1. **Bounds for Probability of Error in Classification**
   a. For a feature, \( x \), state the Bayesian minimum error-rate classification rule for a 2-class problem (related probabilities are given as, \( P(x), p(x|w_i), p(w_i| x) \), and \( P(w_i) \), for \( i=1,2 \)).
   b. Assuming that the probability of classification error is defined as
      \[
      P(error) = \int_{-\infty}^{\infty} P(error \mid x) p(x) dx ,
      \]
      find and explain \( P(error \mid x) \) for a minimum error-rate classifier and clearly show that this classifier minimizes \( P(error) \).
   c. Given the following inequality, \( \min[a, b] \leq a^\beta b^{1-\beta} \ a, b \geq 0, \ 0 \leq \beta \leq 1 \), find an upper bound for \( P(error) \) (i.e. \( P(error) \leq \text{Bound} \)), where this upper bound value is a function of a priori class probabilities, as well as class conditional densities (Chernoff Bound).
   d. Assume that two classes have equal a priori probabilities and their class conditional densities are univariate Gaussian with mean values as 0 and 2, which also have unit variances. Find this upper bound in part (c) for a particular value of \( \beta=1/2 \) (Do not calculate this value to the end) (Bhacatarya Bound).

2. **Component Analysis for Feature Dimension Reduction:** Assume there are \( n \) \( d \)-dimensional feature vectors, \( \{x_1, \ldots, x_n\} \), such that these samples will be represented in lower dimensional spaces via some representations.
   a. Find a single vector, \( x_0 \), in terms of sample vectors, such that the sum of the squared-error between this representative and all these samples is minimized [Hint: Add and subtract the sample mean, \( m \), within the error expression].
   b. Now, assume there is a line passing through this representative vector, \( x_0 \), in part (a), whose direction is determined by an arbitrary unit vector, \( e \). Given that any vector, representing a point on this line, can be written as \( x = x_0 + \alpha e \), where scalar \( \alpha \) determines the distance of the point \( x \) from the \( x_0 \), find the set of values \( \{\alpha_1, \ldots, \alpha_n\} \), such that the sum-of-squared distance between the samples, \( \{x_1, \ldots, x_n\} \), and the vectors pointing to the line (represented by \( \alpha_k \)) are minimized.
   c. Finally, by using the results in (a) and (b), show that the optimum direction, \( e \), which minimizes the sum-of-squared distance between the samples and the representative vectors on the line can be obtained as the eigenvector, corresponding to the largest eigenvalue of the scatter matrix,
      \[
      S = \sum_{i=1}^{n} (x_i - m)(x_i - m)^T .
      \]

3. **Attributed Graph Matching:** For the following 3 scenes, which will be described by using the single relation “ontop” (Note that there are 4 geometric figures, as well as a Ground)
   a. Draw 3 attributed graphs, while indicating all assignments as well as compatibilities
   b. Draw 3 match graphs between the attributed graph pairs, while making comments on the similarity between scenes using match graphs (You should not try to complete all edges within your match graphs)

4. **Feedforward network:** Assume the 3-layer network below, whose weights are also denoted in the same figure. Input and output neuron relations are given for input, hidden and output layers of this network, respectively, as:
   \[
   \begin{align*}
   out_{input} &= in_{input} \\
   out_{hidden} &= \begin{cases} +1 & in_{hidden} > 1 \\
   1 & in_{hidden} > -1 \\
   -1 & in_{hidden} < -1 \end{cases} \\
   out_{output} &= \begin{cases} +1 & in_{output} > 0 \\
   1 & in_{output} < 0 \end{cases}
   \end{align*}
   \]
   For a two-class problem, (assuming \( z>0 \) for class-1), determine the decision regions for these two classes on the 2-D input feature \( (x_1; x_2) \) space.