Anisotropic Skeletal Muscle Conductivity in Electrical Impedance Imaging

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Abstract– **This work presents an analysis of the detectability of an anisotropic skeletal muscle conductivity in electrical impedance imaging, with a given measurement accuracy. A concentric circle model is used to develop a relationship between the anisotropic conductivity parameters and the measurement accuracy required to detect these parameters.**

I. Introduction

The goal of Electrical Impedance Imaging (EII) is to determine the spatial distribution of the electrical conductivity inside a conductor, by applying a limited electrical power to the conductor. EII methods are currently used both to measure impedances and to assign impedance values to various anatomic regions. For a given power, the larger the variation in the boundary potentials as a result of a deviation of the conductivity distribution from a homogeneous and isotropic conductivity, the higher the distinguishability of the deviation in conductivity. EII reconstruction algorithms assume that the conductor region has a piecewise homogeneous and an isotropic conductivity [1]. However, the level of anisotropy is significant in some biological tissues especially, in the skeletal muscle and the myocardium [2,3].

We are interested in determining the *invivo* tissue conductivities from EII measurements using the anatomical information determined from high resolution magnetic resonance and/or ultrasound imaging [4].

The first purpose of this study is to investigate the significance of the skeletal muscle anisotropy and its detectability in EII when the structural information about the conductor region is available.

II. Method of Simulation

Assume a circular inhomogeneity with radius R and homogeneous, isotropic conductivity, σ_1, is centered inside another circular conductor, bounded by surface S with radius a. The conductivity of the outher ring is σ_2. Electric current is injected into the circular region on S. Assuming an applied current density, of the form $j(\theta) = \sum_{n=1}^{\infty}(A_n Cosn\theta + B_n Sinn\theta)$ on S, the electric potential function on S can be expressed as [5]

$$V_1(a,\theta) = \sum_{n=1}^{\infty} \frac{1}{n} \frac{a^n(a^{2n}\sigma_1 - R^{2n}\sigma_1 + a^{2n}\sigma_2 + R^{2n}\sigma_2)}{a^{2n}\sigma_1 + R^{2n}\sigma_1 + a^{2n}\sigma_2 - R^{2n}\sigma_2} [A_n Cos(n\theta) + B_n Sin(n\theta)]. \quad (1)$$

Here σ_1 and σ_2 are homogeneous and isotropic. Previous studies on the forward problem of the electrocardiography suggest that the skeletal muscle anisotropy has a significant effect on the thoracic electric field distribution[2, 3]. Conductivity along a skeletal muscle fiber, σ_h, is higher than the conductivity normal to the muscle fiber, σ_l. The skeletal muscle conductivity tangential to the torso wall, σ_m, has been assummed to be equal to the mean of the normal and longitudinal conductivities and the conductivity normal to the torso wall is assummed to be equal to σ_l, by previous researchers [2,3]. The skeletal muscle anisotropy ratio σ_m/σ_l may typically be about 8 [3]. Using the boundary extension method, i.e. radially streching the outher layer by a factor of $\sqrt{\sigma_m/\sigma_l}$, the anisotropic skeletal muscle layer can be replaced by an isotropic conductivity layer. An isotropic conductivity value $\sigma_2 = \sqrt{\sigma_m \cdot \sigma_l}$ is assigned to the outher region. The electric potential function on S, then becomes

$$V_2(b,\theta) = \sum_{n=1}^{\infty} \frac{1}{n} \frac{b^n(b^{2n}\sigma_1 - R^{2n}\sigma_1 + b^{2n}\sigma_2 + R^{2n}\sigma_2)}{b^{2n}\sigma_1 + R^{2n}\sigma_1 + b^{2n}\sigma_2 - R^{2n}\sigma_2} [A_n Cos(n\theta) + B_n Sin(n\theta)]. \quad (2)$$

Where $b = \sqrt{\sigma_m/\sigma_l} \cdot a$. We then calculated the mean squared difference between V_2 and V_1:
$\gamma = \|V_2 - V_1\|$, where $\|f\| = (\int_S |f(r,\theta)|^2 dS)^{1/2}$. For $j(\theta) = \sum_{n=1}^{\infty}(A_n Cosn\theta + B_n Sinn\theta)$ applied to the conductor, mean squared current density is $\|j\| = 1$ then,

$$\gamma = \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \left[\frac{b^n(b^{2n}\sigma_1 - R^{2n}\sigma_1 + b^{2n}\sigma_2 + R^{2n}\sigma_2)}{b^{2n}\sigma_1 + R^{2n}\sigma_1 + b^{2n}\sigma_2 - R^{2n}\sigma_2} - \frac{a^n(a^{2n}\sigma_1 - R^{2n}\sigma_1 + a^{2n}\sigma_2 + R^{2n}\sigma_2)}{a^{2n}\sigma_1 + R^{2n}\sigma_1 + a^{2n}\sigma_2 - R^{2n}\sigma_2} \right]^2 \right\}^{1/2}. \quad (3)$$

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The summation in equation (3) is a maximum when n=1. For the anisotropy in skeletal muscle to be detected with an EII system having a measurement accuracy ϵ, $\gamma|_{n=1}$ must greater than ϵ:

$$\left[\frac{b(b^2\sigma_1-R^2\sigma_1+b^2\sigma_2+R^2\sigma_2)}{b^2\sigma_1+R^2\sigma_1+b^2\sigma_2-R^2\sigma_2} - \frac{a(a^2\sigma_1-R^2\sigma_1+a^2\sigma_2+R^2\sigma_2)}{a^2\sigma_1+R^2\sigma_1+a^2\sigma_2-R^2\sigma_2}\right] > \epsilon.$$

III. Results

We assumed that the internal inhomogeneity has a constant radius, $R = b/10$. First, radial conductivity of the skeletal muscle, σ_l, is kept constant at $\sigma_l = 1$. The conductivity of the outher layer, σ_2, is set to $\sigma_2 = \sqrt{\sigma_m}$, where $\sigma_m = (b/a)^2$ and $b = 1$. Fig.1. shows the mean squared potential change as a function of the conductivity of the internal inhomogeneity and ratio of unextended radius to the extended radius ($\sqrt{\sigma_l/\sigma_m}$) calculated from equation (3).

Then, σ_m is kept constant at $\sigma_m = 1$ and as the level of anisotropy varied σ_2 is varied as $\sigma_2 = \sqrt{\sigma_l}$ where $\sigma_l = (a/b)^2$ and b=1 (Fig. 2). In Fig. 1.(a) and Fig. 2.(a), the shaded planes represent the detectability plane of the skeletal muscle anisotropy. Measurement noise should be below the detectability plane, for any anisotropy in the skeletal muscle conductivity to be detected.

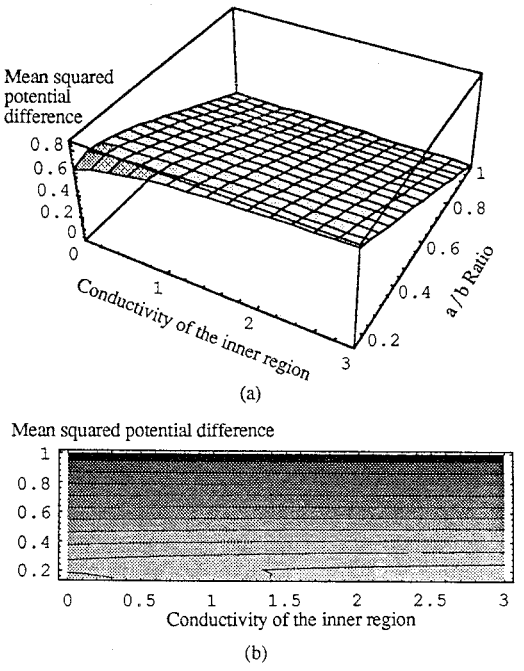

Fig. 1. (a) Mean Squared potential change as a function of σ_1 and a/b. (b) Isopotential map of the mean squared potential. $\sigma_l = 1$ is kept constant and $\sigma_m = (b/a)^2$.

For $a = 0.3536$, which corresponds to anisotropy ratio of 8 for skeletal muscle, and $\sigma_1 = \sigma_l = 1$, the measurement precision must be better than 0.63. For $a = 0.3536$ and $\sigma_1 = \sigma_m = 1$, the measurement precision must be better than 0.66. These signal levels are sufficiently high to be detected, therefore the contribution of skeletal muscle anisotropy to the boundary measurements must not be neglected during the reconstruction of electrical impedance images.

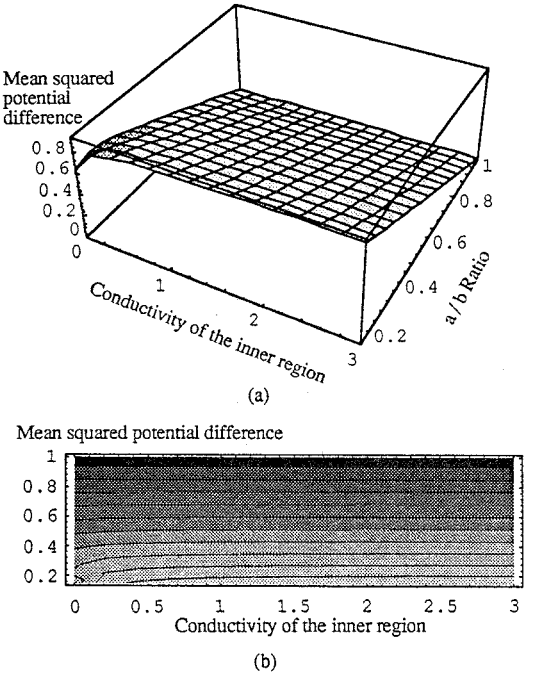

Fig. 2. (a) Mean Squared potential change as a function of σ_1 and a/b. (b) Isopotential map of the mean squared potential. $\sigma_m = 1$ is kept constant and $\sigma_l = (a/b)^2$.

This paper builds on the ideas of Isaacson (1986) [5], which considered the detection of an inhomogeneity inside an otherwise homogeneus isotropic circular conductor. Here, we assumed that the size of the internal inhomogeneity and its conductivity are known and studied the detectability of the anisotropy in the skeletal muscle conductivity.

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