Incorporating *A-Priori* Information in EIT

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Abstract: Use of *a-priori* knowledge on the geometry, the upper and the lower bounds of the tissue resistivities in order to estimate *in vivo* resistivities, in Electrical Impedance Tomography (EIT), is assessed. For the central regions, estimation error is higher due to decreased sensitivity at the centerr. At the peripheral regions the sensitivity is higher, resulting in lower estimation errors. Increasing the number of drive pairs from a single drive pair (as implemented in [1]) to 8 pairs improves the estimation error upto 37 times for the insulator and upto 5 times for the conductor regions. Our results suggests that the MiMSEE algorithm of [1] can be improved further, including all possible drive pairs.

INTRODUCTION

Electrical impedance measurements from a multiple electrode array have been used in [1], to estimate tissue resistivities, in a boundary element canine torso model, when *a-priori* knowledge on the geometry and the upper and the lower bounds of the tissue resistivities are available. This study presents some follow up result of the work described in [1] . The work of [1] uses only a single pair of current drive electrodes. Here, current is applied succesively between all of the opposite pairs of electrodes.

METHODS

A sixteen electrode Electrical Impedance Tomography (EIT) measurement set-up is adapted in simulations. Electrodes are equally spaced around a circular region of conductivity distribution. For simulations of the EIT forward problem, a custom made Finite Element (FE) package [2] is used. Current is injected between opposite (180° apart) pairs of electrodes. The work described in [1] uses only a single pair of current drive electrodes. In this study, current is applied successively between all of the opposite pairs of electrodes. For each drive electrode pair, the potentials were measured between adjacent pairs of electrodes excluding the electrodes through which current is applied. The basic relation between the regional electrical resistivities and the simulated surface potentials is assumed linear:

$$\boldsymbol{v} = \boldsymbol{M} \cdot \boldsymbol{\rho} + \boldsymbol{\eta}_o + \boldsymbol{q}_\eta. \qquad (1)$$

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Where $\boldsymbol{v}$ is a vector with m entries representing the potentials at the measurement electrodes, $\boldsymbol{\rho}$ is a vector with n entries representing the resistivities of n internal regions. $\boldsymbol{q}_\eta$ is a vector with m entries reflecting linearization errors.The linearization offset for the mean resistivity distribution, $\boldsymbol{\eta}_o$, can be expressed as a vector with m entries: $\boldsymbol{\eta}_o = \boldsymbol{v}(\boldsymbol{\rho} = \boldsymbol{\rho}_{mean}) - \boldsymbol{M} \cdot \boldsymbol{\rho}_{mean}$. The forward transformation matrix $\boldsymbol{M}$ is an (mxn) matrix and $\boldsymbol{\rho}_{mean}$ is a vector with n entries, each of which represents the mean value of each region's resistivity. The range of electrical resistivity of tissues is known [3]. It is assumed that tissues may have resistivities between a minimum and a maximum value with equal probability. Based on this *a priori* knowledge of the range of the regional resistivity vector $\boldsymbol{\rho}$, the (n x n) variance-covariance matrix

$$\boldsymbol{S} = \langle(\boldsymbol{\rho} - \langle\boldsymbol{\rho}\rangle)(\boldsymbol{\rho} - \langle\boldsymbol{\rho}\rangle)^T\rangle \qquad (2)$$

is constructed. Each of the regional resistivities are assumed to be not correlated, hence the matrix $\boldsymbol{S}$ is a diagonal matrix. Instrumentation noise of Gaussian probability distribution with zero mean and variance equal to 0.016 is generated and superimposed to the potential measurements. The instrumentation noise is assumed to be uncorrelated. Therefore, variance-covariance matrix of the instrumentation noise is

$$\boldsymbol{J}_\eta = \sigma_{ins}^2 \boldsymbol{I}, \qquad (3)$$

is a diagonal matrix. The total noise covariance terms are assumed to arise from two sources: instrumentation noise and linearization error. The linearization error is assumed to be correlated noise. Details of calculating the entries of the variance-covariance matrix, $\boldsymbol{Q}_\eta$, of the linearization error can be found in [1]. The total noise was assumed as the superposition of the linearization and the instrumentation noise, so that the variance-covariance matrix of the noise is

$$\boldsymbol{N} = \boldsymbol{Q}_\eta + \boldsymbol{J}_\eta. \qquad (4)$$

Minimizing the mean square error between the true solution vector $\boldsymbol{\rho}$, and its estimate $\widehat{\boldsymbol{\rho}}$,

$$\Phi(B, b) = \langle(\boldsymbol{\rho} - \widehat{\boldsymbol{\rho}})^T \boldsymbol{G}(\boldsymbol{\rho} - \widehat{\boldsymbol{\rho}})\rangle = minimum, \qquad (5)$$

and assuming a linear estimator,

$$\widehat{\boldsymbol{\rho}} = \boldsymbol{B} \cdot (\boldsymbol{v} - \boldsymbol{\eta}_o) + \boldsymbol{b}, \qquad (6)$$

gives the optimum inverse $\boldsymbol{B}$ of the (m x n) matrix $\boldsymbol{M}$. Where, $\boldsymbol{G}$ is any arbitrary metric and $\boldsymbol{b} = \langle \boldsymbol{\rho} \rangle - \boldsymbol{B} \cdot \boldsymbol{M} \langle \boldsymbol{\rho} \rangle$.

Strand and Westwater [4] developed an expression for the optimum inverse:

$$B = (S^{-1} + M^T \cdot N^{-1} \cdot M)^{-1} \cdot M^T \cdot N^{-1}, \quad (7)$$

If the linearization error and the instrumentation noise are uncorrelated, then the matrix N is a diagonal matrix. Equation (7) requires inversion of an (nxn) matrix when m>n [4,5,6].

The algorithm summarized above is named as Minimum Mean Squared Error Estimator (MiMSEE) by [1]. The measured potentials are calculated using the FE model for a range of resistivities assigned to each region. The FE model has five regions which have resistivity different than the background resistivity. One of these regions is located at the center (region Z), the other four regions (Z1, Z2, Z3, and Z4) are centered equiangularly spaced at 0.64 of the radius from the center of the circular conductor. Regions Z1, Z2, Z3, and Z4 are located at 0, $\pi/2$, π, and $3\pi/4$ respectively. Resistivity of the background region was kept constant at 500 Ω-cm. Resistivities of the of the other regions are either allowed to range in between 100 Ω-cm and 200 Ω-cm (conductor perturbation case) or in between 600 Ω-cm and 2000 Ω-cm (insulator perturbation case).

In determining the entries of matrix M, it is assumed that a particular entry of matrix M is a function of only the resistivity of corresponding region when the resistivity of other regions are equal to their mean values and is not influenced by the deviation of the resistivity of other regions from their mean values. To obtain the entries of the matrix M, a linear regression fit is utilized.

RESULTS AND DISCUSSION

The fractional errors for region i,

$$\varepsilon_i = \sqrt{\frac{\sum_{n=1}^{N}(\rho_i^n - \widehat{\rho_i^n})^2}{\sum_{n=1}^{N}(\rho_i^n)^2}}, \quad (8)$$

were calculated and given in Table 1 and Table 2 when a single drive pair or 2, 4, and 8 drive pairs are activated. In equation (8), n = 1,2,...,N represents different resistivity distributions, ρ_i and $\widehat{\rho}_i$ are respectively the original resistivity value and the estimated resistivity value of region i.

Table 1: Percentage fractional errors in regional resistivity estimates for insulator perturbation case

Drive-Pair #	Z	Z1	Z2	Z3	Z4
1	9.56	5.41	6.31	5.39	6.07
2	9.31	3.02	1.87	2.60	0.66
4	9.31	2.74	0.88	2.10	1.57
8	9.28	0.56	0.17	2.11	0.49

When the object is located at the center the noise is higher due to decreased sensitivity (lower current density) at the center. At the periphery the sensitivity

Table 2: Percentage fractional errors in regional resistivity estimates for conductor perturbation case

Drive-Pair #	Z	Z1	Z2	Z3	Z4
1	1.48	1.00	1.99	1.80	1.76
2	1.49	0.96	0.77	1.76	0.92
4	1.45	0.78	0.67	1.51	1.04
8	1.39	0.59	0.38	1.26	0.79

is higher, due to increased current density, resulting in lower estimation errors. Increasing the number of drive pairs improves the estimation errors. This improvement is not significant at the center (i.e. region Z). For the peripheral regions (i.e. Z1, Z2, Z3, and Z4), improvements upto 37 times in the insulator perturbation case and upto 5 times for the conductor perturbation case are obtained, by activating 8 drive pairs instead of a single drive pair. Estimation errors in the case of insulator perturbations are larger than those of the conductor perturbations case. Differences in the estimation errors corresponding to symmetrical regions are due to the random nature of the instrumentation noise. As the instrumentation noise goes to zero, these differences disappear.

CONCLUSION

This simulation study combines the knowledge on geometry, the upper and the lower bounds of tissue resistivity values, and the statistical information on the instrumentation noise with EIT measurements in order to enhance the quantitative accuracy in determining tissue resistivities. The results of this study demonstrates that the MiMSEE algorithm developed in [1] can be further improved by including all possible drive electrode pairs.

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