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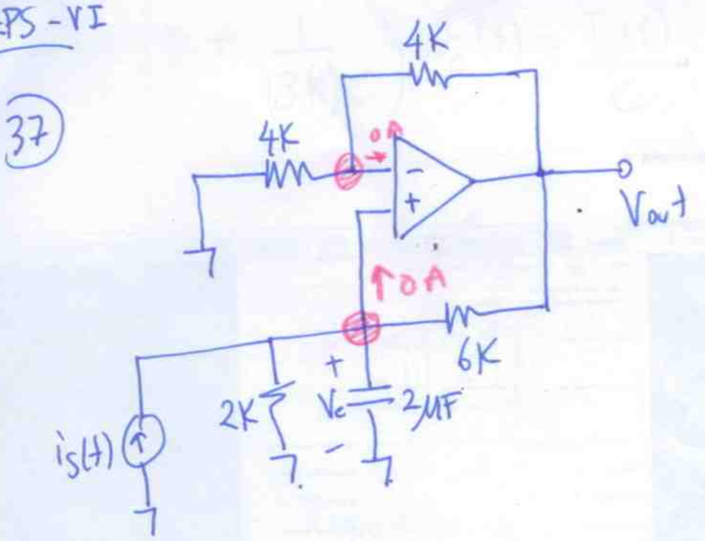
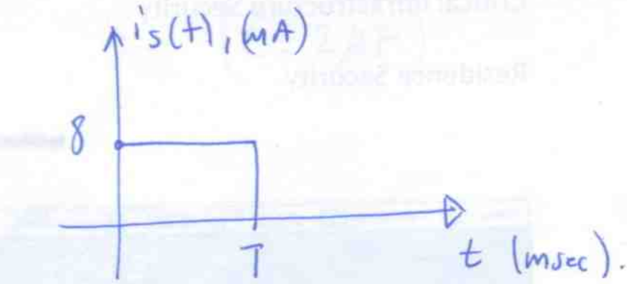




$E_{sat} = 16\,V$, Op-amp Ideal.

$\vartheta_c(0) = 0$

$\vartheta_c(T) = 12\,V$.

Since Op-amp is ideal, $(R_{in} = \infty)$; the following equations are valid in all operational regions of op-amp.

KCL at V_+: $\boxed{\dfrac{V_+}{2K} + \dfrac{V_+ - V_{out}}{6K} + C\underbrace{\dot{\vartheta}_c(t)}_{\vartheta_c(t) = \vartheta_+(t)} = I_s(t)}$; (I)

KCL at V_-: $\dfrac{V_-}{4K} + \dfrac{V_- - V_{out}}{4K} = 0 \longrightarrow \boxed{2V_- = V_{out}}$; (II)

The next step is the investigation of operational regions of op-amp

Case (A): Linear Region

① $|V_{out}| < 16 \xrightarrow{(II)} |V_-| < 8\,V \xrightarrow{②} |V_+| < 8\,V \rightarrow |\vartheta_c(t)| < 8\,V$

$V_+ = \vartheta_c(t)$

Condition for Linear Region

② $V_+ = V_-$

$\xrightarrow{(II)} V_{out} = 2V_- = 2V_+$ ($V_+ = V_c$) substitute in I'

$V_c \rightarrow \dfrac{V_+}{2K} + \dfrac{V_+ - V_{out}}{6K} + C\dot{\vartheta}_c = I_s(t) \longrightarrow \dfrac{V_c}{3K} + C\dot{\vartheta}_c = I_s(t) \longrightarrow$

$$\rightarrow \underbrace{\left(D + \frac{1}{(3K)C}\right)\vartheta_c(t) = \frac{I_s(t)}{C}}_{\text{Dynamic of Cap. in Linear Region}} \qquad (C = 2\mu F)$$

Case (B): +Sat Region

① $V_{out} = 16 \xrightarrow{(II)} V_- = 8V \xrightarrow{②} V_+ > V_-$ (V_c)

$\downarrow$

② $V_+ > V_-$

$\underbrace{V_c > 8V}_{\text{Cond. for +SAT.}}$

16V

$$(I) \rightarrow \frac{V_+}{2K} + \frac{V_+ - V_{out}}{6K} + C\dot{\vartheta}_c(t) = I_s(t)$$

$\downarrow$

$$\underbrace{\left(D + \frac{1}{(\frac{3}{2}K)C}\right)\vartheta_c(t) = \frac{I_s(t)}{C} + \frac{8}{(3K)C}}_{\text{Dynamics of Cap. in +SAT Region.}}$$

Case (C): −SAT Region

① $V_{out} = -16\,V \rightarrow V_- = -8V \xrightarrow{②} V_+ < -8$

② $V_+ < V_-$

$\downarrow$

$\underbrace{V_c < -8}_{\text{Cond. for −SAT}}$

−16V

$$(I) \rightarrow \frac{V_+}{2K} + \frac{V_+ - V_{out}}{6K} + C\dot{\vartheta}_c = I_s(t)$$

$\downarrow$

$$\underbrace{\left(D + \frac{1}{(\frac{3}{2}K)C}\right)\vartheta_c(t) = \frac{I_s(t)}{C} - \frac{8}{(3K)C}}_{\text{Dynamics of Cap in −SAT Region.}}$$

c

Now, we are ready to start integrating all pieces of analysis together. 3)

Step 1 $v_c(0^-) = 0V \longrightarrow v_c(0^+) = 0V$

from Case A

Op-Amp in Linear Region at $t=0^+$

$$\left(D + \frac{1}{(3K)C}\right) v_c(t) = \frac{I_s(t)}{C}$$ } Dynamics of Op-amp in Linear Region.

$$\dot{v}_c(0^+) = \underbrace{\frac{I_s(0^+)}{C}}_{\text{positive}} - \frac{1}{(3K)C} \overset{0}{v_c(0^+)}$$

Cap. Voltage increases at $t=0^+$

~~Solving~~

Capacitor voltage keeps on increasing until T'th sec; ~~or~~ provided that op-amp doesnot change its state from Linear to smth. else.

Solving for $v_c(t)$

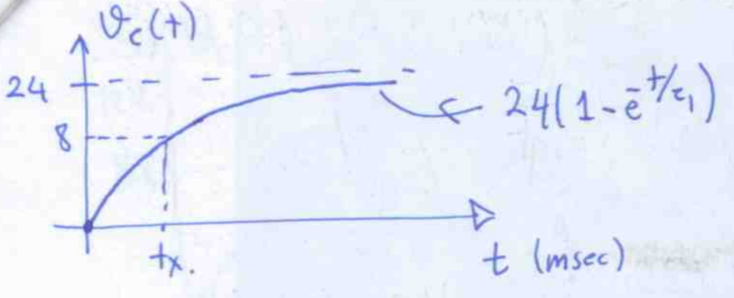
$$\left(D + \frac{1}{(3K)C}\right) v_c(t) = \frac{I_s(t)}{C} = 4000$$

8mA, 2μF

$\tau_1 = 6$ msec.

$$v_c(t) = 24 - 24e^{-t/\tau_1}, \quad t > 0$$

(provided that op-amp is in linear

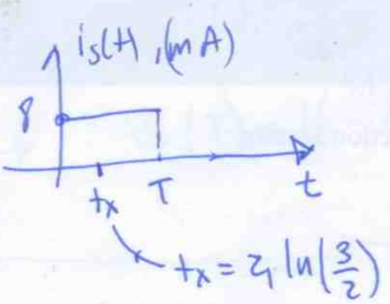


$\vartheta_c(t)$ approaches 24 V asymptotically as ~~...~~ t increases.

But, note that linear region ~~assum~~ condition is violated, when $\vartheta_c(t) \geq 8$; so the graph given above is valid until

$\vartheta_c(t_x) = 8 \rightarrow t_x = \tau_1 \ln\left(\frac{3}{2}\right)$. $\quad (\tau_1 = 6\,\text{msec})$.

Also note that



where $\vartheta_c(T) = 12V$ So input of 8 mA is provided until cap. voltage is 12V.

For $8 \leq \vartheta_c(t) < 12$ $\left(\text{or } t_x \leq t \leq T\right) \rightarrow$ Op-amp in +SAT $\rightarrow$

$$\rightarrow \vartheta_c(t_x^+) = 8V \quad \text{and} \quad \Big(D + \frac{1}{\underbrace{\frac{3}{2}KC}_{\tau_2 = 3\,\text{msec}}}\Big)\vartheta_c(t) = \frac{I_s(t)}{C} + \frac{8}{\frac{3}{2}KC}$$

$\cancel{C}$ ← 2μF, 8mA

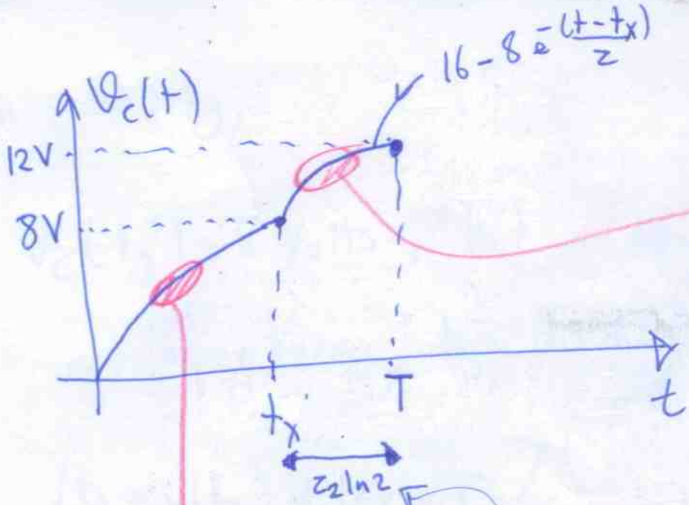
Op-amp Dynamics in +SAT

$$\left(D + \frac{1}{\tau_2}\right)\vartheta_c(t) = \frac{4}{3}\cdot 4000$$

$$\vartheta_c(t) = \overbrace{\frac{4}{3}\cdot 4000\,\tau_2}^{16V} - 8e^{-\frac{(t-t_x)}{\tau_2}}, \quad t \geq t_x.$$

$$= 16 - 8e^{-\frac{(t-t_x)}{\tau_2}}, \quad t \geq t_x.$$

Note: $\vartheta_c(t_x^+) = 8$ is satisfied!



Cap. Charges in +SAT Region ($\tau_2 = 3\,msec$)

Cap. Charges in Linear Region (Op-amp)
(Time-constant, $\tau_1 = 6\,msec$)

To find T $\longrightarrow$ $v_c(T) = 16 - 8e^{-\frac{(T-t_x)}{\tau_2}}$ $\longrightarrow$ $T = t_x + \tau_2 \ln 2$

When $t > T$, the input is OFF! ($i_s(t)=0$), but $v_c(T^+) = 12V$

$\downarrow$

Op-amp is in +SAT

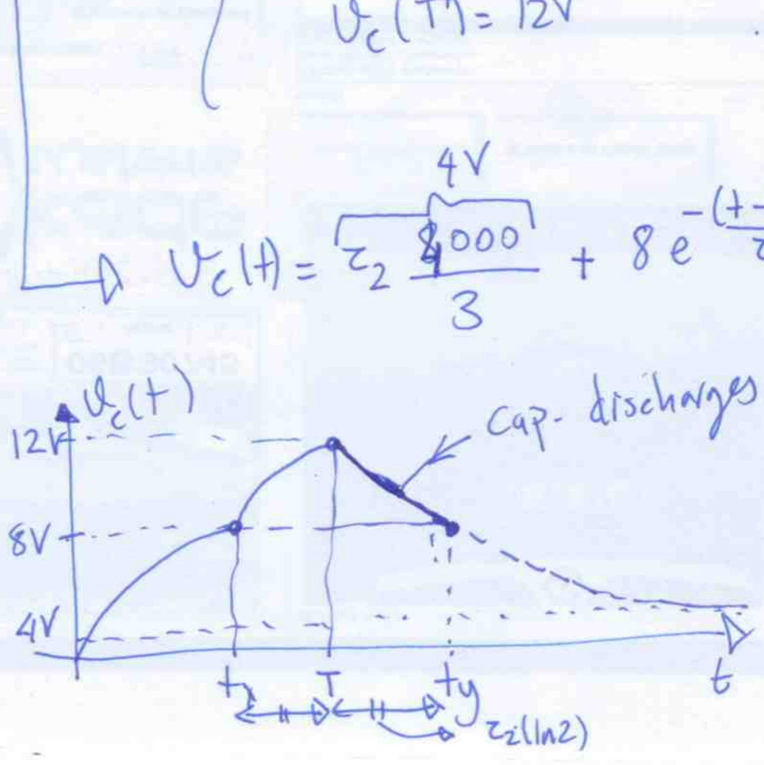
$$\begin{cases} \left(D + \frac{1}{\tau_2}\right) v_c(t) = \frac{8}{(3K)C} \\ v_c(T^+) = 12V \end{cases}$$

3msec

(2μF)

$$v_c(t) = \overbrace{\tau_2 \frac{8000}{3}}^{4V} + 8e^{-\frac{(t-T)}{\tau_2}}, \quad t > T.$$

Cap. discharges in +SAT. ($\tau_2 = 3\,msec$).



$v_c(t_y) = 8V$

$t_y = T + (\ln 2)\tau_2$

When $t > t_y$,

$\vartheta_c(t_y^+) = 8$ Volts, but op-amp enters into linear region and cap continues to discharge in linear region. →

$$\rightarrow \left(D + \frac{1}{\tau_1}\right)\vartheta_c(t) = 0 \qquad \vartheta_c(t) = 8e^{-\frac{(t-t_y)}{\tau_1}}, \quad t > t_y.$$

$$\vartheta_c(t_y^+) = 8V$$