EE 584
MACHINE VISION

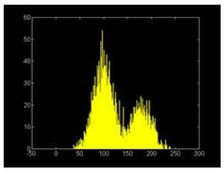
Binary Images Analysis
- Geometrical & Topological Properties
- Connectedness
- Binary Algorithms
- Morphology

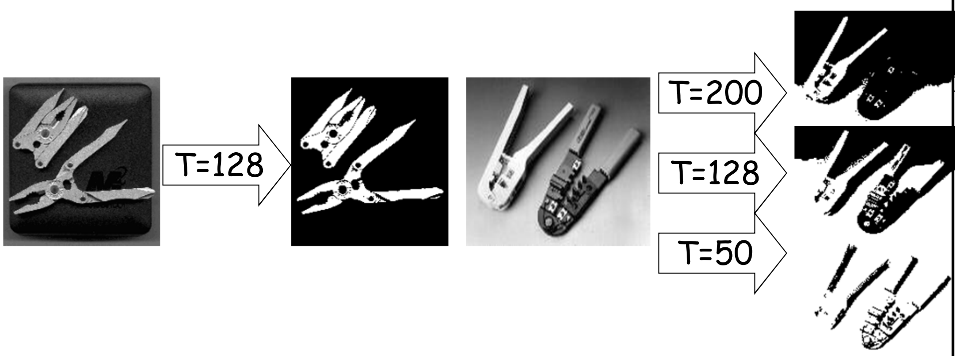
Binary Images

- Binary (two-valued; black/white) images gives better efficiency in acquiring, storage, processing and transmission.
  - simplified description in terms of shape
- Obtained by *thresholding* or *segmentation* of a gray-level image
- One can compute some geometrical (e.g. area, orientation) and topological (connectedness) properties using binary images
- Remember that all we have is a "silhouette", but good recognition by humans is encouraging

Binarization

- Effect of different thresholds T on the image histogram





Binary Images : Geometrical Properties (1/4)

- Assume *continuous* binary image, $b(x,y)$ (background 0, foreground 1)

<u>Area</u> : $A = \iint b(x,y)dxdy$

<u>Position</u> : $$\bar{x} = \frac{\iint x b(x,y)dxdy}{\iint b(x,y)dxdy}$$

$$\bar{y} = \frac{\iint y b(x,y)dxdy}{\iint b(x,y)dxdy}$$

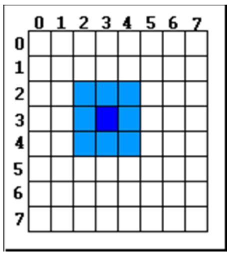
Center of mass (1^{st} moment along one of the axis)

Binary Images : Geometrical Properties (2/4)

For discrete images,

- We denote the set of pixels in a region by R
- Assuming square pixels, we obtain
  - *Area:*

$$A = \sum_{(x,y)\in R} 1$$



  - *Centroid:*

$$\bar{x} = \tfrac{1}{A} \sum_{(x,y)\in R} x$$

$$\bar{y} = \tfrac{1}{A} \sum_{(x,y)\in R} y$$

Source: Shapiro & Stockman

Binary Images : Geometrical Properties (3/4)

<u>Orientation</u> : Find the line for which the integral of the square of the distance to the object points is a minimum

$$E = \iint r^2\, b(x,y)\,dx\,dy$$

r is a perpendicular distance from (x,y) to orientation line

The line passes thru *center of mass* with an angle given by :

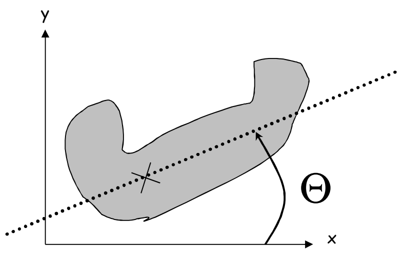
[*See the derivations on the distributed notes*]

$$\tan 2\Theta = \frac{b}{a-c}$$

$$a = \iint (x')^2\, b(x',y')\,dx'dy'$$

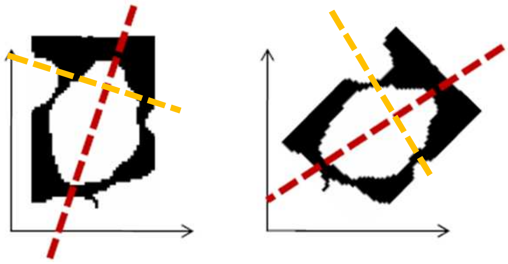
$$b = 2\iint (x'y')\, b(x',y')\,dx'dy'$$

$$c = \iint (y')^2\, b(x',y')\,dx'dy'$$



Binary Images : Geometrical Properties (4/4)

- Another technique to calculate the axis of the second least moment



Axis for which the squared distance to 2D object points is minimized (maximized).

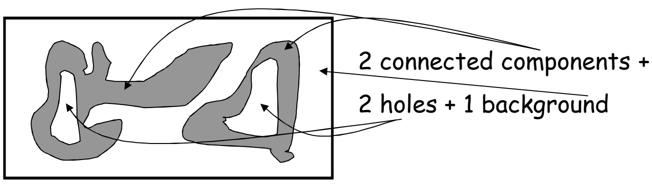
- Compute the eigenvectors of 2nd moment matrix

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} = VDV^T = \begin{bmatrix} v_{11} & v_{12} \\ v_{22} & v_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}^T$$

Binary Images : Topological Properties

In case of more than one object is in the field of view:

- Area, center of mass or orientation produce an *average* value
- Different components should be separated by defining the meaning of *connectedness* between two binary cells
- Two points on an image are connected, if a path can be found along which $b(x,y)$ is constant
- A connected component is the maximal set of connected points



Topological Properties: Discrete Connectedness

- Neighbors : pixels having boundaries
  - 4-neighbors : only edge-adjacent pixels
  - 8- neighbors : edge- & corner-adjacent pixel

B	C	B
C	H	C
B	C	B

4-connected

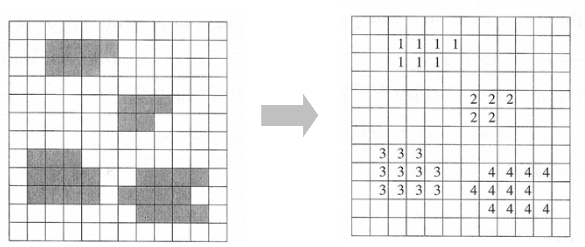
C	C	C
C	H	C
C	C	C

8-connected

- Path : a sequence of neighboring pixels

Topological Properties : Connected Components

- Dividing a binary image into connected components: *Connected component labeling* (CCL)

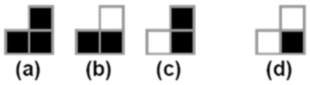


- Two approaches for CCL
  - Recursive
  - Sequential (2-pass)

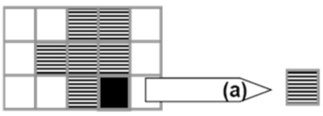
Sequential Connected Components (1/2)

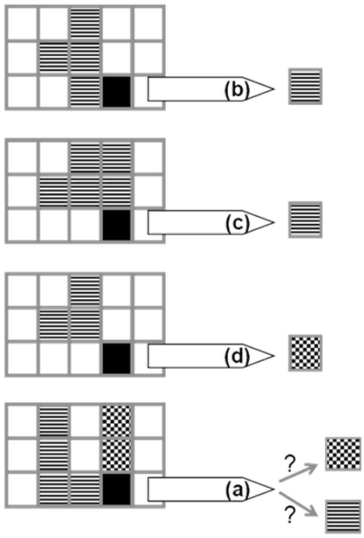
- Labeling a pixel only requires to consider its prior and superior neighbors.
- It depends on the type of connectivity used for foreground (4-connectivity here).

Same object | New object



What happens in these cases?





Equivalence table

Sequential Connected Components (2/2)

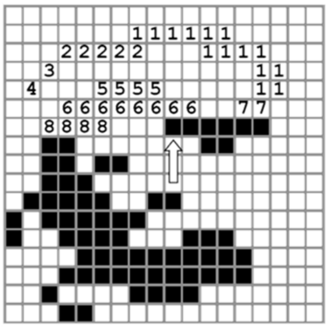
- Process the image from left to right, top to bottom:
  1.) If the next pixel to process is 1

i.) If only one of its neighbors (top or left) is 1, copy its label.

ii.) If both are 1 and have the same label, copy it.

iii.) If they have different labels
– Copy the label from the left.
– Update the equivalence table.

iv.) Otherwise, assign a new label.



- Re-label with the smallest of equivalent labels

```
{1, 2, 7}
{3}
{4}
{5, 6, 8}
{}
```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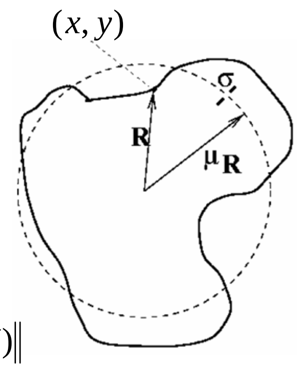
Binary Properties:

- Some properties obtained using connected components analysis
  - Area : Usually used in "size filters"
  - Perimeter : Boundary following algorithms
  - Compactness : $(\text{Perimeter})^2$ / Area
    - *circle* : the most compact
    - *line* : the least compact
  - Circularity:

Circularity



- Measure the deviation from a perfect circle
  - *Circularity:* $C = \dfrac{\mu_R}{\sigma_R}$

    where μ_R and σ_R^2 are the mean and variance of the distance from the centroid of the shape to the boundary pixels (x_k, y_k).
    - *Mean radial distance:*

$$\mu_R = \frac{1}{K}\sum_{k=0}^{K-1}\left\|(x_k, y_k) - (\bar{x}, \bar{y})\right\|$$

    - *Variance of radial distance:*

$$\sigma_R^2 = \frac{1}{K}\sum_{k=0}^{K-1}\left[\left\|(x_k, y_k) - (\bar{x}, \bar{y})\right\| - \mu_R\right]^2$$

Invariant Descriptors

- S is a subset of pixels (region).
- Central $(j,k)^{th}$ moment defined as:

$$\mu_{jk} = \sum_{(x,y)\in S} (x-\bar{x})^j (y-\bar{y})^k$$

- μ_{jk} is invariant to translation of S.

- Interpretation:
  - μ_{00} : 0th central moment: *area*
  - $\mu_{02,20}$: 2nd central moment: *variance*
  - $\mu_{03,30}$: 3rd central moment: *skewness*
  - $\mu_{04,40}$: 4th central moment: *kurtosis*

Moment Invariants

- Normalized central moments

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}, \qquad \gamma = \frac{p+q}{2} + 1$$

- A set of *invariant moments* can be defined for object description:

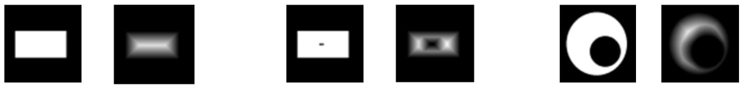
$$\phi_1 = \eta_{20} + \eta_{02}$$
$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$
$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

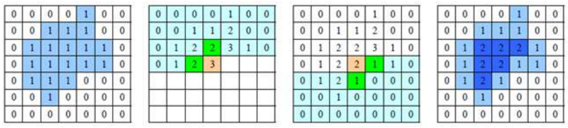
(Additional invariant moments ϕ_5, ϕ_6, ϕ_7 can be found in the literature).

- Invariant to rotation, scaling & translation (RST)

Binary Algorithms : Distance Transform

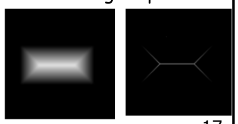
- A measure is necessary to find the distance between two pixels on a binary image, $b(i,j)$
  - City-block, chessboard $d_1(k,l) = |k| + |l|$
  - Euclidean $d_2(k,l) = \sqrt{k^2 + l^2}.$
- **Distance Transform**, $D(i,j)$: *Minimum* distance between an <u>object and all background points</u> $D(i,j) = \min_{k,l:b(k,l)=0} d(i-k, j-l),$





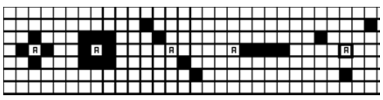
City-block distance transform computation:
Minimum of N&W (S&E) neighbor + 1 and merge 2 passes

- Points that give the locally maximum (ridges) of such distances are called *medial axis (skeleton)*



Binary Algorithms : Morphological Operations

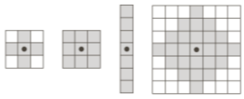
- Mathematical morphology is a set-theoretical approach to image analysis, based on shape.
- Signals are locally compared with *structuring elements* S of arbitrary shape with a reference point R



- <u>Aim</u> : Transforming signals into simpler ones by removing irrelevant information
- Usually used for binary images, but gray-level extensions also exist

Morphological Operators

- Basic idea
  - Scan the image with a structuring element
  - Perform set operations (intersection, union) of image content with structuring element



- Two basic operations
  - Dilation
  - Erosion
- Several important combinations
  - Opening
  - Closing
  - Boundary extraction

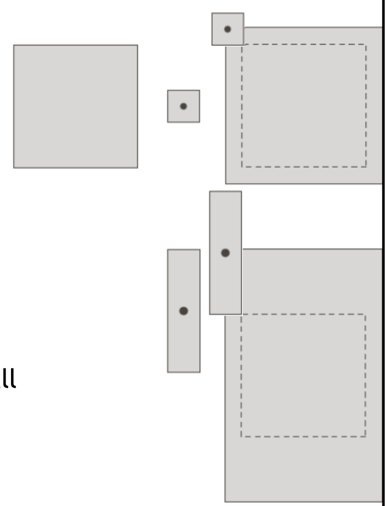
Dilation

- Definition
  - *"The dilation of A by B is the set of all displacements z, such that $(\hat{B})_z$ and A overlap by at least one element".*
  - ($(\hat{B})_z$ is the mirrored version of B, shifted by z)
- Effects
  - If current pixel z is foreground, set all pixels under $(B)_z$ to foreground.

  ⇒ Expand connected components

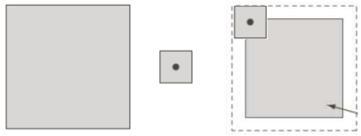
  ⇒ Grow features

  ⇒ Fill holes



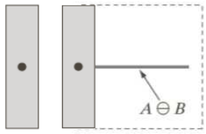
Erosion

- <u>Definition</u>
  - *"The erosion of A by B is the set of all displacements z, such that $(B)_z$ is <u>entirely contained</u> in A".*



$A \ominus B$

- <u>Effects</u>
  - If not every pixel under $(B)_z$ is foreground, set the current pixel z to background.
  - ⇒ Erode connected components
  - ⇒ Shrink features
  - ⇒ Remove bridges, branches, noise



$A \ominus B$

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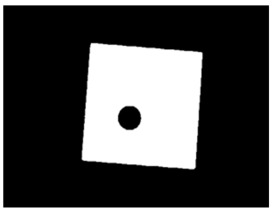
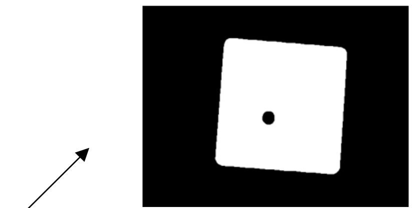
Effects of Dilation and Erosion



Original Image

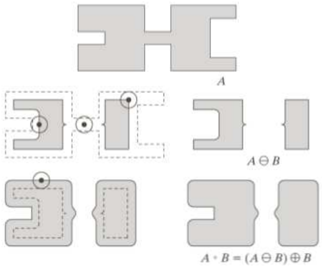


Dilation with circular SE

Erosion with circular SE

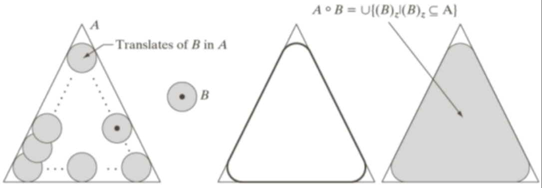
Opening

- Definition
  - Sequence of Erosion and Dilation
    $A \circ B = (A \ominus B) \oplus B$



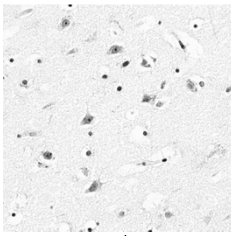
- Effect
  - $A \circ B$ is defined by the points that are reached if B is *rolled around inside* A.

  $\Rightarrow$ Remove small objects, keep original shape.

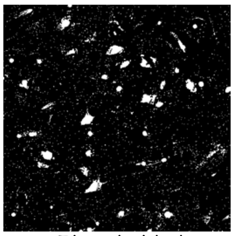


Effect of Opening

- Feature selection through *size* of structuring element



Original image



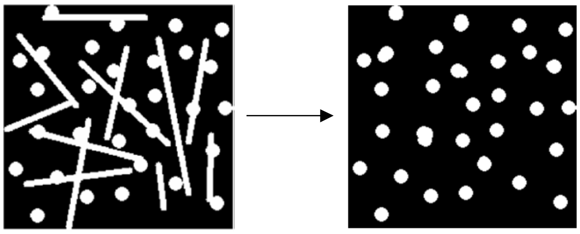
Thresholded

Opening with smaller SE

Opening with larger SE

Effect of Opening

- Feature selection through *shape* of structuring element (SE)

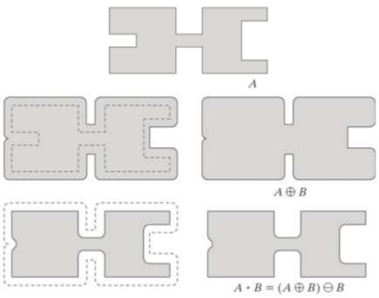


Opening after circular SE

Closing

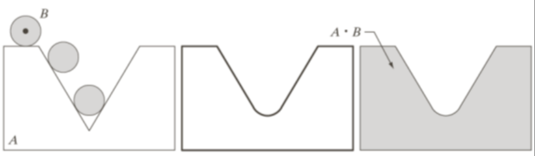
- <u>Definition</u>
  - Sequence of Dilation and Erosion

    $A \bullet B = (A \oplus B) \ominus B$



- <u>Effect</u>
  - $A \bullet B$ is defined by the points that are reached if B is *rolled around on the outside* of A.

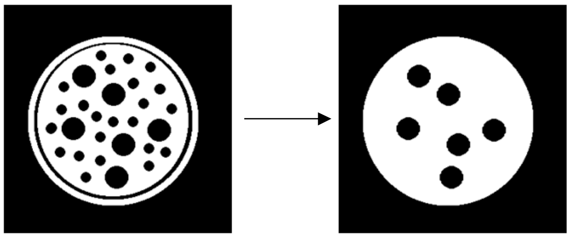
  ⇒ Fill holes, keep original shape.



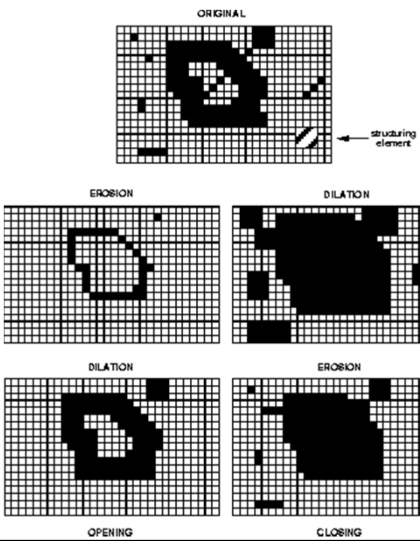
Effect of Closing

- Fill holes in thresholded image (*e.g.* due to specularities)

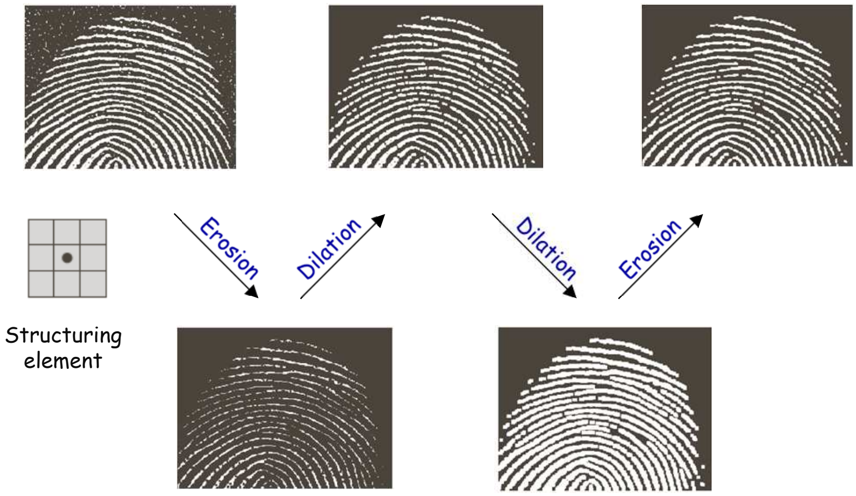
Size of structuring elements determines which structures are selectively filled



Binary Algorithms : Morphological Operations



Example Application: Opening + Closing



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Morphological Boundary Extraction

- Definition
  - First erode A by B, then subtract the result from the original A.

    $\beta(A) = A - (A \ominus B)$

- Effects
  - If a 3×3 structuring element is used, this results in a boundary that is exactly 1 pixel thick.

Morphology Operators on Grayscale Images

- Dilation and erosion typically performed on binary images.
- If image is grayscale:
  - for dilation take the neighborhood max,
  - for erosion take the min.