13.10.2011

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EE 201

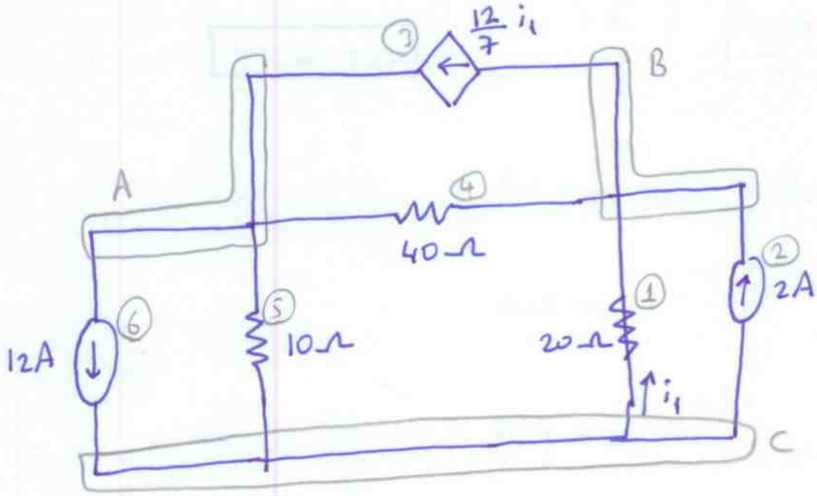
paper - 1/2

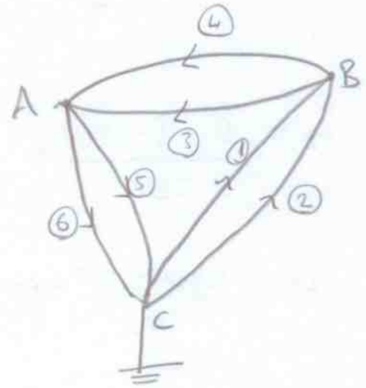
1741297

HOMEWORK #1

section: 3

1)





a) $A = \begin{bmatrix} 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 \end{bmatrix}$ $J = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix}$

① $A . J = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (KCL)

② $V = A^T . e = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \Rightarrow$ $V_1 = -e_B$, $V_2 = -e_B$, $V_3 = e_B - e_a$, $V_4 = e_B - e_a$, $V_5 = e_A$, $V_6 = e_A$

③ $J = G . V + J_s \xrightarrow{\text{multipy by } A} A . J = A . G . V + A . J_s$

using ② and ③ $A . J = A . G . A^T . e + A . J_s = 0$ (by # ①)

so; $e = -(AGA^T)^{-1} . A . J_s$

$$e = -\left(\begin{bmatrix} 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \right) . A . J_s$$

$$-\left(\frac{1}{40}\begin{bmatrix}5 & -1\\ -1 & 3\end{bmatrix}\right)^{-1} \cdot A \cdot J_s = -\frac{20}{7}\begin{bmatrix}3 & 1\\ 1 & 5\end{bmatrix}\begin{bmatrix}0 & 0 & -1 & -1 & 1 & 1\\ -1 & -1 & 1 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\ 2\\ 0\\ -\frac{3e_B}{35}\\ 0\\ 12\end{bmatrix}$$

$$e = \begin{bmatrix}e_A\\ e_B\end{bmatrix} = \begin{bmatrix}34 + \frac{6e_B}{35}\\ 2 - \frac{12e_B}{35}\end{bmatrix}\cdot\left(-\frac{20}{7}\right)$$

$\boxed{e_B = -280V}$

$\boxed{e_A = 40V}$

$i_1 = \dfrac{0-(-280)}{20} = 14A$

$i_2 = 2A$

$i_3 = \dfrac{-280-40}{40} = -8A$

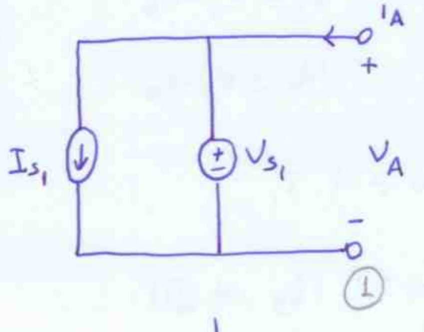
$i_4 = 24A$

$i_5 = \dfrac{40-0}{10} = 4A$

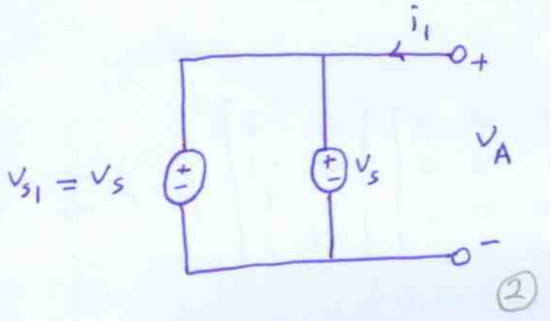
$i_6 = 12A$

b) Since the circuit in part a has on unique soln., we may change any of the circuit element with an appropriate one. In part A since $V_A - V_B = 40V$, if we change the 10Ω resistance with 40V voltage source, there will not any change for other values. Therefore; i_1 is still $\underline{\underline{14A}}$.

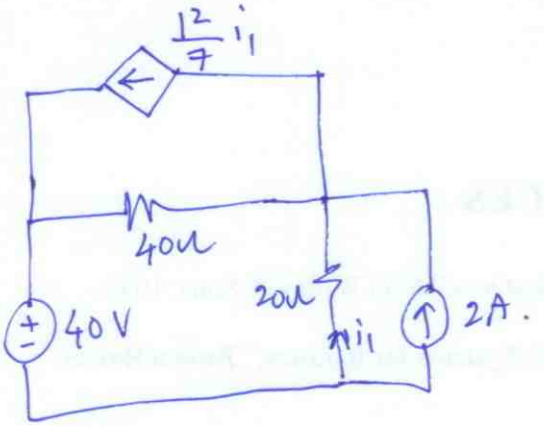
c)



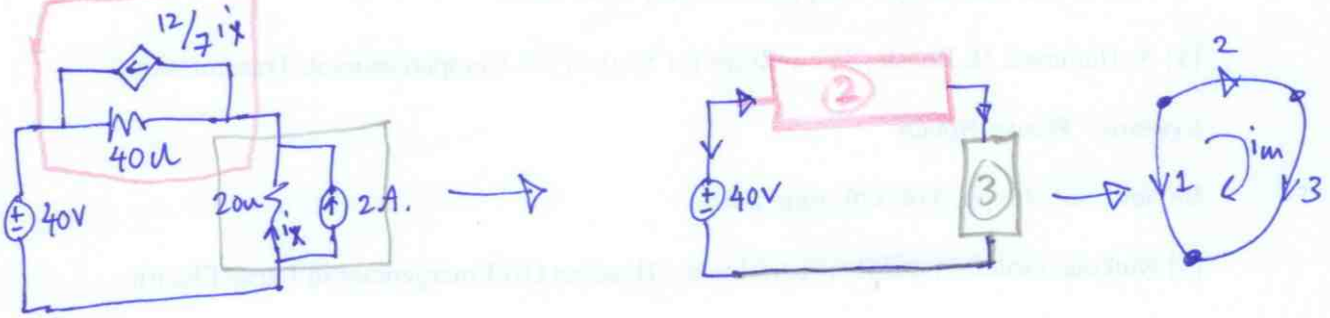
Like part b, we can use substitution theorem for this part. We can change current source with a voltage source without affecting other circuit elements. (figure 2). After changing it, we have two paralel voltage sources. The final voltage source is equal to both of them (because they are parallel)



d)



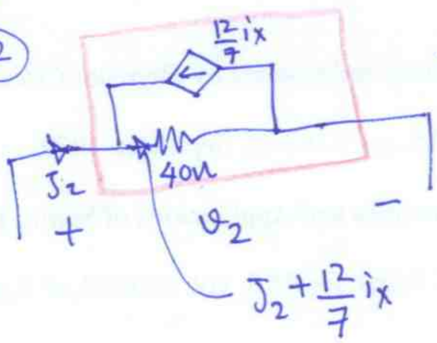
Mesh Analysis:



① $-v_1 + v_2 + v_3 = 0 \longrightarrow [-1 \;\; 1 \;\; 1]\, \underline{v} = 0 \longrightarrow \underline{\underline{M}} = [-1 \;\; 1 \;\; 1]$

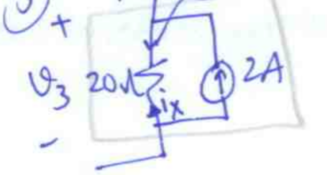
② $\underline{J} = \underline{\underline{M}}^T i_m$ (i_m: mesh current, the unknown)

③ Branch eq.: ① $v_1 = 40$ ✓

② 

$\longrightarrow v_2 = 40\left(J_2 + \frac{12}{7} i_x\right)$ ✗

? (i_x should be expressed in terms of J_k's)

③ 

$v_3 = (J_3 + 2)\, 20$ ✓

$i_x = -(J_3 + 2)$

$v_2 = \cancel{40}(J_2 \cancel{\;\;}) \; 40\,\frac{12}{7}(J_3+2)$ ✓

Thus
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 40 & -480/7 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} + \begin{bmatrix} 40 \\ \frac{-960}{7} \\ 40 \end{bmatrix} ; \quad \underline{v} = \underline{\underline{R}}\,\underline{J} + \underline{v_s}$$

① $\underline{\underline{M}}\,\underline{v} = 0$

② $\underline{J} = \underline{\underline{M}}^T \underline{i_m}$

③ $\underline{v} = \underline{\underline{R}}\,\underline{J} + \underline{v_s}$

$$\longrightarrow \quad \underline{\underline{M}}\,\underline{v} = 0 = (\underline{\underline{M}}\,\underline{\underline{R}}\,\underline{\underline{M}}^T) i_m + \underline{\underline{M}}\,\underline{v_s}$$

$$(\underline{\underline{M}}\,\underline{\underline{R}}\,\underline{\underline{M}}^T) = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 40 & -480/7 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 60 - \frac{480}{7}$$

$$\underline{\underline{M}}\,\underline{v_s} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 40 \\ -960/7 \\ 40 \end{bmatrix} = -\frac{960}{7}.$$

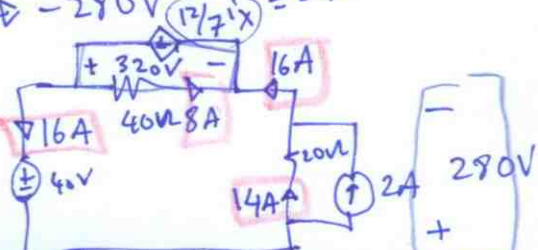
$$(\underline{\underline{M}}\,\underline{\underline{R}}\,\underline{\underline{M}}^T)\,\underline{i_m} = -\underline{\underline{M}}\,\underline{v_s} \longrightarrow \left(60 - \frac{480}{7}\right) i_m = \frac{960}{7}$$

$$\boxed{i_m = -16\ A.}$$

Then ② $\underline{J} = \underline{\underline{M}}^T i_m = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} 16\ A. = \begin{bmatrix} +16 \\ -16 \\ -16 \end{bmatrix} A.$

③ $\underline{v} = \underline{\underline{R}}\,\underline{J} + \underline{v_s} = \begin{bmatrix} 40 \\ \frac{40\cdot(-16)}{7} - \frac{960}{7} \\ 20(-16) + 40 \end{bmatrix}$ → 320V ; → $-280V$ $\quad \frac{12}{7} i_x = 24A.$

$i_x = -(J_3 + 2) = 14A.$ So



All satisfied!