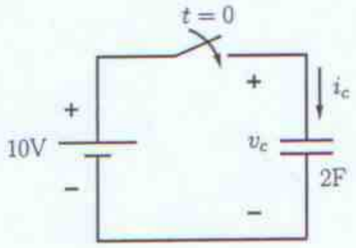
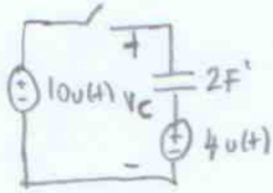
Question 3 (20 pts)

(a)



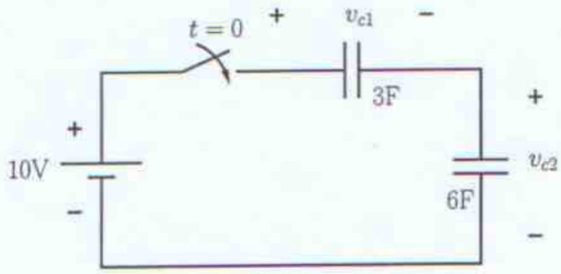
Given $v_c(0^-) = 4$ V. Find $i_c(t)$.



$i_c(t) = C \frac{d\vartheta_c}{dt}$ (2F; $6u(t)$; empty)

$i_c(t) = 12\delta(t)$

(b)



Given $v_{c1}(0^-) = 3$ V, $v_{c2}(0^-) = 1$ V. Find $v_{c2}(t)$ for $t > 0$.

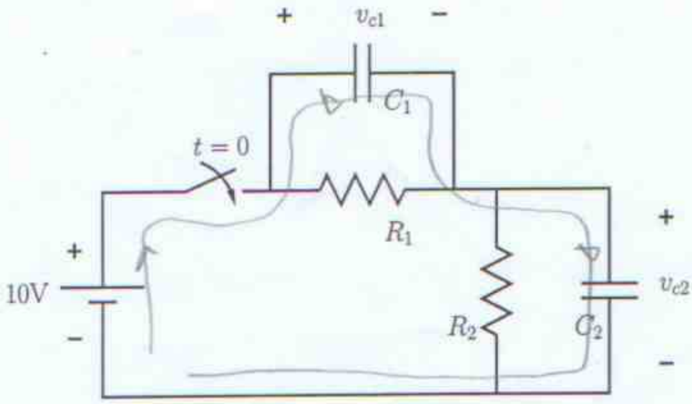
$i_c(t) = 12\delta(t)$

$\vartheta_{c_2}(t) = \vartheta_{c_2}(0^-) + \frac{1}{C_2}\int_{0^-}^{t} i_c(\tau)d\tau$

$= 1 + 2, \quad t > 0$

$= 3, \quad t > 0.$

(c)



Given $v_{c1}(0^-) = 0$, $v_{c2}(0^-) = 0$.

(i) Express $v_{c2}(0^+)$ and $v_{c2}(+\infty)$ in terms of the source voltage and R_1, R_2, C_1, C_2.

(ii) Given $v_{c2}(t) = 6 - 2e^{-t/6}$ V for $t > 0$, find suitable R_1, R_2, C_1, C_2 values.

i) $\vartheta_{c_2}(0^+) = \frac{1/C_2}{1/C_1 + 1/C_2} \cdot 10 = \frac{C_1}{C_1 + C_2} \cdot 10$, $\vartheta_{c_2}(\infty) = 10 \cdot \frac{R_2}{R_1 + R_2}$

ii) $\frac{R_2}{R_1 + R_2} = \frac{3}{5}$, $\frac{C_1}{C_1 + C_2} = \frac{2}{5}$, $\tau = (C_1 + C_2) \cdot R_1 \| R_2 = 6$

$R_2 = \frac{3}{2}R_1$, $C_2 = \frac{3}{2}C_1$, $\tau = \left(\frac{5}{2}C_1\right) \cdot R_1 \cdot \frac{3}{5} = \frac{C_1 R_1}{2} \cdot 3 = 6$

$R_1 = 1\Omega$
$R_2 = \frac{3}{2}\Omega$ (↑)
$C_1 = 4F$ (↓)
$C_2 = 6F$ (↓)

(d) Let R_1, R_2, C_2 be as in part **(c-ii)** and $v_{c2}(t) = 6$ V for $t > 0$. Find C_1.

$6 = \vartheta_{c_2}(0^+) = \vartheta_{c_2}(\infty) \rightarrow \frac{C_1}{C_1 + C_2} \cdot 10 = 10 \cdot \frac{R_2}{R_1 + R_2} = 6 \rightarrow \frac{C_1}{C_1 + C_2} = \frac{3}{5} \rightarrow C_1 = \frac{3}{2}C_2$

$C_1 = 9F$