EE 584
MACHINE VISION

Photometric Stereo
- Radiometry
- BRDF
- Reflectance Map
- Recovering Surface Orientation

Photometric Stereo

- It is possible to recover the orientation of surface patches from a number of images taken under different lightening conditions
- Although the method is simple to implement, it requires control of lightening
- Understanding these methods require detailed analysis of "brightness"

Image Brightness

- Image of a 3-D object depends on
  - Shape
  - Reflectance properties
  - Distribution of light sources

- Under different lightening, some edges having strong contrast in one image, may not be visible in another

- *Radiometry* is the <u>science of radiation measurement</u> and it is necessary to completely understand image formation
- *Brightness* is determined by the amount of <u>energy</u> an imaging system receives per unit apparent area

Radiometry : Definitions

- *<u>Solid angle of a cone</u>* : Area cut out by a cone on a unit sphere

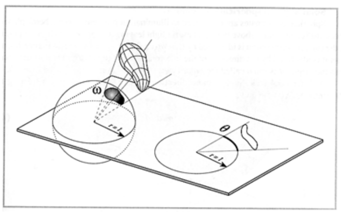


Fig. 7.1

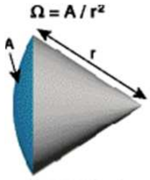
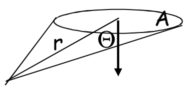


Fig. 7.2 Solid angle

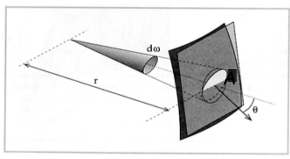
$$Hemisphere \quad area = 2\pi r^2 \quad \rightarrow \Omega \equiv 2\pi \text{ radiance}$$

$$Area \; cut \; out \; by \; cone = A \rightarrow \Omega = ? \text{ steradiance}$$



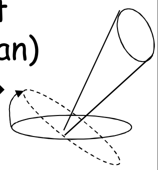
$$\Rightarrow \Omega = \frac{A}{r^2} \xrightarrow{Angle\,\Theta} \Omega = \frac{A\cos\Theta}{r^2}$$

$$d\omega = \frac{dA\cos\theta}{r^2}$$

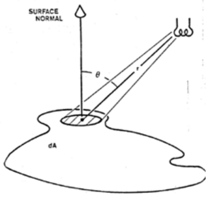


Radiometry : Definitions

- *Radiant flux:* power propagated as light radiation (W)
- *Irradiance* : amount of light falling on a unit surface (W/m2)
- *Radiance* : amount of light radiating from a unit surface towards a “solid” angle (W/m2 steradian) [surface must be perpendicular to the angle direction]➔



- *Radiant exitance (radiosity)* : amount of light radiating from a unit surface (W/m2)
- *Radiant intensity:* amount of light radiating towards a “solid” angle (W/steradian)



Radiant intensity of light source : I

→ Radiant flux, $\Phi = I\,\Omega = I\,A\cos\theta / r^2$

→ Irradiance, $E = \Phi / A = I\cos\theta / r^2$

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Image Formation (1/4)

- Aim is to find the relation between the radiance at an object point (scene radiance) and the corresponding point in the image (image irradiance)

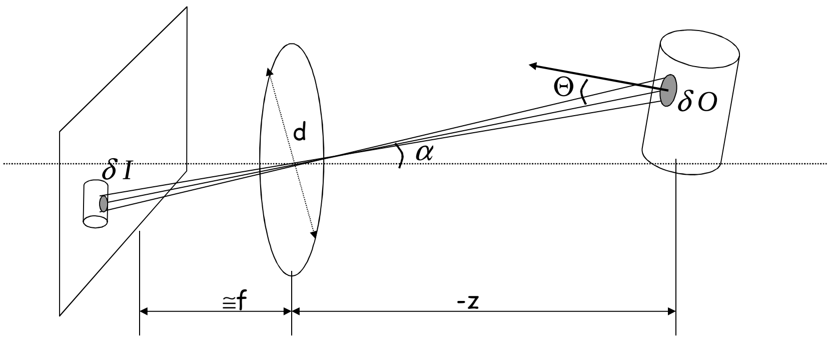
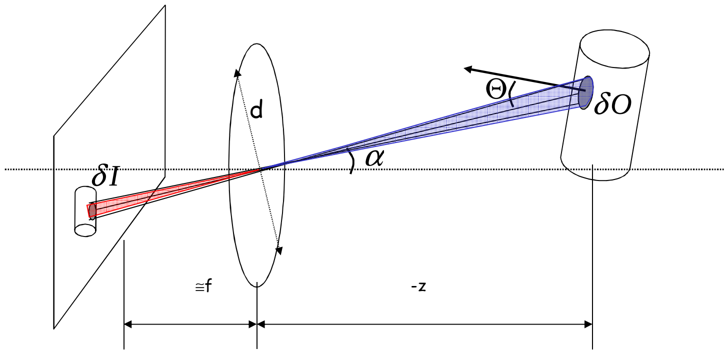


Image Formation (2/4)

- Rays passing thru the center of lens are not deflected
- Solid angles of cones on both sides of the lens are equal to each other

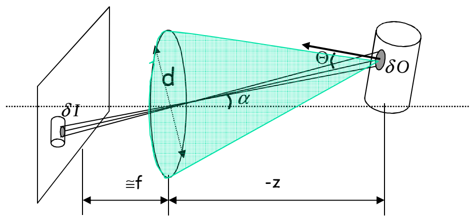


$$\Omega_i = \frac{\delta I \cos\alpha}{(f/\cos\alpha)^2}, \quad \Omega_o = \frac{\delta O \cos\Theta}{(z/\cos\alpha)^2} \quad \xrightarrow{\Omega_i = \Omega_o} \quad \frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\Theta}\left(\frac{z}{f}\right)^2$$

Image Formation (3/4)

- Solid angle for the lens from the object patch :

$$\Omega_l = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2}$$



- Power of light, δP, from object patch, passing thru lens :

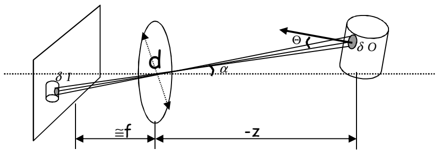
$$\delta P = L\,\delta O\,\Omega_l\cos\Theta = L\,\delta O\,\frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3\alpha\cos\Theta \qquad L: \textit{surface radiance}$$

- Finally, the irradiance, E, on the image patch :

$$E = \frac{\delta P}{\delta I} = L\frac{\delta O}{\delta I}\frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3\alpha\cos\Theta = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cos^4\alpha$$

Image Formation (4/4)

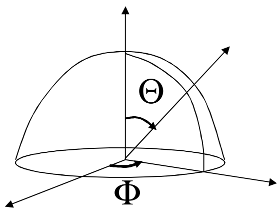
$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$



- Image irradiance is proportional to scene radiance by
  - square of effective *f-number*, f/d
    This ratio is critical while taking shots with a camera, if the scene is less illuminated
  - 4th power of cosine of the angle between optical axis and the ray passing thru the optical axis
    In most cases, this effect is not that severe, since this angle is usually small

Bidirectional Reflectance Distribution Function [BRDF] (1/3)

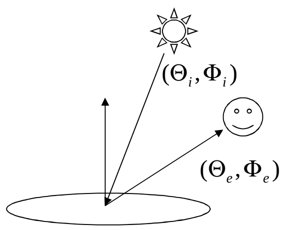
- What determines scene radiance, L ?
  - Amount of light falls on the surface at the scene
  - Fraction of incident light that is reflected
  - Viewing angle



Θ : *polar angle*

Φ : *azimuth angle*

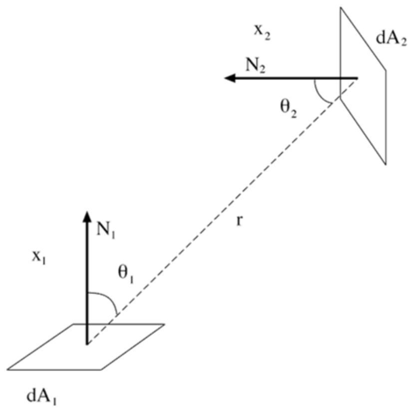
Surface-specific coordinate system



Incident light and emitted light can be defined using azimuth and polar angles

Bidirectional Reflectance Distribution Function [BRDF] (2/3)

- Radiance is constant along straight lines



- Power 1→2, leaving patch-1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1→2, arriving at patch-2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

- These quantities must be equal to each other
  - ➔ two radiances are equal

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Bidirectional Reflectance Distribution Function [BRDF] (3/3)

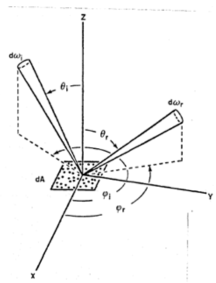
- BRDF, *f(.)*, captures the information how bright a surface appears when viewed from one direction while light falls on it from another

$$f(\Theta_i,\Phi_i;\Theta_r,\Phi_r)=\frac{dL(\Theta_r,\Phi_r)}{dE(\Theta_i,\Phi_i)},\ \frac{1}{steredian}$$

$$d\Phi_i = dL_i\, dA\cos\Theta_i\, d\omega_i = dE(\Theta_i,\Phi_i)dA$$

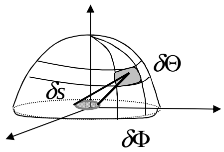
$$d\Phi_r = dL(\Theta_r,\Phi_r)\, dA\, \cos\Theta_r\, d\omega_r$$

$$\Rightarrow f(\Theta_i,\Phi_i;\Theta_r,\Phi_r)=\frac{d\Phi_r/(dA\cos\Theta_r\, d\omega_r)}{d\Phi_i/dA}$$



- For many surfaces (e.g. matte, specular) , radiance is a function of the difference between the azimuth angles : (Φ_r - Φ_i)
  - ➔ BRDF has 3 independent variables

Extended Light Sources (1/2)



- Find the total irradiance on a surface, resulting from extended light sources
- Assume the area of an infinitesimal patch of the sky is given as :

$$du\,dv = R^2 \sin\Theta d\Theta\, d\Phi$$

- Total power emitted from the patch towards the surface, δs :

$$P_{emit} = \underbrace{L_{sky}(\Theta_i,\Phi_i)}_{radiance}\underbrace{\sin\Theta_i\,\delta\Theta_i\;\;\delta\Phi_i\,R^2}_{Area}\underbrace{\cos\Theta_i\frac{\delta s}{R^2}}_{solid\ angle}$$

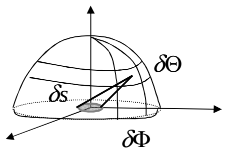
- Irradiance on the surface, δs, resulting from patch on the sky:

$$E(\Theta_i,\Phi_i) = P_{emit}/\delta s = L_{sky}(\Theta_i,\Phi_i)\sin\Theta_i\delta\Theta_i\;\delta\Phi_i\cos\Theta_i$$

Extended Light Sources (2/2)

- Total irradiance on surface, δs, from all patches of the sky:

$$E_0 = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L_{sky}(\Theta_i,\Phi_i)\sin\Theta_i\cos\Theta_i d\,\Theta_i\;d\,\Phi_i$$



- Radiance from surface , δs, towards Θ_e,Φ_e resulting from patch

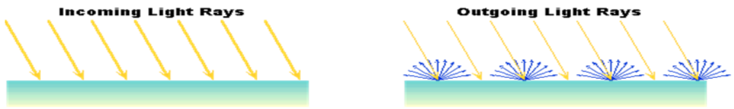
$$L_{\Theta_i\Phi_i}(\Theta_e,\Phi_e) = f(\Theta_i,\Phi_i;\Theta_e,\Phi_e)L_{sky}(\Theta_i,\Phi_i)\sin\Theta_i\cos\Theta_i\delta\Theta_i\;\delta\Phi_i$$

- Total radiance from surface , δs, towards Θ_e , Φ_e :

$$L(\Theta_e,\Phi_e) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\Theta_i,\Phi_i;\Theta_e,\Phi_e)L_{sky}(\Theta_i,\Phi_i)\sin\Theta_i\cos\Theta_i d\,\Theta_i\;d\,\Phi_i$$

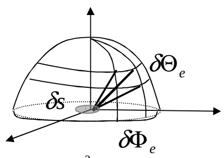
Surface Reflectance Properties (1/2)

- *Lambertian surface* : appears equally bright from all angles; reflects all incident light; i.e. BRDF constant



Total irradiated power on surface = Total radiated power from surface

$$E_0\,\delta s = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} \underbrace{\delta s \cos\Theta_e}_{area}\,\underbrace{L(\Theta_e,\Phi_e)}_{f\,E_o}\,\underbrace{\sin\Theta_e d\,\Theta_e\; d\,\Phi_e}_{differential\ solid\ angle}$$



$$du\,dv = R^2 \sin\Theta_e d\Theta_e\; d\Phi_e$$

$$2\pi f \int_{0}^{\pi/2} \sin\Theta_e \cos\Theta_e d\,\Theta_e = 1 \quad\Rightarrow\quad f = \frac{1}{\pi}$$

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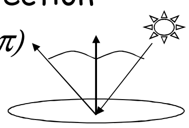
Surface Reflectance Properties (2/2)

- *Specular surface* : reflects all the light arriving from one direction to the reverse direction

➔ $f(\Theta_i,\Phi_i;\,\Theta_e,\Phi_e) = k\,\delta(\Theta_e-\Theta_i)\delta(\Phi_e-\Phi_i-\pi)$



Total irradiated power on surface = Total radiated power from surface

$$E(\Theta_i,\Phi_i)\delta s = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} \underbrace{\delta s\cos\Theta_e}_{area}\,\underbrace{k\delta(\Theta_e-\Theta_i)\delta(\Phi_e-\Phi_i-\pi)}_{f(\Theta_i,\Phi_i;\Theta_e,\Phi_e)}E(\Theta_i,\Phi_i)\underbrace{\sin\Theta_e d\,\Theta_e\; d\,\Phi_e}_{differential\ solid\ angle}$$

$$\int_{-\pi}^{\pi}\int_{0}^{\pi/2} k\delta(\Theta_e-\Theta_i)\delta(\Phi_e-\Phi_i-\pi)\sin\Theta_e\cos\Theta_e d\,\Theta_e\; d\,\Phi_e = 1$$

$$\Rightarrow f(\Theta_i,\Phi_i;\Theta_e,\Phi_e) = \frac{1}{\sin\Theta_i\cos\Theta_i}\delta(\Theta_e-\Theta_i)\delta(\Phi_e-\Phi_i-\pi)$$

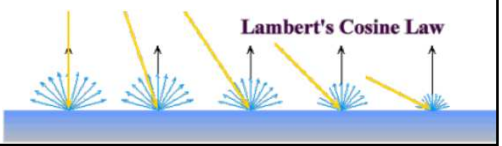
Lambert's Cosine Law

- Assume
  - a Lambertian surface
  - illuminated by a single distant point source (distant ➔ light waves arrive from a single angle)
    - produces an irradiance E on a surface orthogonal to the angle (Θ_S,Φ_S)
- What is radiance of this point source, $E(\Theta,\Phi)$?

$$\text{Since } \int_{-\pi}^{\pi}\int_{0}^{\pi/2} E(\Theta_i,\Phi_i)\sin\Theta_i d\,\Theta_i\, d\,\Phi_i = E \Rightarrow E(\Theta_i,\Phi_i) = E\,\frac{\delta(\Theta_i-\Theta_s)\delta(\Phi_i-\Phi_s)}{\sin\Theta_i}$$

$$L(\Theta_e,\Phi_e) = \iint \underbrace{f(\Theta_i,\Phi_i;\Theta_e,\Phi_e)}_{1/\pi} E(\Theta_i,\Phi_i)\sin\Theta_i\cos\Theta_i\, d\Theta_i d\Phi_i$$

$$\Rightarrow L = \frac{1}{\pi} E\cos\Theta_s \quad \textit{for } \cos\Theta_s > 0$$



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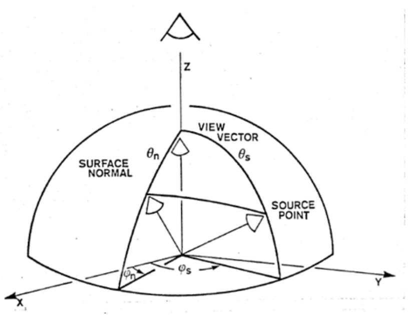
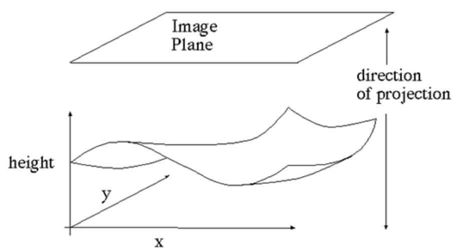
Surface Orientation (1/3)

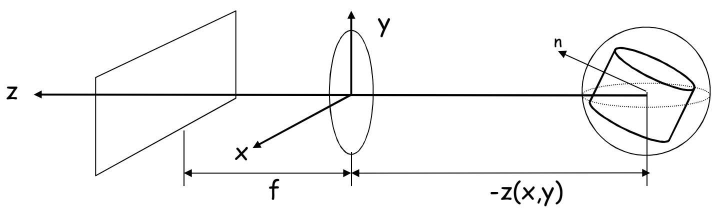
- Assume a viewer-oriented coordinate system
  - previously, coordinate system is erected on a hypothetical surface
  - z-axis : viewing direction or optical axis of camera
- Describe the surface of an object in term of its perpendicular distance -z(x,y) from some reference plane parallel to an image plane for a camera





Surface Orientation (2/3)

- Assume <u>orthographic projection</u>

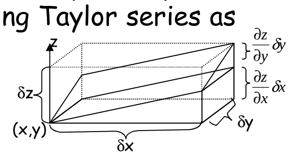


- Surface orientation at any point can be determined by the orientation of the tangent plane at that point
- Normal to this tangent plane represents the orientation

Surface Orientation (3/3)

- On the surface, for a small change in x and y, the spatial change in z can be approximated by using Taylor series as

$$\delta z \approx \underbrace{\frac{\partial z}{\partial x}}_{p}\delta x + \underbrace{\frac{\partial z}{\partial y}}_{q}\delta y \Rightarrow (\delta x \;\; \delta y \;\; \delta z)(-p \quad -q \quad 1)^t = 0$$



- Unit normal vector to this tangent plane is given by

$$\hat{n} = \frac{n}{|n|} = \frac{(-p, -q, 1)^T}{\sqrt{p^2 + q^2 + 1}}$$

- If the object is far away from the reference plane, the angle between the surface normal and direction view (0,0,1) :

$$\cos \Theta_e = \frac{1}{\sqrt{p^2 + q^2 + 1}}$$

- Location of light sources can also be specified using a similar notation, $(-p_s, -q_s, 1)$: using a surface perpendicular to this ray

Reflectance Map (1/2)

- *Reflectance Map*, $R(p,q)$, should be defined to relate surface orientations $(-p,-q,1)$ to the brightness by taking into account surface reflectance properties and light distributions $(-p_s,-q_s,1)$
- For a source of radiance, E, illuminating a Lambertian surface,

$$L = \frac{1}{\pi} E \cos\Theta_s \quad \text{for } \cos\Theta_s > 0$$

- The angle between illuminating ray and surface normal :

$$\cos\Theta_s = \frac{1 + pp_s + qq_s}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^2 + q_s^2 + 1}}$$

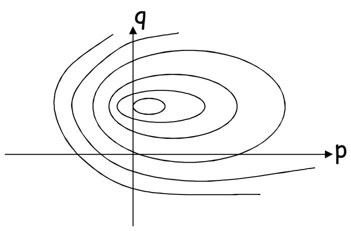
- Since L is proportional to image irradiance, $R(p,q)$ can be defined as above so that it relates surface orientation to brightness

$$R(p,q) = \frac{1 + pp_s + qq_s}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^2 + q_s^2 + 1}}$$

Reflectance Map (2/2)

- Note that if we plot $R(p,q)$ as a function of (p,q), every point in pq-plane (gradient space) corresponds to a particular surface orientation
- For $R(p,q)=c$, contour maps can be used to understand the reflectance map

$$(1 + pp_s + qq_s)^2 = c^2 \left(p^2 + q^2 + 1\right)\left(p_s^2 + q_s^2 + 1\right)$$



For different values of c, we have different contours on which $R(p,q)$ is constant

Over these contours, it is not possible to find the orientation of a surface (p,q) by only observing a radiance value on the image

Shading in Images (1/2)

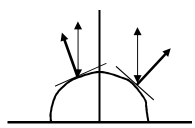
- *Shading* is the variation in brightness for the image of an object due to surface patches with different orientations appearing in different brightness

- Brightness is proportional to image irradiance, E(x,y)
- E(x,y) is proportional to scene (object) radiance, L
- L is proportional to $R(p,q)$ (for surface gradient (p,q))
- Hence, after setting constant of proportionality to 1 :

$$E(x,y) = R(p,q) \quad : \textit{image irradiance equation}$$

Shading in Images (2/2)

- Consider a sphere with Lambertian surface illuminated by a point source at the same place with viewer



- Since $(-p_s,-q_s)$ =(0,0) $\Rightarrow R(p,q) = \dfrac{1}{\sqrt{1+p^2+q^2}}$
- If the sphere is on z-axis : $z = z_0 + \sqrt{r^2 - (x^2+y^2)} \quad for\ (x^2+y^2) < r^2$
- Surface gradients, (p,q) and image brightness can be obtained as

$$p = -\frac{x}{z-z_0} \quad q = -\frac{y}{z-z_0} \Rightarrow \frac{1}{\sqrt{1+p^2+q^2}} = \frac{z-z_0}{r} \Rightarrow E(x,y) = R(p,q) = \sqrt{1-\frac{x^2+y^2}{r^2}}$$

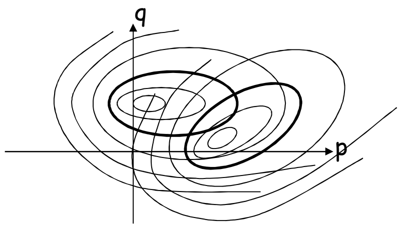
- In computer graphics applications, in order to obtain $E(x,y)$ quickly, (p,q) values are quantized and $R(p,q)$ values are stored in a "look-up table"
- Given z, E(x,y) is determined uniquely. How about the reverse ?

Photometric Stereo (1/2)

- There is a unique mapping from surface orientation to the radiance, given by $R(p,q)$, but inverse mapping is not unique
- In order to determine two unknowns, p&q, we need to equations for each image point → two images taken with different lighting
- Solve the nonlinear equations below to find (p,q)

$$R_1(p,q) = \frac{1 + p_1 p + q_1 q}{\sqrt{1 + p_1^2 + q_1^2}\sqrt{1 + p^2 + q^2}} = E_1$$

$$R_2(p,q) = \frac{1 + p_2 p + q_2 q}{\sqrt{1 + p_2^2 + q_2^2}\sqrt{1 + p^2 + q^2}} = E_2$$



- There can be either zero or several or even infinite number of solutions

Photometric Stereo (2/2)

- In contrary to our assumptions, in practice all the incident light does not radiate back from the surface, but this effect can be easily incorporated into the image radiance equation by a factor

$$E(x,y) = \underbrace{\rho}_{albedo} R(p,q) \quad 0 < \rho < 1$$

- For a surface with varying albedo, both surface orientation and albedo can be recovered using 3 image measurements

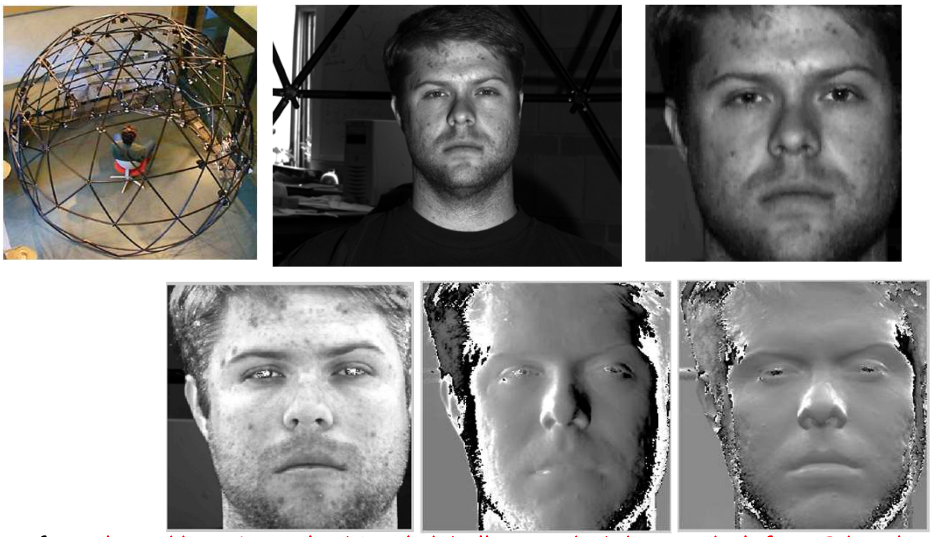
$$E_i = \rho(\vec{s}_i . \vec{n}_i) \quad i = 1,2,3 \quad s_i \text{ represent } i^{th} \text{ light source location}$$

$$\text{where} \quad \vec{s}_i = \frac{(-p_i, -q_i, 1)^t}{\sqrt{1 + p_i^2 + q_i^2}} \text{ and } \quad \vec{n}_i = \frac{(-p, -q, 1)^t}{\sqrt{1 + p^2 + q^2}}$$

- These three equations are combined to obtain a solution :

$$E = \rho S \vec{n} \quad \text{where } E \equiv \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}, S \equiv \begin{bmatrix} \vec{s}_1 \\ \vec{s}_2 \\ \vec{s}_3 \end{bmatrix} \Rightarrow \rho \vec{n} = S^{-1} E$$

Photometric stereo example



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