MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 201 Circuit Theory I

Midterm Examination 1

November 4, 2013

Duration: 120 minutes

Q1	15 pts	
Q2	15 pts	
Q3	20 pts	
Q4	15 pts	
Q5	15 pts	
Q6	20 pts	
Total	100 pts	

Name:

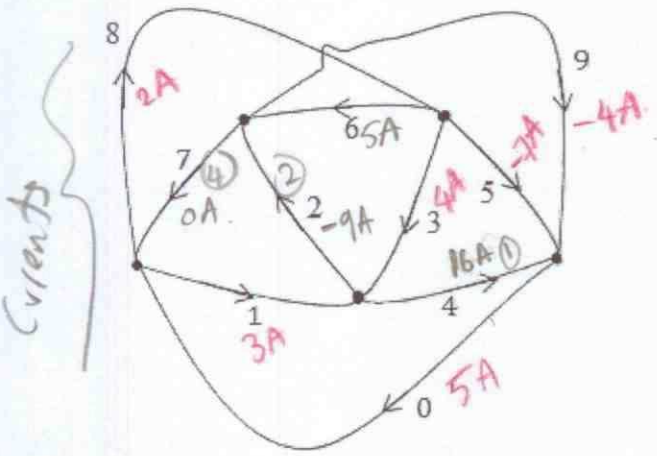
Surname:

ID No: Solutions

Section:

Signature:

Question 1 (15 pts)



The figure on the left shows the oriented graph of a lumped circuit.

Some of the branch currents (in Amperes) and voltages (in Volts) are given below.

Find the missing currents and voltages, and write them in the table.

$i_0 = 5A$	$i_1 = 3A$	$i_2 = -9A$	$i_3 = 4A$	$i_4 = 16A$	$i_5 = -7A$	$i_6 = 5A$	$i_7 = 0A$	$i_8 = 2A$	$i_9 = -4A$
$v_0 = 4V$	$v_1 = 3V$	$v_2 = 6V$	$v_3 = 5V$	$v_4 = -7V$	$v_5 = -2V$	$v_6 = 11V$	$v_7 = -9V$	$v_8 = -2V$	$v_9 = -13V$

Currents:

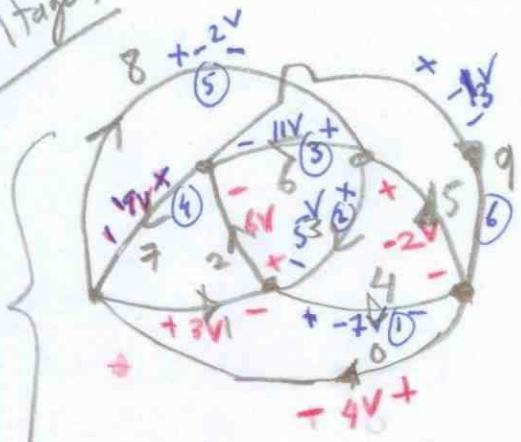
(1) From given currents $i_4 = -i_5 - i_9 + i_0 = 16A.$

(2) $i_2 = i_3 - i_4 + i_1 = -9A.$

(3) $i_6 = i_8 + i_3 - i_5 = 5A.$

(4) $i_7 = i_2 + i_6 - i_9 = 0A.$

Voltages:



(1) $v_4 = -v_3 - v_0 = -7V$

(2) $v_3 = v_5 - v_4 = 5V$

(3) $v_6 = v_3 + v_2 = 11V$

(4) $v_7 = -v_2 - v_1 = -9V$

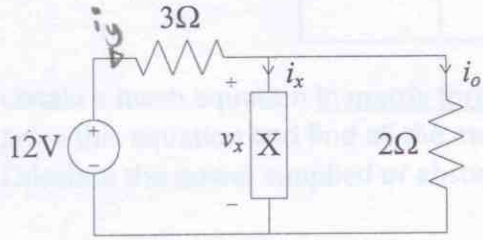
(5) $v_8 = -v_7 - v_6 = -2V$

(6) $v_9 = -v_2 + v_4 = -13V$

Question 2 (15 pts)

Consider the circuit shown in the figure. The element "X" is a resistive element. The current through the 2Ω resistor is measured to be $i_o = 3\cos t$ A.

a) Determine the current i_x.
b) Determine the power given to the element "X".
c) Is the element "X" time invariant? Why?
d) Is the element "X" passive? Why?



$v_o = 6\cos t$ volts

a) $i_x = i_y - i_o = \frac{12 - v_o}{3} - i_o = 4 - 5\cos t$ A.

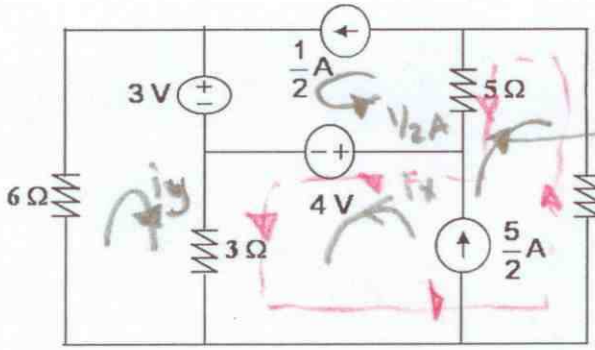
b) $p_x(t) = v_x \cdot i_x = (6\cos t)\cdot(4 - 5\cos t)$ A.

c) Time-Varying: X is a resistive element, for DC input a time-invariant resistance would result in a DC valued current and voltage. Since this is not the case, X is time-varying.

d) Active: A resistive element can not store energy; hence if $P_x(t) < 0$ $\exists\, t$ for some $(i_x(t), v_x(t))$ pair then the component must be active

$P_x(0) = -6$ watts → Component is active.

Question 3 (20 pts) Consider the following circuit.



a) Obtain a mesh equation in matrix form.
b) Solve this equation and find all the mesh currents.
c) Calculate the power supplied or absorbed by the 4V source.

a) KVL around i_y: $6i_y + 3 + 3(i_y + i_x) = 0$

KVL around supermesh: $3(i_x + i_y) + 4\left(i_x - \frac{5}{2}\right) + 5\left(-\frac{1}{2} + i_x - \frac{5}{2}\right) + 4 = 0$

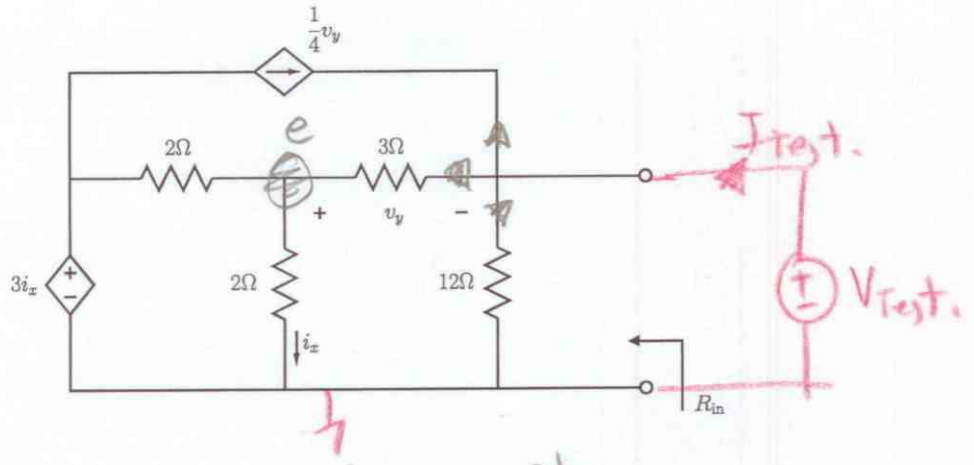
$$\left[\begin{array}{c|c} 3 & 9 \\ \hline 12 & 3 \end{array}\right] \begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} -3 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} +2A \\ -1A \end{bmatrix}$$

b) $i_y = -1A$, $i_x = 2A$, $i_{1/2A} = \frac{1}{2}A$, $\frac{5}{2} - i_x = \frac{1}{2}A$.

c) $P_{4V}^{absorbed} = 4V \times (i_x - \frac{1}{2}) = 4 \times \frac{3}{2} = 6$ Watts (absorbed)

Question 4 (15 pts)



Find the input resistance R_{in}.

$i_x = e/2$

KCL at e :

$$\frac{e}{2} + \frac{e-3i_x}{2} + \frac{e-v_{Test}}{3} = 0$$

$$\left(\frac{1}{2}+\frac{1}{2}-\frac{3}{4}+\frac{1}{3}\right)e = \frac{v_{Test}}{3} \rightarrow e = \frac{v_{Test}/3}{7/12}$$

$$e = v_{Test}\cdot\frac{4}{7}$$

$(e - V_{Test})$

$$I_{Test} = -\frac{1}{4}v_y + \frac{v_{Test}-e}{3} + \frac{v_{Test}}{12}$$

$v_{Test}\frac{4}{7}$

$$= \frac{2}{3}v_{Test} - \frac{7}{12}e$$

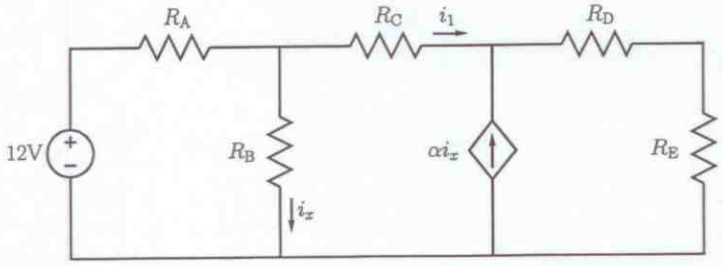
$$= \frac{2}{3}v_{Test} - \frac{7}{12}\cdot\frac{4}{7}v_{Test}$$

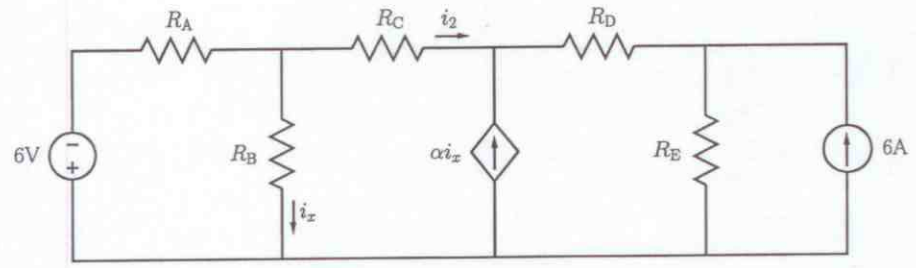
$$= \frac{1}{3}v_{Test}$$

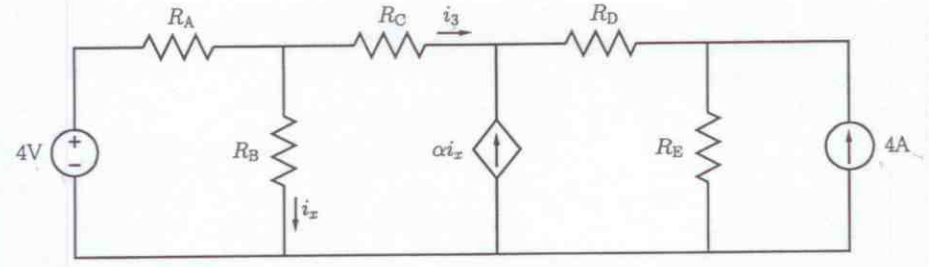
$$R_{IN} = \frac{v_{Test}}{I_{Test}} = 3\,\Omega$$

Question 5 (15 pts)

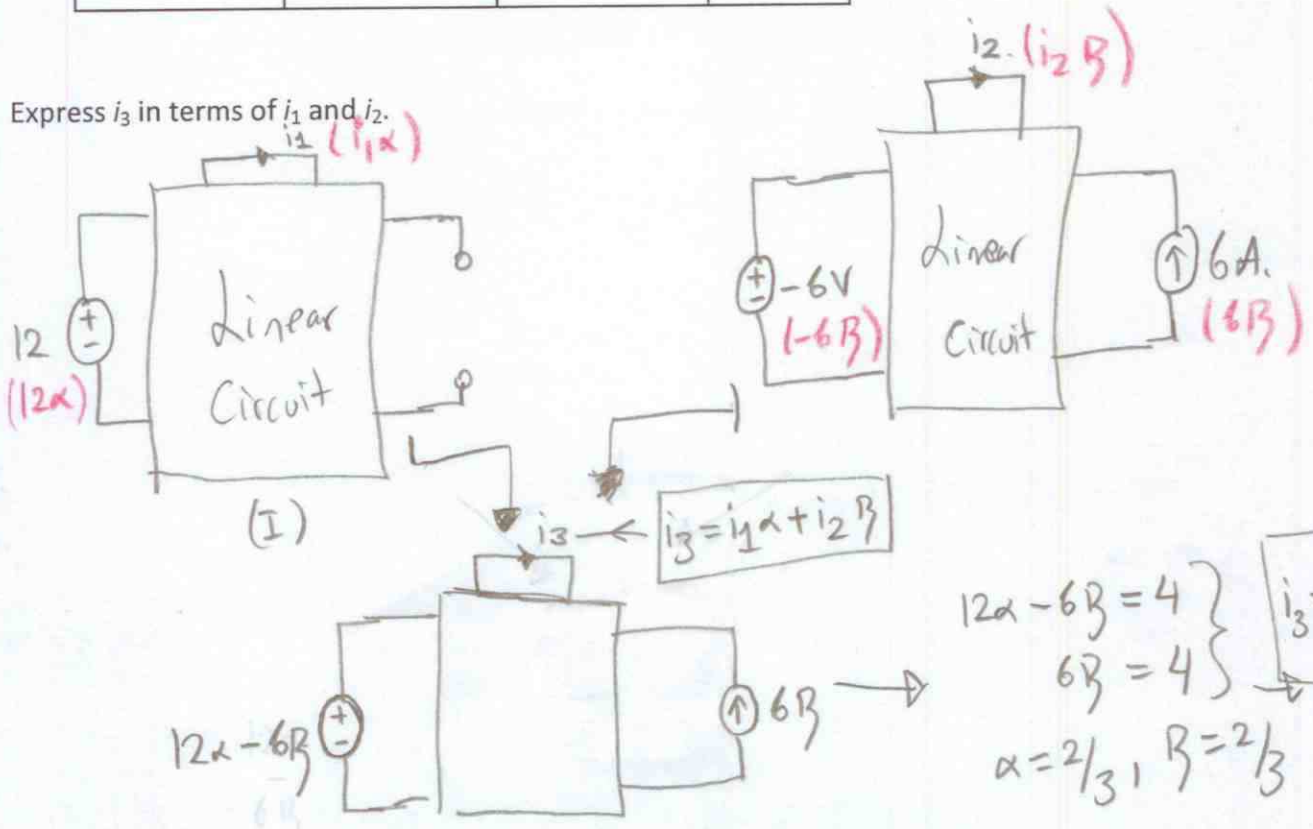
Consider the three circuits below.



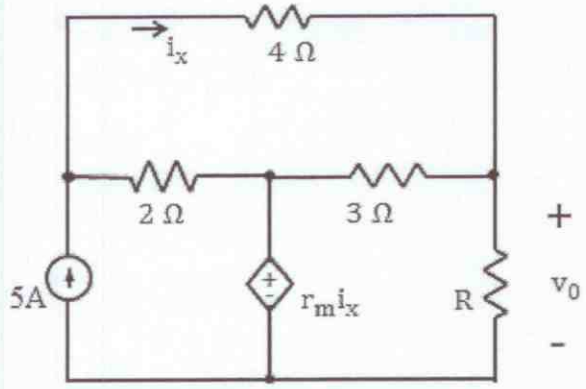




Express i_3 in terms of i_1 and i_2.



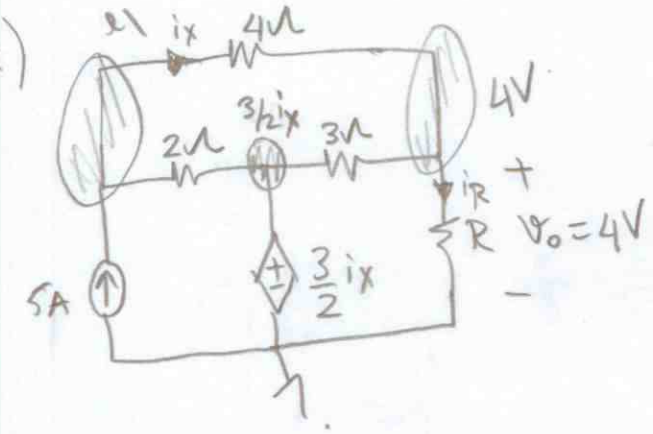
Question 6 (20 pts) In the given circuit, transfer resistance r_m is in the range $(-\infty, \infty)$.



a) Given $v_0 = 4V$, $r_m = 3/2\ \Omega$, find R.

b) r_m is so adjusted that $v_0 = 0$. Find this value of r_m.

a)



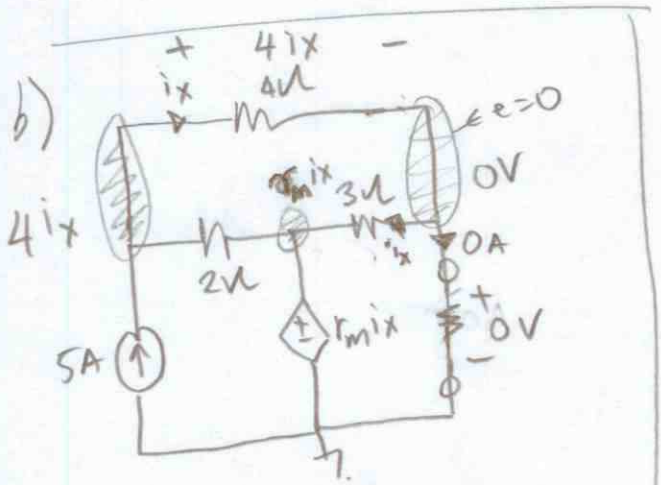
$i_x = \frac{e_1 - 4}{4}$

KCL at e_1: $\frac{e_1 - 4}{4} + \frac{e_1 - \frac{3}{2}i_x}{2} - 5 = 0$

$\left(\frac{1}{4} + \frac{1}{2} - \frac{3}{16}\right) e_1 = 1 - \frac{3}{4} + 5$

$e_1 = \frac{21/4}{9/16} = \frac{28}{3} V$

b)



KCL at $4i_x$ node:

$\frac{4i_x - 0}{4} + \frac{4i_x - r_m i_x}{2} - 5 = 0$ (I)

KCL at e=0:

$\frac{0 - 4i_x}{4} + \frac{0 - r_m i_x}{3} = 0$ (II)

$i_R = \frac{\frac{3}{2}i_x - v_0}{3} + \frac{e_1 - v_0}{4}$

$= -\frac{7}{12}v_0 + \frac{1}{2}i_x + \frac{e_1}{4}$ ← $\frac{e_1 - 4}{4}$

$= -\frac{7}{12}v_0 + \frac{3}{8}e_1 - \frac{1}{2}$ ← $\frac{28}{3}$, ← 4

$= -\frac{7}{3} + \frac{7}{2} - \frac{1}{2}$

$i_R = \frac{2}{3} A. \rightarrow R = \frac{v_0}{i_R} \overset{v_0 = 4}{=} 6\Omega$

Solve for r_m and i_x

From (II): $r_m = -3\ \Omega$

From (I): $i_x = \frac{10}{7} A$