := \left| \frac{1}{R_1 + R_2} \cdot \frac{\beta - 1}{\beta - \gamma} \right|$

+ sat region :  $V_+ > V_-$  &  $V_o = E_s$

from (1) & (2)  $\Rightarrow \beta V_o > \gamma V_o + (1-\gamma)V_{in}$

$$\Rightarrow \beta E_s > \gamma E_s + (1-\gamma)V_{in} \Rightarrow V_{in} < \frac{\beta - \gamma}{1 - \gamma} E_s$$

$$\text{from (3)} \Rightarrow I = \frac{1}{R_1 + R_2} (V_{in} - E_s)$$

- sat region :  $V_+ < V_-$  &  $V_o = -E_s$

from (1) & (2)  $\Rightarrow \beta V_o < \gamma V_o + (1-\gamma)V_{in}$

$$\Rightarrow -\beta E_s < -\gamma E_s + (1-\gamma)V_{in} \Rightarrow V_{in} > -\frac{\beta - \gamma}{1 - \gamma} E_s$$

$$\text{from (3)} \Rightarrow I = \frac{1}{R_1 + R_2} (V_{in} + E_s)$$

Cases :  $\beta > \gamma$

